Modelling sustainable development

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Sustainable development

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- representation of future generations: what will their needs be, and who speaks for them now ?
- implementation: we cannot commit future generations, and this causes immediate policy problems.

Optimal Growth

An infinitely-lived individual or a benevolent government tries to maximize intertemporal welfare

$k(t)$ **capital** at time t, $c(t)$ **consumption** at time t

$f(k)$ instantaneous **production** if capital is k

$$f(k(t)) = c(t) + \frac{dk}{dt} \quad \text{balance equation}$$

If interest rate is $\delta > 0$, and **utility of consuming** c is $u(c)$, then the problem at time $t = 0$ is:

$$\max \int_0^\infty e^{-\delta t} u(c(t))\, dt,$$

$$\frac{dk}{dt} = f(k(t)) - c(t) \text{ and } k(0) = k_0$$

There is a unique *optimal strategy* $c(t) = \sigma(k(t))$ which is given by the Hamilton-Jacobi-Bellman (HJB) equation:

$$\max_c \left\{ u(c) + (f(k) - c) V'(k) \right\} = \rho V(k)$$

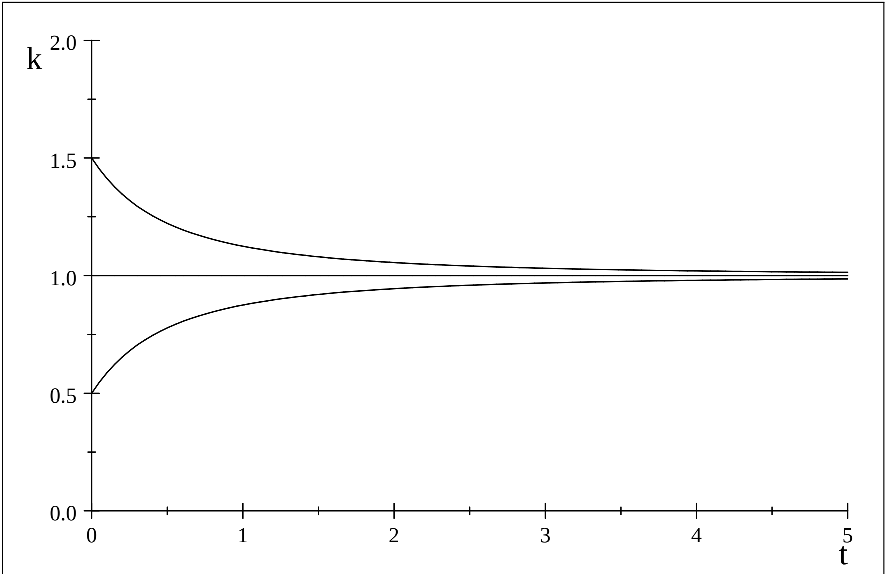
The optimal path converges to a *stationary point*

$$k(t) \longrightarrow k_\infty, \ \ c(t) \longrightarrow c_\infty = f(k_\infty) \text{ when } t \longmapsto \infty$$

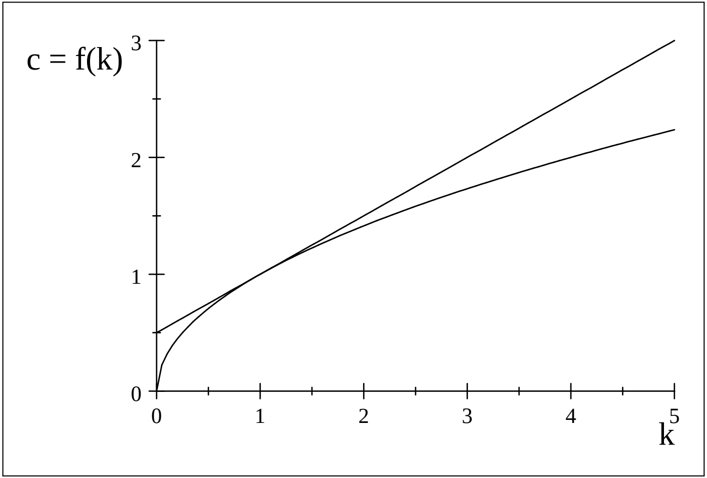
The limit k_∞ is is independent of k_0 and $u(c)$. It is characterized by

$$f'(k_\infty) = \delta$$

Convergence to the stationary state



The production function and the stationary point



$$f'(k) = \delta$$

Sustainable development

We present three different criteria, all of which share the same state equation:

$$\frac{dk}{dt} = f\left(k\left(t\right)\right) - c\left(t\right) \text{ and } k\left(0\right) = k_0$$

