Hedging pressure and speculation in commodity markets*

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Abstract

We propose a micro-founded equilibrium model to examine the interactions between the physical and the derivative markets of a commodity. This model provides a unifying framework for the hedging pressure and storage theories that comprises four types of traders: short and long hedgers, both of which operate on all markets; speculators who operate only on the futures market; and spot traders. We provide the necessary and sufficient conditions on the fundamentals of this economy for a rational expectation equilibrium to exist, and we show that it is unique. The model shows a variety of behaviors at equilibrium that can be used to analyze price relations for any commodity. The regimes are determined by basic characteristics of the commodity under consideration as well as by the present and expected supply conditions on the physical market. Further, through a comparative statics analysis, we precisely identify the losers and winners in the financialization of the commodity markets. Therefore, this paper clarifies the political economy of regulatory issues, like speculators' influence on prices.

JEL Codes: D4; G13; Q02.

Keywords: Equilibrium model; commodity; speculation; regulation; futures markets

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1 Introduction

In the field of commodity derivatives, some questions are as old as the markets themselves, and they remain open today. Speculation is a good example. In his famous article about speculation and economic activity, KALDOR (1939) wrote: "Does speculation exert a price-stabilizing influence, or the opposite? The most likely answer is that it is neither, or rather that it is both simultaneously." More than 70 years later, in June 2011, a G20 report (FAO et al. 2011) stated: "The debate on whether speculation stabilizes or destabilizes prices resumes with renewed interest and urgency during high price episodes. [...] More research is needed to clarify these questions and in so doing to assist regulators in their reflections about whether regulatory responses are needed and the nature and scale of those responses." In this paper, we develop a model of commodity trading that provides insights into this question. To overcome the discouraging conclusion that everything is possible, the model divides markets into four categories. We show that the consequence of financialization becomes predictable once the market's category is acknowledged.

This division is based on two markers: the sign of the basis (the difference between the futures price and the current spot price) and the sign of the risk premium (the difference between the expected spot price and the futures price). Commodity markets have the following major types of actors: hedgers of future sales (producers and storers), hedgers of future purchases (processors and final consumers), speculators, and spot traders. Most formal and informal analyses ignore long hedgers. Although their positions are often small compared to short positions, their role can occasionally be strong. Overall, the relative weights of the actors define the category in which a market belongs. Our focus is on the basic diversity of the objectives of the players in the field.

The model embeds the storage theory and the hedging pressure theory. The storage theory focuses on the basis. The hedging pressure theory (DE ROON et al. 2000), an extension of the normal backwardation theory proposed by KEYNES (1930), analyzes the risk premium (or the expected basis). The literature usually focuses on the storage theory or the normal backwardation theory but not both simultaneously. The choice of one of the two theories depends on whether the focus is on physical markets and the costs of storage or on futures and the reallocation of risk between agents. Although these theories are complementary, the two strands have usually remained apart, with a few exceptions focusing mainly on short hedgers.

The model is built to represent a wide variety of commodities. There is a spot market at times $t = 1$ and $t = 2$, and a futures market in which contracts are initiated at $t = 1$ and settled at $t = 2$. The spot market is physical, which implies a nonnegativity constraint on inventories, while the futures market is financial and allows shorting. Beside the mean-variance maximizers (the hedgers of buy and sell positions and the speculators), the motivation of the spot traders is their immediate needs, and they are sensitive to the current price. The uncertainty in the market comes from not knowing the amount of the commodity produced and the demand of the spot traders. The solution is a rational expectation equilibrium with incomplete markets. The equilibrium's uniqueness stems from the fact that the four market categories do not overlap: a given distribution of the shocks leads to one and only one category. Negative prices are mathematically possible because in certain markets, like electricity, the commodity is not storable and cannot be thrown away. In contrast, the free-disposal condition prevents this case in agricultural products. To guarantee nonnegative prices, we use a validity condition for the distribution of the shocks.

Because the model is tractable, it allows for new types of comparative statics. We focus the analysis on financialization that we define as an increase in the speculative activity. Financialization is a metaphor for diverse exogenous events. It could refer to more speculators in search of profit opportunities or diversification or to an increase in the apparent risk tolerance of speculators due to a smaller correlation between the commodity and other (unmodeled) investment vehicles. Another scenario is the relaxation of limit positions on the futures markets.

The comparative statics is on the major economic indicators: risk premiums, prices (in particular levels and variances), and quantities. The financialization has predictable effects that are based on the four market categories, which in turn depend on expectations and the balance between the types of agents in the economy. The typical reasoning is that speculators facilitate hedging, and therefore increased speculation generates more storage. In a period when storers buy and store, these actions stabilize prices (volatility diminishes); but when the storers release their inventories, these bigger quantities destabilize prices. Finally, we compare the cases with and without futures markets. The analysis clarifies the distinction between the debates on financialization and those on the desirability of futures markets. We show that futures markets per se help the economy find an equilibrium.

Beyond the descriptive predictions, we develop a political-economic approach to the financialization of commodity futures markets. We use our model to perform a welfare analysis and to draw regulatory implications. This question, again, is as old as the derivatives markets. NEWBERY (2008) aptly summarizes the usual yet ambivalent appreciation of the impact of derivative markets on welfare. The author distinguishes between what he calls the "layman" and "the body of informed opinion." He explains that to the first, "the association of speculative activity with volatile markets is often taken as proof that speculators are the cause of the instability"; whereas to the second, "volatility creates a demand for hedging or insurance." Our results make a case for prudence in the face of the "evidence" of malfunctioning futures markets. Because volatilities and their evolution are the object of much discussion, our model is useful when it comes to interpreting these changes normatively.

Our analysis demands a basic distinction: regulation affects the profits from speculation and the benefits from hedging differently. As far as speculation only is concerned, all incumbent agents prefer that the number of speculators does not increase. We find reasons why hedgers have opposite views as to whether speculation should be encouraged or restricted. The economic determinant of political positions appears to be the weight the agents have in the market under consideration and not their specific economic role as buyer or seller. For example, an industrialist can both support an increase in the speculative activity in a market even when hedging is costly to him or her and be against speculation in another market where hedging is profitable. To the best of our knowledge, this paper is the first to reveal this political-economic tension.

The remainder of the paper is as follows. Section 2 is the literature review. Section 3 presents the model, the definition of the equilibrium and shows existence and uniqueness of the equilibrium. Section 4 analyzes the economic mechanisms behind the possible equilibrium regimes. In Section 5, we disclose the comparative statistics with respect to financialization and the political-economic propositions. Most of the proofs are in the Appendix. The Appendix also offers complementary results on the no-futures scenario.

2 Literature review

In this article, we focus on four fundamental economic actors in commodity and futures markets: hedgers of future sales, hedgers of future purchases, speculators, and spot traders. Their relative weights are the basis of a vast variety of cases we observe in reality and the comparative statics we propose.

The literature on commodity prices usually separates the following two questions. The first one is the link between the current spot and futures prices. This link is usually associated with the storage theory initiated by KALDOR (1940), WORKING (1949), and BRENNAN (1958). The second question is the link between the expected future spot price and the futures price (the risk premium). KEYNES (1930) was the first to investigate this question through the theory of normal backwardation, nowadays more frequently referred to as the hedging pressure theory (DE ROON et al. 2000). Most of the time, the equilibrium models acknowledge the same separation.

Our model shares an interest in spot prices and the role of nonnegative inventories with the equilibrium models based on the storage theory. However, our emphasis on the presence of diverse categories of hedgers creates a distinction with DEATON and LAROQUE (1992) and CHAMBERS and BAILEY (1996), because these two models have a representative agent. Besides, their setting uses complete markets whereas ours uses incomplete ones. Focusing on risk aversion creates an additional distinction with ROUTLEDGE et al. (2000), who work with an implicit intertemporal risk-neutral agent. Finally, the quest for a tractable two-period model explains that the convenience yield does not enter in our analysis, though an ad hoc specification could be added.

Like the equilibrium models based on the hedging pressure theory, we focus on the risk premium (or bias in the futures price) and the risk transfer function of the derivative market. From that viewpoint, our model is related to ANDERSON and DANTHINE (1983), HIRSHLEIFER (1988), HIRSHLEIFER (1989), GUESNERIE and ROCHET (1993), BOONS et al. (2014), and BASAK and PAVLOVA (2016). The studies by ANDERSON and DANTHINE (1983) and NEWBERY (1987) are important sources of modeling ideas for our paper.

A few recent equilibrium models have attempted to gather the two theories of commodity prices. We are close to this strand of the literature. Compared to what we do, GORTON et al. (2013) put emphasis on the role of the stock-outs for the basis and the risk premium. ACHARYA et al. (2013) focus on the relation between the risk premium and the risk tolerance of the hedgers. However, in all of these models, with the exception of BAKER (2016) who investigates the effect of financialization in a simulation-based model, only one category of hedgers is considered: producers, with short hedging positions.

We do not consider the literature on informational asymmetries as in VIVES (2008), where different behaviors come from different information, not different goals. Such a question is investigated in the context of commodity markets by SOCKIN and XIONG (2015), LECLERCQ and PRAZ (2014), and GOLDSTEIN and YANG (2016) among others.

We use our model to examine the destabilizing effect of speculation and to analyze welfare issues. This is also the case in HART and KREPS (1986), NEWBERY (1987), GREGOIR and SALANIÉ (1991), GUESNERIE and ROCHET (1993) and, more recently, in BAKER and ROUTLEDGE (2016). Our contribution to this literature is that we have precise predictions on the losers and winners among hedgers of the financialization in commodity markets. Indeed, financialization is not a pure Pareto improvement of the risk-sharing technologies but rather a redistribution of welfare.

These political economic considerations are complementary to the recent empirical literature on financialization that focuses on the contagion between financial and commodity markets. Compared with what we do, BOONS et al. (2014) and HAMILTON and WU (2014) analyze the portfolio effects, and BRUNETTI and REIFFEN (2014), HAMILTON and WU (2015) and BRUNETTI et al. (2015), among others, focus on index traders.

The empirical tests performed on the basis of equilibrium models are rare. The tests undertaken by ACHARYA et al. (2013) could be used as a fruitful source of inspiration for further developments. For the analysis of the risk premium, the empirical tests performed by HAMILTON and WU (2014) and SZYMANOWSKA et al. (2014) as well as the calibrations and simulations proposed by BESSEMBINDER and LEMMON (2002) and BAKER (2016), are other possible directions.

3 The model

3.1 The framework

We simplify time by taking two periods where actions take place; period 0 is used for ex ante evaluations. There is one commodity and a numéraire. Two markets exist: a spot market for the commodity, where a random exogenous production $\tilde{\omega}_t$ arrives at times $t = 1$ and $t = 2$, and a futures market in which contracts are initiated at $t = 1$ and settled at $t = 2$. The futures market allows for both short and long positions. However, the spot market only allows long positions: inventories cannot be negative.

There are four types of traders. The first three make intertemporal decisions, that is, they decide at period $t = 1$ conditionally on their expectations for period $t = 2$. They are industrial operators with a selling or a buying position to hedge, and speculators. The fourth type (spot traders) is interested in the current spot price only.

- Hedgers of future sales.

  We call these hedgers *storers* or *inventory holders (I)*. They have storage capacity that they can use to buy the commodity on the spot market at $t = 1$ and release it at $t = 2$. Thus, they have a long position on the physical market that they can hedge on the futures market.

We can also describe these storers as agricultural producers. They plant seeds in period 1 and commit to selling their crops in period 2. Therefore, they might want to hedge these future sales.

- Hedgers of future purchases.

  We call these hedgers *processors (P)*, or industrial users, because they use the commodity to produce other goods that they sell to final consumers. They cannot store and their production process is rigid: they commit at $t = 1$ to buy their input on the spot market at $t = 2$. Thus, they have an implicit commitment to buy on the spot market that they can hedge on the futures market. For the sake of simplicity we assume that the production process is instantaneous so that they deliver their final product at $t = 2$.

  Final consumers could also hedge their future consumption plan, so that the so-called processors also comprise final consumers.