- Chichilnisky (1996, 1997): the C-criterion

$$\int_0^\infty e^{-\delta t} u\left(c\left(t\right)\right) dt + \alpha \lim_{t\to\infty} \tilde{u}\left(c\left(t\right)\right)$$

- Ekeland and Lazrak (2006, 2006, 2010), Ekeland and Zhou (2012): the E(r)-criterion

$$\int_0^\infty u(c(t))e^{-\delta t} dt + \alpha r \int_0^\infty \tilde{u}(c(t))e^{-rt} dt,\ r \to 0$$

- Ekeland and Zhou (2012): the H(r)-criterion

$$\int_0^\infty u(c(t))e^{-\delta t} dt + \alpha \int_0^\infty \left[\tilde{u}(c(t)) - \tilde{u}(c_\infty)\right] dt$$

Time inconsistency

All three models show time inconsistency. It is most easily seen in the first one:

$$\int_T^\infty e^{-\delta(t-T)} u(c(t))\, dt + \alpha \lim_{t\to\infty} \tilde{u}(c(t)) =$$

$$e^{\delta T} \left(\int_T^\infty e^{-\delta t} u(c(t))\, dt + e^{-\delta T} \alpha \lim_{t\to\infty} \tilde{u}(c(t)) \right) =$$

so the optimality criterion at time T is different from the optimality criterion at time 0 restricted to $[T, \infty]$.

Since the decision-maker at time T cannot commit its successors, he is facing a leader-follower game. The rational outcome is a Stackelberg equilibrium.

Equilibrium strategies

We consider only Markov strategies $c = \sigma(k)$ such that $k(t)$ converges to a limit $\bar{k}$ when $k \to \infty$

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- σ is an equilibrium strategy if the answer is yes.

The C-criterion

Theorem

The only equilibrium strategy for the C-criterion is the optimal strategy in the neo-classical optimal growth model

Proof.

The decision-maker, acting on any finite time interval, cannot influence $\lim_{t\to\infty} \tilde{u}\left(c\left(t\right), k\left(t\right)\right)$. As far as she is concerned, only the integral term is relevant □

Note also that there is no optimal solution for any decision-maker:

$$\sup_c \left\{ \begin{array}{c} \int_0^\infty e^{-\delta t} u\left(c\left(t\right), k\left(t\right)\right) dt + \alpha \lim_{t\to\infty} \tilde{u}\left(c\left(t\right), k\left(t\right)\right) \\ \frac{dk}{dt} = f\left(k\left(t\right)\right) - c\left(t\right) \text{ and } k\left(0\right) = k_0 \end{array} \right. = +\infty$$

So the C-criterion shows no concern at all for future generations, and in no way can represent sustainable development

The E(r) criterion: characterisation of equilibrium strategies

We denote by i the inverse of $\bar{u}'$, with $\bar{u} = u + \alpha\delta\tilde{u}$. Set $a = (\delta + r)/2$ and $b = (\delta - r)/2$. Here $v(k)$ is the value function.

Theorem

Suppose there is a C^1 function $w(k)$ and a C^2 function $v(k)$ satisfying the system:

$$\begin{aligned} v'(k)\left(f(k) - i\left(v'(k)\right)\right) + \bar{u}\left(i\left(v'(k)\right)\right) &= av(k) + \\ w'(k)\left(f(k) - i\left(v'(k)\right)\right) + u\left(i\left(v'(k)\right)\right) - \alpha r\tilde{u}\left(i\left(v'(k)\right)\right) &= bv(k) + \end{aligned}$$

with the boundary conditions:

$$v(k_\infty) = \delta^{-1}u(f(k_\infty)) + \alpha\tilde{u}(f(k_\infty)), \quad w(k_\infty) = \delta^{-1}u(f(k_\infty)) - \alpha\tilde{u}(f(k_\infty$$

and suppose the strategy $\sigma(k) := i(v'(k))$ converges to k_∞. Then σ is an equilibrium strategy.