- *Speculators (S)* use the commodity price as a source of risk and make a profit through their positions in futures contracts. They do not trade on the spot market. Nor do they handle other financial assets.

The futures and spot markets operate in a partial equilibrium framework: in the background, there are other price-sensitive sellers and processors of the commodity. We call these additional agents *spot traders*, and a linear net demand function that is exogenous and random describes their global effect at each period. The demand can be either positive or negative.

3.2 The prices

We study the relations between prices in the spot and the futures markets through a rational expectation equilibrium (REE) analysis. We model uncertainty as the probability space $(\Omega, \mathcal{A}, \mathbb{P})$.

The realized prices are denoted by F, P_1 and P_2. The prices are only functions of known shocks and expectations about the future. These functions are denoted by F_1 for the futures price in period 1, and p_t for the spot prices at times $t = 1$ and $t = 2$. Given the shocks $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ realized in periods 1 and 2, respectively and that $(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2)$ follows a known distribution, we have:

$$F := F_1(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2], \mathrm{Var}_1[\tilde{\varepsilon}_2]), \tag{1}$$

$$P_1 := p_1(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2], \mathrm{Var}_1[\tilde{\varepsilon}_2]), \tag{2}$$

$$P_2 := p_2(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \mathbb{E}_1[\tilde{\varepsilon}_2], \mathrm{Var}_1[\tilde{\varepsilon}_2]). \tag{3}$$

These structural relations provide statistical predictions, like the prices' expectations and variances, the risk premia ($\mathbb{E}_0[P_2 - F]$ and $\mathbb{E}_1[P_2] - F$), and the conditional expectations (e.g., $\mathbb{E}_1[P_2|P_1]$).

Using a microeconomic model with rational expectations, we find F, P_1 and P_2 as solutions of the nonlinear system of equations:

$$\begin{cases} mP_1 - n_I \max\left[F - P_1, 0\right] = \tilde{\varepsilon}_1, & (4) \\ mF + \gamma\left(n_I \max\left[F - P_1, 0\right] - n_P \max\left[Z - F, 0\right]\right) = \mathbb{E}_1[\tilde{\varepsilon}_2]\,. & (5) \end{cases}$$

where the real parameters m, γ, Z, and where P_1 and F are the fundamental unknowns from which we derive the other endogenous variables: P_2, inventories X, and commitments Y.

Besides the statistical properties and their relations with fundamental variables and with the distribution of shocks (descriptive comparative statics), we use the microeconomic foundation to evaluate the structural or regulatory changes in the agents' ex ante welfare (normative comparative statics), which has an important application to the financialization of the commodity markets.

3.3 The optimal positions

There are n_P processors, n_S speculators, and n_I storers. All of these intertemporal traders make their decisions at time $t = 1$ conditionally on the information available at $t = 2$. They are risk averse and intertemporal utility maximizers. We retain a mean-variance framework. We set the risk-free interest rate at 0. Appendix A contains the details on profit maximization and optimal positions.

On the physical market. The aggregate inventories X depend on the difference between the futures and the spot prices, and on the number of storers n_I:

$$X = n_I \max\left[F - P_1, 0\right]. \tag{6}$$

The storers hold inventories if the futures price is higher than the current spot price. This position is the only one in the model that directly links the spot and futures prices. This is consistent with the theory of storage and, more precisely, its analysis of contango and the informational role of futures prices.

The aggregate committed demand Y depends on the profits of the processors who hedge themselves via futures at price F and some reference value Z interpretable as the marginal profit on the processing operations:

$$Y = n_P \max\left[Z - F, 0\right], \tag{7}$$

where n_P is the number of processors.

On the futures market. When an agent i sells (buys) futures contracts, his or her position is short (long), and the number of futures contracts he or she holds is negative (positive). We assume that there is a perfect convergence between the basis at the expiration of the futures contract, thus at time $t = 2$ the positions are settled at the spot price P_2.

All agents can undertake speculative operations in the futures market. The speculative positions depend negatively on variance and positively on the risk premium and on the collective risk tolerance T_i of each category. For example, f_S^{Agg}, the aggregate speculative position on futures of the n_S speculators is:

$$f_S^{\text{Agg}} = T_S \frac{\mathbb{E}_1[P_2] - F}{\text{Var}_1[P_2]}, \tag{8}$$

with $T_S = \frac{n_S}{\alpha_S}$, α_S being the risk aversion of the speculators.

The hedging positions are directly linked to the physical quantities. Storers take short positions that match their physical inventory, and processors take long positions that match their commitment to buy in period 2. Having hedged 100 percent of their physical positions, hedgers add a speculative component to their position on the futures market, according to their risk aversions and expectations (as described in equation 8). The presence of a speculative component in all optimal positions is consistent with the volatile positions that CHENG and XIONG (2014) and KANG et al. (2014) have recently found in a large number of commodity markets.

$$f_I^{\text{Agg}} = -X + T_I \frac{\mathbb{E}_1[P_2] - F}{\text{Var}_1[P_2]} \qquad \text{with } T_I = \frac{n_I}{\alpha_I}; \tag{9}$$

$$f_P^{\text{Agg}} = Y + T_P \frac{\mathbb{E}_1[P_2] - F}{\text{Var}_1[P_2]} \qquad \text{with } T_P = \frac{n_P}{\alpha_P}. \tag{10}$$

DANTHINE (1978) motivates this separation of the physical and the futures decisions. In the absence of a futures market (a scenario analyzed in the Appendix D), this separation property disappears: the quantities held on the physical market necessarily have a speculative dimension. Further, as ANDERSON and DANTHINE (1983) show, this property does not hold if the final good's price (or Z here) is endogenous and stochastic unless a second futures market for the final good, or a forward market, exists.

3.4 Clearing conditions

The market clearing conditions on the spot market have the supply on the left-hand side, and the demand on the right-hand side:

$$\begin{cases} \tilde{\omega}_1 = X + \tilde{\mu}_1 - mP_1, \\ \tilde{\omega}_2 + X = Y + \tilde{\mu}_2 - mP_2. \end{cases}$$

where $\tilde{\omega}_t$ is the random production, and m is the price sensitivity of the linear net demand of the spot traders $\tilde{\mu}_t - mP_t$; the intercept $\tilde{\mu}_t$ takes into account the random nature of this net demand. In period 1, there is the demand for storage X to be released (supply) in period 2. In period 2, on the demand side, the commitments Y of the processors must be served.

Because both $\tilde{\mu}_t$ and $\tilde{\omega}_t$ are exogenous processes, we retain the notation: $\tilde{\varepsilon}_t = \tilde{\mu}_t - \tilde{\omega}_t$. This yields:

$$\begin{cases} mP_1 - X = \tilde{\varepsilon}_1, & (11) \\ mP_2 + X - Y = \tilde{\varepsilon}_2. & (12) \end{cases}$$

The clearing of the futures market adds the following equation to the analysis :

$$(T_S + T_P + T_I)\,\frac{\mathbb{E}_1[P_2] - F}{\mathrm{Var}_1[P_2]} = X - Y.$$

Because $\mathrm{Var}_1[P_2]$ is given by the spot market clearing equation (12) and by rearranging, we obtain:

$$\mathbb{E}_1[P_2] - F = \beta\,\mathrm{HP}, \qquad \text{where} \quad \beta := \frac{1}{m^2}\frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{T_S + T_P + T_I} > 0. \tag{13}$$

β, a known real, is the sensitivity of the risk premium to the hedging pressure.

The net hedging demand is called the hedging pressure $\mathrm{HP} := X - Y$. This definition relies on an absolute rather than a relative value, as done by De Roon et al. (2000) or Acharya et al. (2013), among others.[1]

The sign of the bias is exactly the sign of the hedging pressure. Because the risk aversion of the operators only influences the speculative part of the futures position, it does not affect the sign of the bias. Furthermore, when $\mathrm{HP} = 0$, there is no bias in the futures price; and the risk transfer function of the market is entirely undertaken by the hedgers because their positions on the futures market are opposite and match exactly. Thus, an absence of bias has plausible explanations other than the unlikely global risk neutrality. In terms of magnitude, equation (13) shows that the bias in the futures price depends on the fundamental economic structures (storage and production costs embedded in the hedging pressure), on the subjective parameters (agents' risk aversion), and on the uncertainty in the economy.

3.5 The rational expectation equilibrium (REE)

We replace the physical quantities X and Y with their values as given by (6) and (7) in the market clearing equations (11) (spot market at $t = 1$) and (13) (futures). Because $\mathbb{E}_1[\tilde{\varepsilon}_2] = m^2\,\mathbb{E}_1[P_2]$ in equation (12), our rational expectation equilibrium in period 1 is reduced to a system of two equations for P_1 and F:

$$\begin{cases} mP_1 - n_I \max[F - P_1, 0] = \tilde{\varepsilon}_1, & (14) \\ mF + \gamma\,(n_I \max[F - P_1, 0] - n_P \max[Z - F, 0]) = \mathbb{E}_1[\tilde{\varepsilon}_2]\,. & (15) \end{cases}$$

[1] These authors divide the net hedging positions by the transactions volume.

where m, n_I, n_P, and γ are known numbers. More precisely:

$$\gamma := 1 + m\beta = 1 + \frac{1}{m} \frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{T_S + T_P + T_I} > 0 \tag{16}$$

This system presents three characteristics. First, the two equations have a microeconomic basis. Second, due to nonnegativity constraints on the inventories and committed inputs, the system is nonlinear. Third, the two prices P_1 and F depend on the current and expected scarcity shocks on the physical market, $\tilde{\varepsilon}_1$ and $\mathbb{E}_1[\tilde{\varepsilon}_2]$. Once the functions p_1 and F_1 are calculated, all the other unknowns (P_2, X, Y in particular) can be derived.

Further, the parameter γ encodes a lot of information about the structure of the market, which is useful for static comparatives. We have $1 \leq \gamma \leq +\infty$. If some agents are risk-neutral (for instance, the processors: $T_P = +\infty$), then $\gamma = 1$. If all of the agents are pure hedgers, so that $T_K = 0$ for all K, then $\gamma = +\infty$.

Definition 1 (Solution)**.** *A* solution *in period 1 is a family of quantities and prices*

$$(X, Y, F, P_1, P_2) \in \mathbb{R}^2_+ \times \mathbb{R}^2 \times L^0(\Omega, \mathcal{A}, \mathbb{P})$$

(P_2 is a random variable at period 1) such that processors, storers, and speculators act as price-takers; all markets clear; and the quantities are nonnegative: $X \geq 0$ and $Y \geq 0$.

This solution allows for negative prices. Negative prices may occur in certain markets, like electricity, because this commodity is non-storable and cannot be thrown away, unlike agricultural products. In the following definition of the equilibrium, we however require nonnegative prices, because these correspond to the vast majority of the situations encountered in commodity markets. Besides, this approach in two steps facilitates the exposition of the results.

Definition 2 (Equilibrium)**.** *An* equilibrium *in period 1 is a solution such that prices are nonnegative:*

$$F \geq 0, \quad P_1 \geq 0, \quad \text{and} \quad P_2 \geq 0 \quad \text{almost surely.}$$

To find the solution, we solve the system (14)–(15). Consider the mapping $\varphi : \mathbb{R}^2_+ \to \mathbb{R}^2$ defined by:

$$\varphi(P_1, F) := \begin{pmatrix} mP_1 - \max[F - P_1, 0] \\ mF + \gamma(\max[F - P_1, 0] - \max[Z - F, 0]) \end{pmatrix}.$$

The system can be rewritten as:

$$\varphi(P_1, F) = \begin{pmatrix} \tilde{\varepsilon}_1 \\ \mathbb{E}_1[\tilde{\varepsilon}_2] \end{pmatrix}$$

and it has a unique solution if and only if the right-hand side belongs in the image of φ.

The function φ is divided into four regions in the space of the prices (P_1, F), as described by Figure 1. P_1 is the horizontal coordinate, and F is the vertical one. We denote the origin in $\mathbb{R}^2_+$ as O, the point $(0, Z)$ as A, and the point (Z, Z) as M. The four regions are separated by the straight lines $F = P_1$ and $F = Z$. We label them as follows:

- Region 1, where $F > P_1$ and $F < Z$, and both X and Y are positive.
- Region 2, where $F > P_1$ and $F > Z$, and $X > 0$ and $Y = 0$.
- Region 3, where $F < P_1$ and $F > Z$, and $X = 0$ and $Y = 0$.
- Region 4, where $F < P_1$ and $F < Z$, and $X = 0$ and $Y > 0$.

In the regions where $X > 0$, we have $X = F - P_1$; and in the regions where $Y > 0$, we have $Y = Z - F$. Therefore, in each region, the mapping is linear and continuous across the boundaries.

The uniqueness of the solution is proved by showing that the four regimes do not overlap. The precise borders of each region are also linear and φ is continuous across boundaries (Appendix B.1). In each region, the prices and quantities can be explicitly computed (Appendix B.2). Figure 4 represents the region of the solution for any given couple of shocks $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$ on the physical market.

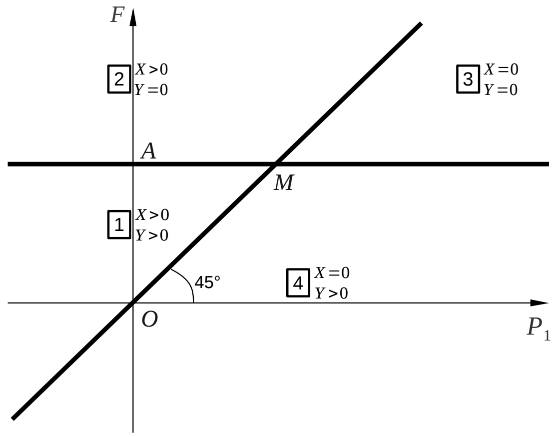
Proposition 1. *A solution exists and is unique.*

The nonnegativity conditions on the deterministic and random prices yield the validity condition in the following proposition.

Proposition 2. *The solution is an equilibrium if and only if $\mathbb{E}_1[\tilde{\varepsilon}_2] \geq B(\tilde{\varepsilon}_1)$, where B is a piecewise-linear continuous decreasing function. The exact expression of B is in Appendix B.3.*

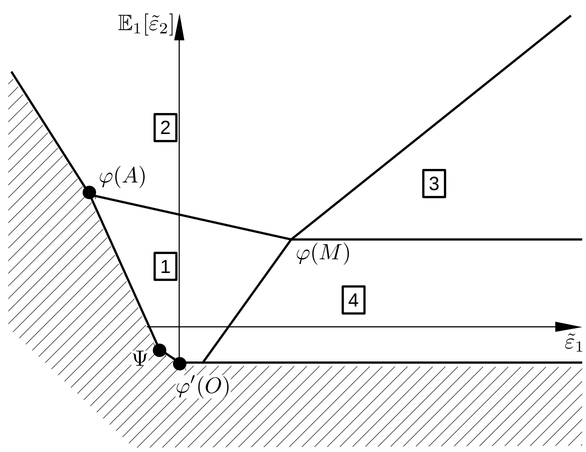
Definition 3. *The condition in Proposition 2 is hereinafter referred to as the* validity condition*.*

Figure 2 represents the equilibrium region for any given couple of shocks on the physical market $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$. Regions numbers are used indifferently in the various representations of the equilibrium. The hatched regions delimit the validity condition of the equilibrium. They correspond to negative values for F or P_1, or P_2. Thus, the states of extreme abundance (low values of $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$) are inconsistent with positive prices. For a low $\tilde{\varepsilon}_1$, the constraint on P_1 matters more, but for a low $\tilde{\varepsilon}_2$ the constraint on P_2 becomes more restrictive.



This figure represents all four possible cases on the physical market. The comparison between the futures price F and the spot price at time 1, P_1, determines the aggregate inventories X. The comparison between the futures price F and the profit on processing activities Z determines the commitments Y. The regions are labeled 1 to 4.

Figure 1: The four regions in the space of the prices (P_1, F)



This figure represents the image of φ. We use Regions 1 to 4 from Figure 1. $\varphi(A)$, $\varphi(O)$, and $\varphi(M)$ are the images of A, O, and M. The supply shock $\tilde{\varepsilon}_1$ represents the scarcity observed at time 1, and $\mathbb{E}_1[\tilde{\varepsilon}_2]$ is the scarcity expected at time 2. Knowing $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$, the relevant region of φ can be read on the diagram, and thus the type of equilibrium. The non-hatched area corresponds to the validity condition (nonnegative P_1, F and P_2).

Figure 2: Equilibrium. The four regions in the space of the supply shocks $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$.

4 Equilibrium analysis

The model provides all of the possible price relations compatible with the theories of storage and hedging pressure in the presence of both short and long hedgers. Meanwhile, it shows how the risk management and price discovery functions of the futures markets interact.

4.1 Compatibility with the two traditional theories of commodity prices

In Regions 1 and 2, the futures market is in contango: $F > P_1$. Inventories are positive, and they are used for intertemporal arbitrages. In Regions 3 and 4, there is no inventory, and the market is in backwardation: $F < P_1$. These configurations are fully consistent with the theory of storage.

The hedging pressure HP explains the sign and magnitude of the risk premium $\mathbb{E}_1[P_2] - F$, that is, the price paid or received by the hedgers to transfer their risk via the futures market (see equation 13). The analysis of the four regions discloses the reasons for the classical conjecture that there is a positive risk premium and the reasons why the reverse is also plausible, as mentioned by several empirical studies.[2] Region 1 illustrates this point: it is the only one where all operators are active, and it has two important subcases, as depicted by Figure 3:

- Δ: there is no risk premium ($\text{HP} = 0$), and the risk is exchanged free of charge between storers and processors who have perfectly matching positions: they hedge each other.
- Subregion 1U (and Region 2): above Δ, $\text{HP} > 0$ and $F < \mathbb{E}_1[P_2]$. The net hedging position is short, and speculators in a long position are indispensable to the clearing of the futures market. In order to induce their participation, there must be a profitable bias between the futures price and the expected spot price.
- Subregion 1L (and Region 4): below Δ, $\text{HP} < 0$, and $F > \mathbb{E}_1[P_2]$. The net hedging position is long and the speculators must be short, which requires that the expected spot price is lower than the futures price and the risk premium is negative.

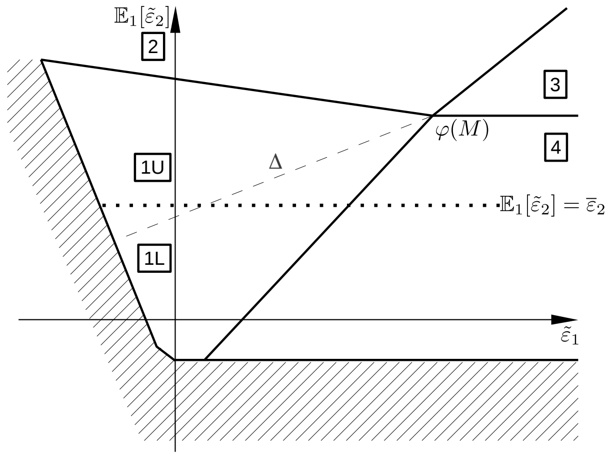
The model is thus consistent with a positive, negative, or null risk premium, as required by the hedging pressure theory.

4.2 A unified framework for prices analysis

The model offers a wide variety of possible situations. For example, the prices in Subregion 1U are simultaneously in contango on the current basis (storage theory) and in backwardation on the expected basis (hedging pressure theory).[3] Thus, $P_1 < F < \mathbb{E}_1[P_2]$. Yet a contango can also coincide with a negative premium, as in Subregion 1L.

[2] For extensive analyses of the risk premium in a large number of commodity markets, see e.g., FAMA and FRENCH (1987), KAT and OOMEN (2007), GORTON et al. (2013), and BHARDWAJ et al. (2015).

[3] KEYNES (1930) himself considers such situations, that is, a contango that coincides with a positive risk premium; however, he assumes that this should be the case for exceptional circumstances only.



This figure is a detailed view of Region 1 as displayed in Figure 2. This region is separated into two subregions, 1U and 1L, by the line Δ. The positions of the operators are organized according to the scarcity, current ($\tilde{\varepsilon}_1$), and expected ($\mathbb{E}_1[\tilde{\varepsilon}_2]$). In Subregion 1U, the inventories X are higher that the physical commitments Y. The reverse is true in Subregion 1L. The horizontal dotted line represents an arbitrary level, $\bar{\varepsilon}_2$, for the scarcity expected at time 2. Along this line, the equilibrium changes when the current shock $\tilde{\varepsilon}_1$ changes.

Figure 3: Physical and financial decisions in space $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$, with details on Region 1.

Figure 3 explains such a result. In this figure, we fix the expected scarcity $\mathbb{E}_1[\tilde{\varepsilon}_2]$ at some value $\bar{\varepsilon}_2$ (moderate scarcity). The horizontal dotted line shows how the current supply situation, $\tilde{\varepsilon}_1$, changes. If period 1 experiences abundance (Subregion 1U), then there is massive storage: the current price is low and the expected profits are attractive; storers need more hedging than processors; thus there is a positive bias in the futures price, and speculators have long positions. For a less marked abundance (Subregion 1L), storage is more limited; the storers' hedging needs diminish while those of the processors increase; the net hedging position becomes long, the bias in the futures price becomes negative, and the speculators have short positions.

Table 1 summarizes all of the possible combinations between prices in equilibrium, for all regions. They explain where a commodity is likely to be found on average, due to its fundamental economic structures. For example, a commodity characterized by high storage costs or a quick depreciation of the inventories should be situated in Region 4, where $X = 0$, prices are in backwardation, and the risk premium is negative. In contrast, for the majority of precious metals, inventories are especially abundant: they are value reserves not necessarily to be used. These commodities are found in Region 2, characterized by a high level of stocks ($X > 0$ and $Y = 0$), prices in contango, and a positive risk premium. Region 3, where $X = Y = 0$, is for commodities whose production is almost immediately consumed by the spot traders. Because Region 1 is the richest, it should gather the majority of commodities.

Naturally, a market does not have to be in the same region all of the time because shocks (current or via expectations) move it from its average position. Markets normally experience changing supply and demand conditions, for example, from spot traders. Moreover, risk aversions also depend on changing factors that are exogenous to the commodity market.

Region	Prices		Positions		
(1) Tag	(2) Basis $F - P_1$	(3) Risk premium $\mathbb{E}_1[P_2] - F$	(4) Storage X	(5) Commitments Y	(6) Speculation
2	> 0	> 0	> 0	$= 0$	Long
1U	> 0	> 0	> 0	> 0	Long
Δ	> 0	$= 0$	> 0	> 0	Inactive
1L	> 0	< 0	> 0	> 0	Short
4	< 0	< 0	$= 0$	> 0	Short
3	< 0	$= 0$	$= 0$	$= 0$	Inactive

This table summarizes all of the possible price combinations obtained with the model. The results are region by region; regions are listed counter-clockwise after the figures (column 1). Columns 2 and 3 refer to the prices' relations to the theory of storage (column 2) and to the hedging pressure theory (column 3). Columns 4-6 display how the prices' relations are linked with positions on inventories (column 4), on physical commitments (column 5), and with speculation in the futures market (column 6).

Table 1: The four types of equilibrium: a theoretical framework for prices analysis.

This unification of the theories of storage and hedging pressure is important for understanding the impact of financialization, because it provides the channel through which financial shocks propagate.

5 The impact of financialization

Increased speculation, henceforth financialization, comes from easier access to the futures markets (extensive effect) or from a sudden rise in the appetite for risk (intensive effect). These events are modeled as an increase in the risk tolerance in the market due either to an increase in the number of speculators n_S, or to a decrease in the risk aversion α_i of any category of participants.