The E(r) criterion: existence of equilibrium strategies

The preceding system of ODEs is in implicit form, and the first one is singular:

$$\bar{u}^* \left(v'(k)\right) + f(k)\, v'(k) = av(k) + bw(k)$$
$$\bar{u}^*(x) := \bar{u}(i(x)) - xi(x) = \max_y \{\bar{u}(y) - xy\}$$
$$\bar{u}^*(x) + f(k)\, x \geq \bar{u}(f(k)) \text{ for all } x$$

Theorem

For any $r > 0$, and any k_∞ such that:

$$\frac{\delta}{1 + \alpha\delta} < f'(k_\infty) < \delta$$

there is an equilibrium strategy for E(r) converging to k_∞

Note that, as $r \to 0$, the intervals of definition of such strategies around k_∞ shrink to 0

The H(r) criterion

For the H(r)-criterion we may conduct a similar analysis

Theorem

For any k_∞ such that:

$$f'(k_\infty) < \frac{\delta}{1+\alpha}$$

there is an equilibrium strategy converging to k_∞

Setting $f'(\bar{k}) = \delta(1+\alpha)^{-1}$, we see that every $k_\infty > \bar{k}$ is a possible sustainable equilibrium. Further analysis shows that if $\bar{k} < k_1 < k_2$, and σ_1 and σ_2 are the equilibrium strategies converging to k_1 and k_2, and starting from some k_0, there will be some time T such that switching from σ_2 to σ_1 is Pareto-improving for all $t \geq T$. In other words, only $\bar{k}$ is renegotiation-proof

A practical case: fisheries

Schaefer's model:

$$\max_h \int_0^\infty e^{-rt} (p - c(x)) h(t) \, dt$$
$$\frac{dx}{dt} = f(x) - h(t)$$

- $x(t) \geq 0$ is the fish population at time t
- $0 \leq h(t) \leq \bar{h}$ is the fishing effort
- $c(x)$ is the unit cost of fishing
- p (constant) is the unit price

Intergenerational equity (Ekeland, Karp, Sumaila)

- the population grows at the rate $g = \alpha - \omega$ and consists of identical individuals
- each generation has a pure rate of time preference δ
- each generation discounts at the rate σ the utility of future generations, evaluated at birth

The resulting discount rate is:

$$D(t) = \frac{\tilde{\delta} - \tilde{\sigma}}{\alpha + \tilde{\delta} - \tilde{\sigma}} e^{-\left(\tilde{\delta} + \omega\right)t} + \frac{\alpha}{\alpha + \tilde{\delta} - \tilde{\sigma}} e^{-(\tilde{\sigma} - \alpha + \omega)t}$$

with $\tilde{\delta} = \delta$ and $\tilde{\sigma} = \sigma$ (public good) or $\tilde{\delta} = \delta + (\alpha - \omega)$ and $\tilde{\sigma} = \sigma + 2(\alpha - \omega)$ (private good shared equally among all individuals alive). The selfish case (no concern for future generations) corresponds to $\sigma = \infty$, so:

$$D(t) = e^{-rt}, \quad r = \delta + \omega$$

The equilibrium strategy (Ekeland, Karp, Sumaila)

- In the Shaefer model, the optimal strategy is a threshold strategy: bring the population to an equilibrium level x^* defined by:

$$\delta + \omega = f'(x^*) - \frac{f'(x^*)\, c'(x^*)}{p - c(x^*)}$$

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$$\tilde{\delta} - (\alpha - \omega) = f'(x^*) - \frac{f'(x^*)\, c'(x^*)}{p - c(x^*)}$$

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- Rule: if there is any concern for future generation, replace the interest rate $\delta + \omega$ of the current generation by $\delta - (\alpha - \omega)$ (public good) or by δ (private good shared equally)

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