5.1 Financialization's impact on prices and quantities

In the equilibrium analysis, the agents know $\tilde{\varepsilon}_1$ when they take their positions. Thus all endogenous variables are ultimately functions of the two numbers revealed at period 1: $\tilde{\varepsilon}_1$ and $\mathbb{E}_1[\tilde{\varepsilon}_2]$. We can then focus on the *unconditional* variance $\mathrm{Var}_0[\cdot]$ of the prices, assuming for simplicity that the two shocks $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are independent.

Proposition 3. *The effects of financialization on risk premiums, price levels, and variances are as in Table 2. The dominant side in hedging, that is, the one that determines the sign of the hedging pressure, is the key factor. The dominant side are inventory holders in Subregions 1U and Region 2, processors in Subregion 1L and Region 4.*

The computations are in Appendix C.1. Table 2 shows that:

Region	Risk Premium	Positions		Prices			Variances		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Tag	$\lvert \mathbb{E}_1[P_2] - F \rvert$	X	Y	F	P_1	P_2	$\text{Var}_0[F]$	$\text{Var}_0[P_1]$	$\text{Var}_0[P_2]$
2	↘	↗	0	↗	↗	↘	↘	↘	↗
1U	↘	↗	↘	↗	↗	↘	↘	↘	↗
1L	↘	↘	↗	↘	↘	↗	↘	↘	↗
4	↘	0	↗	↘	⟷	↗	⟷	⟷	⟷
3	0	0	0	⟷	⟷	⟷	⟷	⟷	⟷

This table describes the impact of financialization, region by region (column 1). Column 2 represents the absolute value of the risk premium; columns 3-4 focus on the physical positions (inventories X and commitments of the processors Y). Columns 5-7 focus on the prices: the futures price F, and the two spot prices P_1 and P_2. Columns 8-10 are devoted to the variances of these prices.
Legend: ↗: variable increases, ↘: decreases; ⟷: no impact; 0: variable is null.

Table 2: The impact of the financialization on the risk premium, the quantities, and the prices.

- The increased speculative activity decreases the risk premium, in absolute value : the hedging costs of all hedgers are reduced. In this sense, speculation facilitates hedging, as in NEWBERY (1987), and, more recently, in ACHARYA et al. (2013).
- The dominant side in hedging increases their physical positions, while the dominated side decrease theirs. The change in the futures price depends on the risk premium's sign that is generated by the hedging pressure HP. This change in F impacts the physical decisions.
- Inventories are a transmission channel for shocks in "space" (between the financial and the physical markets) and in time (between periods 1 and 2).
- In the period where storers buy and store, financialization stabilizes spot prices. When they release their inventories, a destabilization effect appears.

The financialization clearly influences the whole functioning of the commodity markets. This influence depends on the market. In each region, the impact depends on the relative importance of short and long hedgers. Moreover, the financialization process amplifies the difference in the relative hedging positions and consequently the market impact of each category of hedger.

Derivative markets might also "destabilize" the underlying markets, although the term is inappropriate because it sounds judgmental while it only refers to a statistical property. Further, higher price volatility does not mean lower welfare; quite the contrary, more volatility means that prices are more informative signals. The impact of financialization on price volatilities is often a naïve (and wrong) substitute for an authentic welfare analysis.

5.2 Welfare analysis

A welfare analysis on the impact of financialization is especially interesting with a model where industrial operators are not only hedgers, but also, somehow, speculators. Given that the model is solved, we express the indirect individual utilities, in equilibrium, of the speculators, the storers, and the processors. Then, we analyze the welfare impact of financialization.

Indirect utilities as functions of the equilibrium prices P_1 and F. We only examine the case of Region 1 where all agents are active. Therefore, we have $F < Z$ and $P_1 < F$. For the sake of simplicity, we again assume that $\tilde{\varepsilon}_1$ is known by the agents.

The speculators' indirect utility is given by

$$U_S = f_S(\mathbb{E}_1[P_2] - F) - \frac{1}{2}\alpha_S f_S^2 \operatorname{Var}_1[P_2]\,,$$

where we substitute the values of f_S and $\mathbb{E}_1[P_2]$ that leads to:

$$U_S = \frac{(\mathbb{E}_1[P_2] - F)^2}{2\alpha_S \frac{\operatorname{Var}_1[\tilde{\varepsilon}_2]}{m^2}}. \tag{17}$$

The storers' indirect utility is given by:

$$U_I = (x + f_I)\ \mathbb{E}_1[P_2] - x\,P_1 - f_I\,F - \frac{1}{2}x^2 - \frac{1}{2}\alpha_I(x + f_I)^2 \operatorname{Var}_0[P_2]\,,$$

where we substitute the optimal values of the individual position f_I, of the individual inventory x, and of $\operatorname{Var}_0[P_2]$:

$$U_I = \frac{(\mathbb{E}_1[P_2] - F)^2}{2\alpha_I \frac{\operatorname{Var}_1[\tilde{\varepsilon}_2]}{m^2}} + \frac{1}{2}(F - P_1)^2. \tag{18}$$

Similarly, the processors' indirect utility is:

$$U_P = \frac{(\mathbb{E}_1[P_2] - F)^2}{2\alpha_P \frac{\operatorname{Var}_1[\tilde{\varepsilon}_2]}{m^2}} + \frac{1}{2}(Z - F)^2. \tag{19}$$

For all agents, the speculative and the hedging components of the indirect utilities are clearly separated. All agents are speculators in some way. The speculative component is all the more important when the risk premium is high, whatever its sign. The hedging component depends on the industrial activity. For the storers, it is directly related to the cost of storage $F - P_1$; and for the processors, it rises with the margin on the processing activity $Z - F$.

In what follows, we denote respectively by U_i^S and U_i^H the speculative and the hedging components of type-i agents.

Welfare impact. Using the explicit expressions of equilibrium prices (Appendix B.2), we write the equilibrium utilities in terms of the fundamentals of the economy. These expressions are differentiated with respect to n_S (computations in Appendix C.2).

Proposition 4. *The effects of the financialization on the equilibrium utilities are in Table 3.*

Region	Speculation	Inventory holders		Processors	
(1)	(2)	(3)	(4)	(5)	(6)
Tag	$U_i^S \quad \forall i$	U_I^H	$U_I^H + U_I^S$	U_P^H	$U_P^H + U_P^S$
2			*Cond.*		
1U	↘	↗	*Cond.*	↘	↘
1U near Δ			↗		
1L near Δ					↗
1L	↘	↘	↘	↗	*Cond.*
4					*Cond.*
3	⟷	⟷	⟷	⟷	⟷

This table summarizes the impact of financialization on the individual welfare. The results are given by region with a special focus around the line Δ where the hedging component U_i^H of the utilities is higher than the speculative component U_i^S. Column 2 gives the impact of financialization on U_i^S for any operator i. Columns 3-4 detail the situation of the inventory holder with respect to the hedging component of his or her utility and to his or her total utility: $U_I^H + U^S I$. Columns 5-6 do the same for the processor.
Legend: ↗ variables increase; ↘ decrease; ⟷ no impact; "Cond.": the impact depends on the relative importance of the hedging and the speculative components in the total utility of the operator.

Table 3: The impact of the financialization on welfare.

Table 3 shows that:

- All agents lose on the speculative component of their utilities with financialization. Consequently, the pure speculator always loses from more competition.
- For the dominated side, a concomitant decrease in the speculative and in the hedging components of the utilities leads to a clear conclusion: losses. This is for example the case for processors in Subregion 1U and Region 2.
- For the dominant side, the two components go in opposite directions, as illustrated by the inventory holders in Subregion 1U and Region 2.

  The net result can be summarized as follows: the derivative of the hedgers' utilities change signs across Δ and along another straight line passing through M. Indeed, near the line Δ that separates Subregions 1U and 1L, speculation is small because the physical positions of the storers and the processors almost match. In other terms, the speculation component is of second order with respect to the hedging component. The dominant side gains. The details are in Appendix C.2.

In terms of political economy – in the sense that economic interests may determine political positions – we can draw the following conclusions. First, as far as the speculative component of their profit is concerned, all agents are against financialization. Not because of a malfunctioning of the futures market engendered by financialization, but because it harms their speculative profits. Second, profit from speculation and hedging are affected differently by regulation. Depending on the economic characteristics of the market where they operate, short and long hedgers have, most of the time, opposite interests. Therefore, we cannot systematically assign a pro or con position to storers nor to processors.

5.3 Financialization and the validity condition

To study how the validity condition of the equilibrium changes with financialization, we examine how the borders of the four regions move with the synthetic index γ. The computations are detailed in Appendix C.3.

How does the set of valid distributions of the physical shocks ($\tilde{\varepsilon}_1$, $\mathbb{E}_1[\tilde{\varepsilon}_2]$) move? The markets are not equal regarding this phenomenon. Regions 2 and 4 enlarge with financialization. In these regions where hedging has only one side, negative values for the three prices become less likely: for the markets situated in these two regions, financialization facilitates the absorption of positive supply shocks. In Region 1, where the physical positions of storers and processors compensate (more or less) for each other, the conclusion is not clear-cut. The changes are especially important when there is a relative abundance in the physical market and for low values of $\tilde{\varepsilon}_1$ and $\mathbb{E}_1[\tilde{\varepsilon}_2]$ (see Figure 7 in the Appendix C.3).

6 Comparison with the no-futures (NF) scenario

In this section, we compare two scenarios: with and without futures markets. We compare optimal positions, the validity condition as well as prices and their volatilities. The details of the computations are given in Appendix D.

Without futures, the separation between the physical decision, the hedging and the speculation is no longer true. As for the validity condition (Proposition 5 and Figure 8, in the appendix), the four regions of the NF scenario are included in those of the base scenario. Unambiguously, in the absence of a futures market, the possibility to absorb positive supply shocks while maintaining positive prices is reduced.

In the same spirit, we find that the variance in the period 1 spot price increases in the NF scenario. Relying on a model with storage as we do brings a different perspective than NEWBERY (1987). The futures market has a stabilizing effect. In period 2, the results are more contrasted. In Region 4, there is no change. Due to the absence of storage, without futures the two periods are statistically independent. In Region 2, the variance is bigger in the base scenario. The facilitation of storage via hedging creates contagion from the first period to the second one. Finally, the results are not clear-cut in Region 1.

The debates on financialization and on the desirability of futures markets must not be confused. First, the NF scenario is not the limit case for the base case where the number of speculators n_S tends to zero. A futures market does allow some risk sharing between hedgers, even if there are no speculators. Second, if financialization might harm certain categories of operators, the opening of a futures market has some very clear positive consequences.

7 Conclusion

Our model shows the interaction between commodity spot and futures markets and it covers the variety of situations observed in markets. In equilibrium, there might be a contango or a backwardation, the risk premium might be positive or negative, inventories might be held or not, the commodity might be processed or not, and adding speculators might increase or decrease the hedging benefits. This variety of situations exists in real commodity markets.

The functioning of a given market depends on the fundamentals and the realization of supply shocks. We show that the predictions on the effect of financialization can be much more precise once the specificities are considered. Whether financialization stabilizes or destabilizes markets by moving expected prices or volatilities can be rationalized by the four regimes we identify.

The diversity of objectives between agents has not received sufficient attention, in particular if we consider that one side of the hedging is generally bigger than the other. The fact that hedgers have clearly opposed views concerning the consequences of financialization sheds light on confused, changing, and even contradictory debates. Economic interest explains the diversity of political postures between agents and across time.

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APPENDIX

A Optimal positions

A.1 Utilities

All agents have mean-variance utilities. For them, a profit $\tilde{\pi}$ brings the utility:

$$\mathbb{E}_1[\tilde{\pi}] - \frac{1}{2}\alpha_i \operatorname{Var}_1[\tilde{\pi}] \tag{20}$$

where α_i is the risk aversion parameter of a type i individual. This choice has a long tradition in the literature (see, e.g., HIRSHLEIFER 1988).

A.2 Profit maximization

Speculators. Their profit resulting from a position in the futures market f_S is the random variable:

$$\tilde{\pi}_S(f_S) = f_S\,(P_2 - F),$$

and the optimal position is

$$f_S = \frac{\mathbb{E}_1[P_2] - F}{\alpha_S \operatorname{Var}_1[P_2]}. \tag{21}$$

This position is purely speculative. It depends mainly on the level and on the sign of the bias in the futures price. The speculator goes long whenever he or she thinks that the expected spot price is higher than the futures price. Otherwise, he or she goes short. Further, the speculator is all the more inclined to take a position if the risk aversion and the volatility of the underlying asset are low.

Storers. They can hold any nonnegative inventory. However, storage is costly: holding a quantity x between $t = 1$ and $t = 2$ costs $\frac{1}{2}Cx^2$. The parameters C (cost of storage) and α_I (risk aversion) characterize the storer. This agent has to decide how much inventory to buy at $t = 1$, if any, and what position to take in the futures market, if any. The storer buys $x \geq 0$ on the spot market at $t = 1$, resells it on the spot market at $t = 2$, and takes the position f_I on the futures market. The resulting profit is the random variable:

$$\tilde{\pi}_I(x, f_I) = x\,(P_2 - P_1) + f_I\,(P_2 - F) - \frac{1}{2}Cx^2.$$

The optimal position on the physical market is:

$$x = \frac{1}{C}\max\left[F - P_1, 0\right]. \tag{22}$$

The storer holds inventories if the futures price is higher than the current spot price. The optimal position on the futures market is:

$$f_I = -x + \frac{\mathbb{E}_1[P_2] - F}{\alpha_I \operatorname{Var}_1[P_2]}. \tag{23}$$

This position is decomposed into two elements. First, a negative position $-x$ that hedges the inventories: the storer sells futures contracts in order to protect him or herself against a decrease in the spot price. Second, a speculative position, structurally identical to that of the speculator, that reflects the storer's risk aversion and his or her expectations about the relative level of the futures and the expected spot prices.

We argue that this modeling embeds producers. Take the case of soybeans and call x the crop. Producers have physical returns on their planting (say g beans per bean) and some financial cost for the whole cycle. The profit is random and is as follows:

$$\tilde{\pi}_I(x, f_I) = x\left(P_2 - \frac{P_1}{g}\right) + f_I\,(P_2 - F) - \frac{1}{2}Cx^2. \tag{24}$$

We get:

$$x = \frac{1}{C}\max\left[F - \frac{P_1}{g}, 0\right]\ ; \tag{25}$$

$$\text{and } f_I = -x + \frac{\mathbb{E}_1[P_2] - F}{\alpha_I \operatorname{Var}_1[P_2]}. \tag{26}$$

The structure of the problem remains the same. The additional parameter adds marginal insights.

Processors. A processor decides at time $t = 1$ how much input y to buy at $t = 2$ and which position f_P to take on the futures market. The revenue from sales at period $t = 2$ is $(y - \frac{\beta}{2}y^2)\,Z$, where Z is our convention for the forward price of the output, and the other factor reflects decreasing marginal revenue. Due to these forward transactions, the processor knows at time $t = 1$ the revenue of these sales. The profit is the random variable:

$$\tilde{\pi}_P(y, f_P) = \left(y - \frac{\beta}{2}y^2\right)Z - y\,P_2 + f_P\,(P_2 - F).$$

Therefore, the processor's optimal decisions are:

$$y = \frac{1}{\beta Z}\max\left[Z - F, 0\right], \tag{27}$$

$$f_P = y + \frac{\mathbb{E}_1[P_2] - F}{\alpha_P \operatorname{Var}_1[P_2]}. \tag{28}$$

The processor also uses the futures market to plan production, particularly if the price of the input F is below that of the output Z. The position on the futures market is decomposed into two elements: a hedge position and a speculative position.

A.3 Simplified definition of X and Y for the equilibrium analysis

Although we assume that all individuals are identical in each category of agents, more subtle assumptions could be retained without much complication. For example, what if the storers have different technologies? If storer i (with $i = 1, \dots, n_I$) has technology C_i, then the total inventories are $(\sum_i 1/C_i) \max[F - P_1, 0]$. In other words, storers can be aggregated.

We can redefine the variables as follows: $C_i = 1$ and n_I is the number of storers, where

$$n_I \quad \text{replaces} \quad \begin{cases} n_I/C & \text{if storers are identical,} \\ \sum_i 1/C_i & \text{otherwise.} \end{cases}$$

Then,

$$x = \max[F - P_1, 0] \quad \text{and} \quad X = n_I \max[F - P_1, 0].$$

Similarly, if processor i (with $i = 1, \dots, n_P$) has technology β_i, then the total input demand is $(\sum_i 1/(\beta_i Z)) \max[Z - F, 0]$. We can redefine the variables as follows: $\beta_i = 1$ and n_P is the number of processors, where

$$n_P \quad \text{replaces} \quad \begin{cases} \dfrac{n_P}{\beta Z} & \text{if processors are identical,} \\ \dfrac{1}{Z} \displaystyle\sum_i \dfrac{1}{\beta_i} & \text{otherwise.} \end{cases}$$

Then

$$y = \max[Z - F, 0] \quad \text{and} \quad Y = n_P \max[Z - F, 0].$$

B The equilibrium: existence, uniqueness, expressions, and validity

The equilibrium is characterized by a system of equations that is piecewise linear. We prove uniqueness by showing that the four regimes do not overlap. To do so, we explicitly compute the precise borders, the prices, and the quantities for each regime.

B.1 Proof of Proposition 1 (existence and uniqueness)

A solution is a family $(X, Y, F, P_1, P_2) \in \mathbb{R}_+^2 \times \mathbb{R}^2 \times L^0(\Omega, \mathcal{A}, \mathbb{P})$ that satisfies the equations (6), (7), (11), (12), and (13). We examine the image by φ of Regions 1 to 4 that are depicted in Figure 1. Figure 4 displays these images.

In Region 1, we have:

$$\varphi(P_1, F) = \begin{pmatrix} mP_1 - n_I(F - P_1) \\ mF + \gamma(n_I(F - P_1) - n_P(Z - F)) \end{pmatrix}.$$

We compute the images $\varphi(O)$, $\varphi(A)$, and $\varphi(M)$ as:

$$\begin{aligned}\varphi(O) &= (0, -\gamma n_P Z),\\ \varphi(A) &= Z\,(-n_I, m + \gamma n_I),\\ \varphi(M) &= m\,Z\,(1,1).\end{aligned}$$

The image of Region 1 is bordered by the angle $\varphi(A)\varphi(M)\varphi(O)$.

The image of Region 2 is bounded by two infinite half-lines: one is the image of $\{P_1 \leq Z, F = Z\}$, the other the image of $\{P_1 = F, F \geq Z\}$. In Region 2, we have:

$$\varphi(P_1, F) = \begin{pmatrix} mP_1 - n_I(F - P_1) \\ mF + \gamma n_I(F - P_1) \end{pmatrix}.$$

The first half-line is the continuation of segment $\varphi(M)\varphi(A)$. The second half-line emanates from $\varphi(M)$ and is represented by the vector $(1, 1)$.

The image of Region 4 is bounded by two half-lines: the continuation of segment $\varphi(M)\varphi(O)$ and the image of $\{P_1 \geq Z, F = Z\}$. In Region 4, we have:

$$\varphi(P_1, F) = \begin{pmatrix} mP_1 \\ mF - \gamma n_P\,(Z - F) \end{pmatrix}.$$

The image of Region 3 is entirely contained in $\mathbb{R}^2_+$ where it is the remainder of the three images in the other three regions.

B.2 Explicit prices and quantities

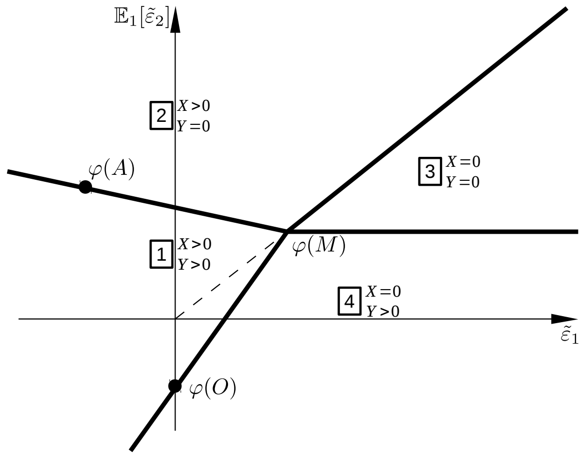
In each of the regions defined in the paragraph 3.5, the final expressions of the equilibrium prices have the following form:

$$F = F_1(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2]\,, \mathrm{Var}_1[\tilde{\varepsilon}_2]), \tag{29}$$

$$P_1 = p_1(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2]\,, \mathrm{Var}_1[\tilde{\varepsilon}_2]), \tag{30}$$

$$P_2 = p_2(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \mathbb{E}_1[\tilde{\varepsilon}_2]\,, \mathrm{Var}_1[\tilde{\varepsilon}_2]). \tag{31}$$

Because of the expressions for Region 1, setting $n_P = 0$ gives the expressions for Region 2 and setting $n_I = 0$ gives expressions for Region 4.



This figure represents the image of φ. We use Regions 1–4 from Figure 1. $\varphi(A)$, $\varphi(O)$, and $\varphi(M)$ are the images of A, O, and M. The supply shock $\tilde{\varepsilon}_1$ represents the scarcity observed at time 1, and $\mathbb{E}_1[\tilde{\varepsilon}_2]$ is the scarcity expected for time 2. Knowing $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$, the diagram shows the relevant region of φ, and thus the type of equilibrium is disclosed.

Figure 4: The four regions in the space of the supply shocks $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$.

Region 1.

$$F = \frac{mn_I\gamma\frac{\tilde{\varepsilon}_1}{m} + m(m+n_I)\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} + (m+n_I)n_P\gamma Z}{mn_I\gamma + m(m+n_I) + (m+n_I)n_P\gamma}, \tag{32}$$

$$P_1 = \frac{m(m+(n_I+n_P)\gamma)\frac{\tilde{\varepsilon}_1}{m} + mn_I\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} + n_In_P\gamma Z}{m(m+(n_I+n_P)\gamma) + mn_I + n_In_P\gamma}, \tag{33}$$

$$P_2 = \frac{\tilde{\varepsilon}_2}{m} + \frac{mn_I\frac{\tilde{\varepsilon}_1}{m} - ((m+n_I)n_P + mn_I)\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} + (m+n_I)n_P Z}{mn_I\gamma + m(m+n_I) + (m+n_I)n_P\gamma}, \tag{34}$$

All denominators are equal. They are written in different ways only to show that F and P_1 are convex combinations of $\frac{\tilde{\varepsilon}_1}{m}$, $\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m}$, and Z. Thus:

$$\mathbb{E}_1[P_2] - F = (\gamma-1)\frac{mn_I\left(\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} - \frac{\tilde{\varepsilon}_1}{m}\right) - n_P(m+n_I)\left(Z - \frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m}\right)}{m(m+n_I+(n_I+n_P)\gamma) + n_In_P\gamma}. \tag{35}$$

Quantities:

$$X = \frac{m(m+n_P\gamma)\left(\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} - \frac{\tilde{\varepsilon}_1}{m}\right) + mn_P\gamma\left(Z - \frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m}\right)}{m(m+n_I+(n_I+n_P)\gamma) + n_In_P\gamma}, \tag{36}$$

$$Y = \frac{mn_I\gamma\left(\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} - \frac{\tilde{\varepsilon}_1}{m}\right) + m(m+n_I(1+\gamma))\left(Z - \frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m}\right)}{m(m+n_I+(n_I+n_P)\gamma) + n_In_P\gamma}. \tag{37}$$

Region 2.

$$F = \frac{n_I \gamma \frac{\tilde{\varepsilon}_1}{m} + (m + n_I)\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m}}{m + n_I(1+\gamma)} \; ; \; P_1 = \frac{(m + n_I\gamma)\frac{\tilde{\varepsilon}_1}{m} + n_I \frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m}}{m + n_I(1+\gamma)} \; ; \; P_2 = \frac{\tilde{\varepsilon}_2}{m} + \frac{n_I\left(\frac{\tilde{\varepsilon}_1}{m} - \frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m}\right)}{m + n_I(1+\gamma)} \; ;$$

$$X = \frac{m\left(\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} - \frac{\tilde{\varepsilon}_1}{m}\right)}{m + n_I(1+\gamma)} \; ; \; Y = 0.$$

Therefore:

$$\mathbb{E}_1[P_2] - F = (\gamma - 1)\frac{n_I\left(\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} - \frac{\tilde{\varepsilon}_1}{m}\right)}{m + n_I(1+\gamma)} > 0. \tag{38}$$

Region 3.

$$F = \frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} \; ; \; P_1 = \frac{\tilde{\varepsilon}_1}{m} \; ; \; P_2 = \frac{\tilde{\varepsilon}_2}{m} \; ; \; X = 0 \; ; \; Y = 0 \; ; \; \mathbb{E}_1[P_2] - F = 0.$$

Region 4.

$$F = \frac{m\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} + n_P\gamma Z}{m + n_P\gamma} \; ; \; P_1 = \frac{\tilde{\varepsilon}_1}{m} \; ; \; P_2 = \frac{\tilde{\varepsilon}_2}{m} + \frac{n_P\left(Z - \frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m}\right)}{m + n_P\gamma} \; ; \; X = 0 \; ; \; Y = \frac{m\left(Z - \frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m}\right)}{m + n_P\gamma}.$$

Therefore:

$$F - \mathbb{E}_1[P_2] = (\gamma - 1)\frac{n_P\left(Z - \frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m}\right)}{m + n_P\gamma} > 0. \tag{39}$$

B.3 Proof of Proposition 2 (validity)

Lemma 1. *The solution verifies $P_1 \geq 0$ and $F \geq 0$ if and only if $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$ verifies:*

$$\mathbb{E}_1[\tilde{\varepsilon}_2] \geq -\frac{m + n_I\gamma}{n_I}\tilde{\varepsilon}_1 \qquad \text{if } \tilde{\varepsilon}_1 \leq -n_I Z, \tag{40}$$

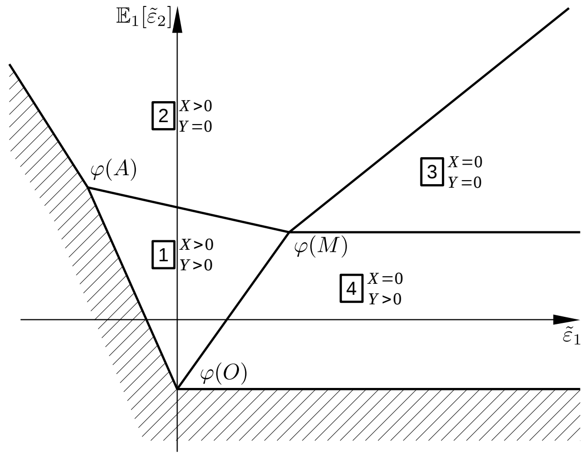
$$\mathbb{E}_1[\tilde{\varepsilon}_2] \geq -n_P\gamma Z - \frac{m + (n_I + n_P)\gamma}{n_I}\tilde{\varepsilon}_1 \qquad \text{if } -n_I Z \leq \tilde{\varepsilon}_1 \leq 0, \tag{41}$$

$$\mathbb{E}_1[\tilde{\varepsilon}_2] \geq -n_P\gamma Z \qquad \text{if } \tilde{\varepsilon}_1 \geq 0, \tag{42}$$

and then it is unique.

Proof. We only have to identify the images by φ of the horizontal and the vertical axis of Figure 1, taking into account the regimes they are in, in order to apply the right expression of φ. The horizontal axis gives the horizontal half-line emanating from $\varphi(O)$ (equation 42), the vertical axis gives the segment $\varphi(O)\varphi(A)$ (equation 41), and the half-line emanating from $\varphi(A)$ (equation 40).

These conditions are pictured in Figure 5. □



This figure illustrates the validity condition of the equilibrium when the positivity constraints on P_1 and F are taken into account for the supply shocks $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$. We use the Regions 1–4 from Figure 4. The hatched area is associated with the negative values for P_1 or F. The points $\varphi(O)$ and $\varphi(A)$ are representative of Lemma 1.

Figure 5: The validity condition of the equilibrium, with positivity constraints on P_1 and F.

We verify that almost surely $P_2 \geq 0$. By equation (12), the exact condition is:

$$\tilde{\varepsilon}_2 \geq X - Y \qquad \text{almost surely.}$$

Lemma 2. *An equilibrium exists if and only if $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$ satisfies (40), (41), and (42); plus the condition of*

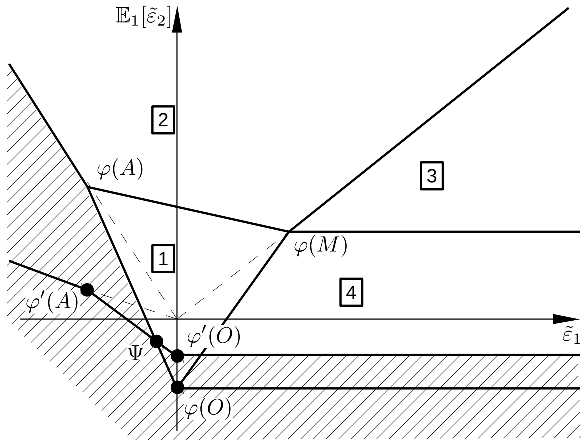
$$\begin{aligned}
&\textit{In Region 1,} && \tilde{\varepsilon}_2 \geq m\frac{mn_I\left(\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} - \frac{\tilde{\varepsilon}_1}{m}\right) - n_P\,(m+n_I)\left(Z - \frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m}\right)}{m(m+n_I+(n_I+n_P)\gamma)+n_In_P\gamma} && \textit{a.s.}\\
&\textit{In Region 2,} && \tilde{\varepsilon}_2 \geq \frac{mn_I\left(\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} - \frac{\tilde{\varepsilon}_1}{m}\right)}{m+n_I(1+\gamma)} && \textit{a.s.}\\
&\textit{In Region 3,} && \tilde{\varepsilon}_2 \geq 0 && \textit{a.s.}\\
&\textit{In Region 4,} && \tilde{\varepsilon}_2 \geq -\frac{mn_P(Z - \frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m})}{m+n_P\gamma} && \textit{a.s.}
\end{aligned}$$

Proof. In Region 1, given equation (34) in Appendix B.2, $P_2 \geq 0$ a.s. is equivalent to:

$$0 \leq \frac{\tilde{\varepsilon}_2}{m} + \frac{mn_I\frac{\tilde{\varepsilon}_1}{m} - ((m+n_I)n_P + mn_I)\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} + (m+n_I)n_PZ}{mn_I\gamma + m(m+n_I) + (m+n_I)n_P\gamma},$$

which gives the expression of the proposition after rearrangement.

Starting from the condition for Region 1, taking $n_P = 0$ yields the condition for Region 2, taking $n_I = 0$ yields the condition for Region 4, and taking $n_I = n_P = 0$ yields the condition for Region 3. □



This figure illustrates how the frontiers of Regions 1 and 4 are modified when the positivity constraints on P_2 are added to the positivity constraints on P_1 and F for the supply shocks $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$. The hatched regions correspond to situations inconsistent with validity. The points $\varphi'(O)$ and $\varphi'(A)$ characterize the new boundaries associated with Lemma 3. The point Ψ marks the intersection of the two sets of constraints.

Figure 6: The validity condition of the equilibrium with positivity constraints on all prices.

Lemma 2 gives restrictions on the joint distribution of $(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2)$ for the existence of an equilibrium. We can go further and answer a complementary question: for a given $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$ is there a distribution of $\tilde{\varepsilon}_2$ such that an equilibrium exists?

Lemma 3. *If $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$ is such that P_1 and F are nonnegative (Lemma 1), then there is a distribution of $\tilde{\varepsilon}_2$ that supports an equilibrium if and only if*

$$\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} \geq -\frac{n_I\tilde{\varepsilon}_1 + (m+n_I)n_P Z}{m(m+(\gamma-1)n_P) + n_I(m\gamma + (\gamma-1)n_P)} \qquad \textit{in Region 1,}$$

$$\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} \geq -\frac{n_I}{m+n_I\gamma}\frac{\tilde{\varepsilon}_1}{m} \qquad \textit{in Region 2,}$$

$$\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} \geq 0 \qquad \textit{in Region 3,}$$

$$\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} \geq -\frac{n_P Z}{m+(\gamma-1)n_P} \qquad \textit{in Region 4.}$$

Proof. Starting from Lemma 2 and because the limit case that allows us to draw the frontier occurs when $\operatorname{ess\,inf}\{\tilde{\varepsilon}_2\} = \mathbb{E}_1[\tilde{\varepsilon}_2]$, we find the conditions above after rearrangement. □

$\varphi'(O)$ and $\varphi'(A)$ are arbitrary names for the kinks of the new three-part linear boundary (there are only three parts because the condition in Region 3 is irrelevant given its position). These characteristic points of the boundary are:

$$\varphi'(O) = \left(0, -\frac{mn_P}{m + n_P(\gamma - 1)} Z\right);$$
$$\varphi'(A) = \left(-n_I Z, \frac{n_I^2}{m + n_I\gamma} Z\right).$$

Further, $\varphi'(O)$ is above $\varphi(O)$ (both have the same null abscissa), and $\varphi'(A)$ is below $\varphi(A)$ (both have the same negative abscissa). The picture shows the constraints of Lemma 1 and Lemma 3. The intersection point Ψ of the two sets of constraints is in Region 1 with:

$$\Psi = \left(-\frac{n_I n_P(\gamma - 1)Z}{m + (n_I + n_P)(\gamma - 1)}, -\frac{mn_P Z}{m + (n_I + n_P)(\gamma - 1)}\right).$$

The whole set of constraints is depicted by Figure 6. The figure displays the existence conditions for the supply shocks $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$. The comparison with Figure 5 shows what Lemma 3 (characteristic points $\varphi'(O)$ and $\varphi'(A)$) adds to Lemma 1 (characteristic points $\varphi(O)$ and $\varphi(A)$).

The upper envelope of the conditions in Lemma 1 and Lemma 3 define function B in Proposition 2.

C The impact of the financialization

We analyze financialization as a decrease in γ as explained at the beginning of Section 5, or directly through changes in the number of speculators n_S. We examine the impact on equilibrium prices and quantities, utilities, and on the validity condition.

C.1 Impact on prices and quantities

We focus first on Regions 2 and 4 in order to examine two basic mechanisms, then discuss Region 1 in which the previous effects are combined. The equilibrium prices are drawn from Appendix B.2.

Region 2. The prices F and P_1 are weighted averages of $\tilde{\varepsilon}_1/m$ and $\mathbb{E}_1[\tilde{\varepsilon}_2]/m$; and $\mathbb{E}_1[\tilde{\varepsilon}_2]/m \geq \frac{\tilde{\varepsilon}_1}{m}$ in Region 2. These two facts determine the variations in the three prices given in Table 2. Thus, we can conclude directly from equation (38) that $\mathbb{E}_1[P_2] - F$ decreases as speculation increases.

Moreover,

$$\text{Var}_0[F] = \left(\frac{n_I\gamma}{m + n_I(1+\gamma)}\right)^2 \frac{\text{Var}_0[\tilde{\varepsilon}_1]}{m^2}, \qquad \text{Var}_0[P_1] = \left(\frac{m + n_I\gamma}{m + n_I(1+\gamma)}\right)^2 \frac{\text{Var}_0[\tilde{\varepsilon}_1]}{m^2},$$
$$\text{Var}_0[P_2] = \frac{\text{Var}_0[\tilde{\varepsilon}_2]}{m^2} + \left(\frac{n_I}{m + n_I(1+\gamma)}\right)^2 \frac{\text{Var}_0[\tilde{\varepsilon}_1]}{m^2}, \quad \text{Var}_0[X] = \left(\frac{-1}{m + n_I(1+\gamma)}\right)^2 \text{Var}_0[\tilde{\varepsilon}_1],$$

with obvious comparative statics.

Region 4. Again, F and P_1 are weighted averages of $\tilde{\varepsilon}_1/m$ and $\mathbb{E}_1[\tilde{\varepsilon}_2]/m$. These averages, in addition to $\mathbb{E}_1[\tilde{\varepsilon}_2]/m \leq Z$ in Region 4, determine the comparative statics on the three prices. Therefore, we can conclude directly from equation (39) that $F - \mathbb{E}_1[P_2]$ decreases with financialization.

Region 1. The two subregions 1U and 1L are separated by the line Δ already defined by $X - Y = 0 \Leftrightarrow \mathbb{E}_1[P_2] - F = 0$ (see equation 35). The prices and quantities are in the form of $\frac{A+B\gamma}{C+D\gamma}$ with positive numerators and denominators. This expression increases with respect to γ if $BC - DA \geq 0$. Subregions 1U and 1L resemble Regions 2 and 4 respectively. This is true for the levels of F, P_1, P_2, X, and Y whose variations are summarized in Table 2. Thus,

$$\begin{aligned}
\mathrm{Var}_0[F] &= \left(\frac{mn_I\gamma}{m(m+n_I+(n_I+n_P)\gamma)+n_In_P\gamma}\right)^2\frac{\mathrm{Var}_0[\tilde{\varepsilon}_1]}{m^2},\\
\mathrm{Var}_0[P_1] &= \left(\frac{m(m+(n_I+n_P)\gamma)}{m(m+n_I+(n_I+n_P)\gamma)+n_In_P\gamma}\right)^2\frac{\mathrm{Var}_0[\tilde{\varepsilon}_1]}{m^2},\\
\mathrm{Var}_0[P_2] &= \left(\frac{mn_I}{m(m+n_I+(n_I+n_P)\gamma)+n_In_P\gamma}\right)^2\frac{\mathrm{Var}_0[\tilde{\varepsilon}_1]}{m^2}+\frac{\mathrm{Var}_0[\tilde{\varepsilon}_2]}{m^2}.
\end{aligned}$$

The position with regard to Δ is not relevant for the variances, which are monotonic in the same way whatever the considered subcase. See Table 2 in subsection 5.1.

C.2 Impact on utilities

In what follows, we explain the results gathered in Table 3. We focus on Region 1, where all operators are active. First, we particularize the indirect utilities (formulas 17, 18, and 19) to the case when the markets are in equilibrium. In that case, P_2 becomes a function of (P_1, F), and the formulas become:

$$U_S = \frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{2m^2\alpha_S\left(\sum\frac{n_i}{\alpha_i}\right)^2}\left(n_I\left(F-P_1\right)-n_P\left(Z-F\right)\right)^2; \tag{43}$$

$$U_I = \frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{2m^2\alpha_I\left(\sum\frac{n_i}{\alpha_i}\right)^2}\left(n_I\left(F-P_1\right)-n_P\left(Z-F\right)\right)^2+\frac{(F-P_1)^2}{2}; \tag{44}$$

$$U_P = \frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{2m^2\alpha_P\left(\sum\frac{n_i}{\alpha_i}\right)^2}\left(n_I\left(F-P_1\right)-n_P\left(Z-F\right)\right)^2+\frac{(Z-F)^2}{2}. \tag{45}$$

Formulas (43), (44), and (45) provide the indirect utilities of the agents at equilibrium in terms of the equilibrium prices P_1 and F. These expressions can in turn be expressed in terms of the fundamentals of the economy, namely $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ (see equations 32-34 in Appendix B.2). The new expressions can be differentiated with respect to the parameters.

To investigate whether an increase in the number of speculators increases or decreases the welfare of speculators, inventory holders, and industry processors; we work directly with formulas

(43), (44), and (45) and take the sensitivities of P_1 and F with respect to parameter n_S. In contrast, the complete substitution of equilibrium values is unworkable.

Sensitivity of U_S. Differentiating formula (43) yields:

$$\begin{aligned}
\frac{dU_S}{dn_S} =& \frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{m^2\alpha_S\left(\sum\frac{n_i}{\alpha_i}\right)^2}\left(n_I(F-P_1)-n_P(Z-F)\right)\left(m+n_P\left(1+\frac{m}{n_I}\right)\right)\frac{dP_1}{dn_S} \\
&-\frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{m^2\alpha_S^2\left(\sum\frac{n_i}{\alpha_i}\right)^3}\left(n_I(F-P_1)-n_P(Z-F)\right)^2 \\
=&-\frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{m^2\alpha_S^2\left(\sum\frac{n_i}{\alpha_i}\right)^3}\left(1-\frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{m\sum\frac{n_i}{\alpha_i}}\frac{\left(m+n_P\left(1+\frac{m}{n_I}\right)\right)}{\left(\frac{m}{n_I}+1\right)(m+\gamma n_P)+\gamma m}\right) \\
&\times\left(n_I(F-P_1)-n_P(Z-F)\right)^2. \\
=&-\frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{m^2\alpha_S^2\left(\sum\frac{n_i}{\alpha_i}\right)^3}\;\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\gamma(n_In_P+m(n_I+n_P))} \\
&\times\left(n_I(F-P_1)-n_P(Z-F)\right)^2.
\end{aligned} \tag{46}$$

The sign of $\frac{dU_S}{dn_S}$ is constant in Region 1: it is negative. Adding speculators decreases the remuneration associated with risk bearing.

Sensitivity of U_I. Differentiating formula (44) yields:

$$\begin{aligned}
\frac{dU_I}{dn_S} =& -\frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{m^2\alpha_S\alpha_I\left(\sum\frac{n_i}{\alpha_i}\right)^3}\;\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\gamma(n_In_P+m(n_I+n_P))} \\
&\times\left(n_I(F-P_1)-n_P(Z-F)\right)^2+(F-P_1)\left(\frac{dF}{dn_S}-\frac{dP_1}{dn_S}\right) \\
=& -\frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{m^2\alpha_S\alpha_I\left(\sum\frac{n_i}{\alpha_i}\right)^3}\;\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\gamma(n_In_P+m(n_I+n_P))} \\
&\times\left(n_I(F-P_1)-n_P(Z-F)\right)^2 \\
&+\frac{F-P_1}{n_I\alpha_S}\frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{\left(\sum\frac{n_i}{\alpha_i}\right)^2}\frac{n_I(F-P_1)-n_P(Z-F)}{\left(\frac{m}{n_I}+1\right)(m+\gamma n_P)+\gamma m}.
\end{aligned} \tag{47}$$

As mentioned before, the utility due to speculative activities decreases when n_S increases. As far as the utility of hedging is concerned, the effect depends on the sign of $n_I(F-P_1)-n_P(Z-F)$. This line has already been encountered; it separates Region 1 into two subcases. In Subregion 1U, the utility of hedging increases for the storers because they need more hedging than processors. The opposite conclusion arises in Subregion 1L.

As far as the total utility is concerned, $n_I(F-P_1)-n_P(Z-F)$ is a common factor. Thus:

$$\frac{dU_I}{dn_S} = A_I(n_I(F-P_1)-n_P(Z-F)) \times (K_1(F-P_1)+K_2(Z-F)),$$

for suitable constants A_I, K_1, and K_2. This equation shows that the sign changes across

- The line Δ that is defined by $n_I\,(F-P_1)+n_P\,(Z-F)=0$
- The line D_I that is defined by the equation $K_1(F-P_1)+K_2(Z-F)=0$

Both Δ and D go through the point M where $P_1=F=Z$. If $K_2/K_1<0$, the line D enters Region 1; if $K_2/K_1>0$, it does not. So, if $K_2/K_1<0$, Region 1 is divided into three subregions by the lines D and Δ, and the sign changes when one crosses from one to the other. If $K_2/K_1>0$, then Region 1 is divided into two subregions by the line Δ, and the sign changes across Δ. In all cases, the response of inventory holders to an increase in the number of speculators depends on the equilibrium.

Sensitivity of U_P. Differentiating formula (45) yields:

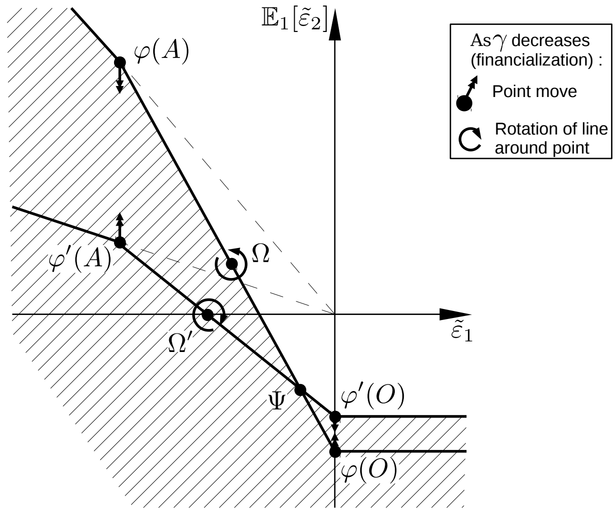
$$\begin{aligned}\frac{dU_P}{dn_S} =& -\frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{m^2\alpha_S\alpha_P\left(\sum\frac{n_i}{\alpha_i}\right)^3}\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\gamma(n_In_P+m(n_I+n_P))}\\ &\times(n_I(F-P_1)-n_P(Z-F))^2+(F-Z)\,\frac{dF}{dn_S}\\ =& -\frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{m^2\alpha_S\alpha_P\left(\sum\frac{n_i}{\alpha_i}\right)^3}\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\gamma(n_In_P+m(n_I+n_P))}\\ &\times(n_I(F-P_1)-n_P(Z-F))^2\\ &+(F-Z)\left(\frac{m}{n_I}+1\right)\frac{1}{m\alpha_S}\frac{\mathrm{Var}_1[\tilde{\varepsilon}_2]}{\left(\sum\frac{n_i}{\alpha_i}\right)^2}\frac{n_I(F-P_1)-n_P(Z-F)}{\left(\frac{m}{n_I}+1\right)(m+\gamma n_P)+\gamma m}.\end{aligned} \tag{48}$$

Again, the utility due to speculation decreases, and the utility linked to hedging depends on the sign of $n_I(F-P_1)-n_P(Z-F)$. In Subregion 1U, the utility of hedging decreases for the processors, and it increases in Subregion 1L.

As far as the total utility is concerned, $n_I(F-P_1)-n_P(Z-F)$ is a common factor. Thus:

$$\frac{dU_P}{dn_S} = A_P(n_I(F-P_1)-n_P(Z-F)) \times (K_3(F-P_1)+K_4(Z-F))$$

for suitable constants A_P, K_3, and K_4. As in the preceding case, there is a line D_p (different from D_I) that enters Region 1 if $K_3/K_4<0$ and does not if $K_3/K_4>0$. In the first case, Region 1 is divided into three subregions by D_P and Δ, in the second it is divided into two subregions by Δ, and the sign of $\frac{dU_I}{dn_S}$ changes when one crosses the frontiers.



This figure shows the change in the validity condition for the supply shocks $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$, when financialization is intensified (a decrease in γ). The hatched regions correspond to situations inconsistent with positive prices. $\varphi(O)$, $\varphi'(O)$, $\varphi(A)$, $\varphi'(A)$, and Ψ are the characteristic points of the validity conditions. Ω and Ω' are fixed points, whose positions do not depend on γ.

Figure 7: Validity of a solution: the impact of financialization.

C.3 Impact on the validity condition

This appendix examines the modifications in the validity condition when the intensity of the speculation changes. We analyze the moves in the boundaries defined by Lemma 1 and Lemma 3 (validity condition). Figure 6 is the starting point. Figure 7 illustrates how the characteristic points of the validity condition move when γ decreases (i.e., n_S increases). The calculations show that $\varphi(O)$, $\varphi'(O)$, $\varphi(A)$, and $\varphi'(A)$ move vertically.

The vertical downward move of $\varphi'(O)$ and $\varphi(A)$ means that the validity condition is relaxed in Regions 2 and 4 when the financialization increases.

The effects on Region 1 are mixed. On the one hand, the segment $\varphi(A)\varphi(O)$ rotates counterclockwise around the fixed point Ω whose position does not depend on γ:

$$\Omega = \left(-\frac{n_I n_P Z}{n_I + n_P}, \frac{m n_P Z}{n_I + n_P}\right)$$

On the other hand, the segment $\varphi'(A)\varphi'(O)$ rotates clockwise around the other fixed point Ω' (i.e., whose position does not depend on γ):

$$\Omega' = \left(-\frac{(m + n_I) n_P Z}{n_I}, 0\right)$$

The comparative statics on γ are now clear: when financialization intensifies, the validity conditions in Region 2 and Region 4, where only one type of actor actually has a physical position, relax. In Region 1, where the physical positions of storers and processors compensate (more or less) for each other, the conclusion is not clear-cut.

D Comparison with the no-futures scenario (NF)

This appendix is devoted to the case where there is no futures market (scenario NF). It is compared with the *base scenario*, that is, the one we explore in the rest of this article. Without futures, the speculators are inactive, and there are three kinds of operators: storers, processors, and spot traders.

We study fundamental equations, the validity of the equilibrium as well as prices and volatilities.

D.1 Fundamental equations

The optimal position of the storer becomes:

$$X_{\text{NF}} = \frac{n_I}{C + \alpha_I \frac{\text{Var}_1[\tilde{\varepsilon}_2]}{m^2}} \max\left[\mathbb{E}_1[P_2^{\text{NF}}] - P_1^{\text{NF}}, 0\right]. \tag{49}$$

The storer holds inventory if the expected price is higher than the current spot price. The processor's activity depends on the fact that the marginal revenue on the processing activity is higher than the expected spot price of the commodity:

$$Y_{\text{NF}} = \frac{n_P}{\beta Z + \alpha_P \frac{\text{Var}_1[\tilde{\varepsilon}_2]}{m^2}} \max\left[Z - \mathbb{E}_1[P_2^{\text{NF}}], 0\right]. \tag{50}$$

In this scenario, the separation between a physical decision, hedging, and speculation is no longer true.

Equation (12), taken in expectations and adapted to the NF scenario, is:

$$\mathbb{E}_1[P_2^{\text{NF}}] = \frac{1}{m}\left(\mathbb{E}_1[\tilde{\varepsilon}_2] - n_I X_{\text{NF}} + n_P Y_{\text{NF}}\right).$$

This expression allows us to follow exactly the same line of reasoning as in the base scenario with the modified mapping φ_{NF} defined by:

$$\varphi_{\text{NF}}(P_1, \mathbb{E}_1[P_2]) = \begin{pmatrix} mP_1 - \frac{n_I C}{C+\alpha_I \frac{\text{Var}_1[\tilde{\varepsilon}_2]}{m^2}} \max\left[\mathbb{E}_1[P_2] - P_1, 0\right] \\ m\,\mathbb{E}_1[P_2] + \frac{n_I C}{C+\alpha_I \frac{\text{Var}_1[\tilde{\varepsilon}_2]}{m^2}} \max\left[\mathbb{E}_1[P_2] - P_1, 0\right] - \frac{n_P \beta Z}{\beta Z+\alpha_P \frac{\text{Var}_1[\tilde{\varepsilon}_2]}{m^2}} \max\left[Z - \mathbb{E}_1[P_2], 0\right] \end{pmatrix}.$$

Formally, the analysis of equilibrium (existence, uniqueness and validity condition) is identical to the one done in the base scenario. Therefore, we reuse the previous calculations by applying the variables and parameters in the transposition given in Table 4.

Base scenario:	F	γ	n_I	n_P
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
NF scenario:	$\mathbb{E}_1[P_2^{\text{NF}}]$	1	$\overbrace{n_I^{\text{NF}} = n_I \times \frac{C}{C + \alpha_I \frac{\text{Var}_1[\tilde{\varepsilon}_2]}{m^2}}}$	$\overbrace{n_P^{\text{NF}} = n_P \times \frac{\beta Z}{\beta Z + \alpha_P \frac{\text{Var}_1[\tilde{\varepsilon}_2]}{m^2}}}$

This table explains the correspondence, in prices and in parameters, established between the base scenario and the no-futures (NF) scenario. This formal trick allows us to perform the equilibrium analysis in the two scenarios in parallel.

Table 4: Correspondance between variables and parameters in the base and in the NF scenarios.

D.2 Equilibrium: the validity condition

Proposition 5. *The* validity condition *on* $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$ *is stricter in the NF scenario than in the base scenario.*

Proof. In this part we investigate the impact of having or not having a futures market on the validity condition.

We use the complete conditions of validity from Lemma 3:

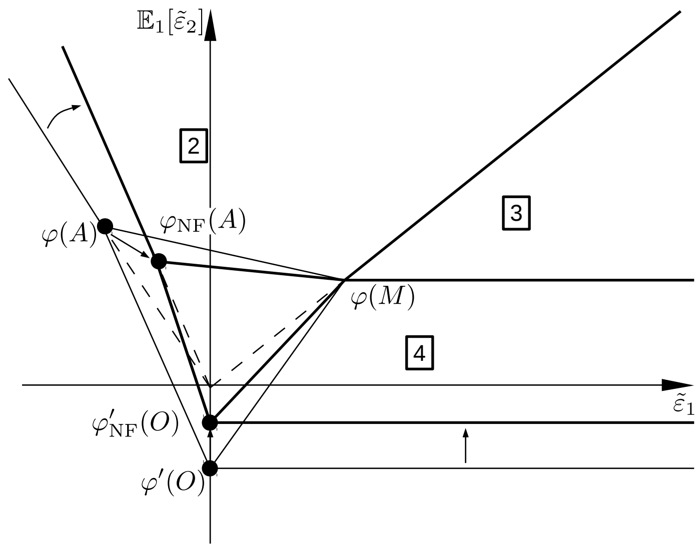
1. $\varphi'_{\text{NF}}(O)$ is above $\varphi'(O)$, where we use the convention that $\varphi'_{\text{NF}}(O)$ is the equivalent in the NF scenario of $\varphi'(O)$ in the basic model. We use similar conventions for the other symbols.
2. $\varphi'_{\text{NF}}(O) = \varphi_{\text{NF}}(O)$, therefore these two points coincide with Ψ_{NF}.
3. The two previous points imply that only the frontier in Lemma 1 matters.
4. The slope of the left frontier of Region 2 is steeper (in absolute value) in the NF case than in the basic model. This increased steepness also restricts the possibilities of an equilibrium where all prices are positive in the NF case.
5. $\varphi_{\text{NF}}(A)$ has a higher abscissa and a lower ordinate than $\varphi(A)$. This also leads to more restrictive conditions in the NF scenario.

□

The results of this analysis are depicted by Figure 8: the four regions of the NF scenario are included in those of the base scenario. When there is no futures market, Region 1 diminishes; Region 2 grows at the expense of the basic Region 1 and it is cut on its left border; Region 3 does not change; Region 4 grows at the expense of basic Region 1 and is cut on its bottom border.

D.3 Prices and volatility

In order to analyze the variances, we look at prices from period 0, where $\tilde{\varepsilon}_1$ is not known and thus treated as random, as in Section 5. Being aware that having futures or not can change the region



This figure shows how the equilibrium changes when there is no futures market (NF scenario) for the supply shocks $(\tilde{\varepsilon}_1, \mathbb{E}_1[\tilde{\varepsilon}_2])$. The regions are labeled 1 to 4. In the presence of a futures market, the validity condition of the equilibrium is delimited by the points $\varphi'(O)$, $\varphi(A)$, $\varphi(M)$. When there is no futures market, the critical points are $\varphi_{\text{NF}}(O)$, $\varphi_{\text{NF}}(A)$ and $\varphi(M)$.

Figure 8: The validity condition of the equilibrium in two scenarios with and without a futures market.

where the equilibrium exists, for simplicity we compare the variances region by region as if the equilibrium where in the same region, whatever the scenario.

The prices in the NF scenario can be retrieved directly, or with Table 4 and the equations of Appendix B.2. In Region 1 (other regimes can be retrieved with the same method as in previous calculations):

$$P_1^{\text{NF}} = \frac{m(m + n_I^{\text{NF}} + n_P^{\text{NF}})\frac{\tilde{\varepsilon}_1}{m} + mn_I^{\text{NF}}\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} + n_I^{\text{NF}} n_P^{\text{NF}} Z}{m(m + n_I^{\text{NF}} + n_P^{\text{NF}}) + mn_I^{\text{NF}} + n_I^{\text{NF}} n_P^{\text{NF}}}, \tag{51}$$

$$P_2^{\text{NF}} = \frac{\tilde{\varepsilon}_2}{m} + \frac{mn_I^{\text{NF}}\frac{\tilde{\varepsilon}_1}{m} - ((m + n_I^{\text{NF}})n_P^{\text{NF}} + mn_I^{\text{NF}})\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} + (m + n_I^{\text{NF}})n_P^{\text{NF}} Z}{mn_I^{\text{NF}} + m(m + n_I^{\text{NF}}) + (m + n_I^{\text{NF}})n_P^{\text{NF}}}. \tag{52}$$

$$\mathbb{E}_1[P_2^{\text{NF}}] = \frac{mn_I^{\text{NF}}\frac{\tilde{\varepsilon}_1}{m} + m(m + n_I^{\text{NF}})\frac{\mathbb{E}_1[\tilde{\varepsilon}_2]}{m} + (m + n_I^{\text{NF}})n_P^{\text{NF}} Z}{mn_I^{\text{NF}} + m(m + n_I^{\text{NF}}) + (m + n_I^{\text{NF}})n_P^{\text{NF}}}. \tag{53}$$

We compare the variances in P_1 and P_1^{NF} in Region 1:

$$\text{Var}_0[P_1] = \left(\frac{m(m + (n_I + n_P)\gamma)}{m(m + n_I + (n_I + n_P)\gamma) + n_I n_P \gamma}\right)^2 \frac{\text{Var}_0[\tilde{\varepsilon}_1]}{m^2};$$

$$\text{Var}_0[P_1^{\text{NF}}] = \left(\frac{m(m + n_I^{\text{NF}} + n_P^{\text{NF}})}{m(m + 2n_I^{\text{NF}} + n_P^{\text{NF}}) + n_I^{\text{NF}} n_P^{\text{NF}}}\right)^2 \frac{\text{Var}_0[\tilde{\varepsilon}_1]}{m^2}.$$

The latter is unambiguously bigger than the former. We analyze the numerator after the reduction in the difference to the same denominator. All of the terms have the same sign. And our calculations are available on request. The futures market stabilizes the price in period 1, as in Subsection 5.1, because inventory purchases are countercyclical. In the absence of a futures market, the stabilizing effect is attenuated.

Concerning period 2:

$$\mathrm{Var}_0[P_2] = \left(\frac{mn_I}{m(m+n_I+(n_I+n_P)\gamma)+n_I n_P \gamma}\right)^2 \frac{\mathrm{Var}_0[\tilde{\varepsilon}_1]}{m^2} + \frac{\mathrm{Var}_0[\tilde{\varepsilon}_2]}{m^2};$$
$$\mathrm{Var}_0[P_2^{\mathrm{NF}}] = \left(\frac{mn_I^{\mathrm{NF}}}{m(m+2n_I^{\mathrm{NF}}+n_P^{\mathrm{NF}})+n_I^{\mathrm{NF}} n_P^{\mathrm{NF}}}\right)^2 \frac{\mathrm{Var}_0[\tilde{\varepsilon}_1]}{m^2} + \frac{\mathrm{Var}_0[\tilde{\varepsilon}_2]}{m^2}.$$

In Region 4, they are identical: due to the absence of storage, the absence of futures leaves the two periods statistically independent. In Region 2, the variance is bigger in the base scenario. This is the effect underlined in NEWBERY (1987): the facilitation of storage transports shocks from the first period to the second one. The comparison is ambiguous in Region 1, and our attempts to factorize the difference does not produce particularly interesting conditions.