Mistakes in mathematics

An introduction to Spinoza

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Two memories

- A professor (1961) : Alexandre Rambaud
  - « *Si je me trompe, vous me prévenez* »
- A student (1982) : Eric Séré
  - « *Je ne comprends pas ce point* »
  - « *Mais la démonstration, je sais la faire* »

Truth as individual experience

- Truth is unique
  - *But falsehood is manifold*
- Can be reached alone
  - *Pascal, Ramanujan*
- It is a source of joy :
  - Ευρηκα !
- It is its own criterion

Ethica II, Prop. 43 *« Qui veram habet ideam, simul scit se verum habere ideam, nec de rei veritate potuit dubitare »*

Scholium *« Sane sicut lux seipsam, & tenebras manifestat, sic veritas norma sui, & falsi est ».*

Truth as social experience

- Because mathematical truth is unique, it can be shared with others

Ethica I, App « *Omnibus enim in ore est, quot capita, quot sensus, suo quemque sensu abundare, non minora cerebrorum, quam palatorum esse discrimina ... Res enim si intellexissent, illae omnes, teste Mathesis, si non allicerent, ad minimum convincerent.* »

- We need others :
  - To share the joy
  - To check the proof
- An empirical test : mathematicians do make mistakes !

Famous mistakes : Poincaré

- In January 1889, Poincaré proves that the three-body system in celestial mechanics is stable, and wins a one-time prize created by king Oskar II

- In July 1889, Phragmén, in charge of reviewing the manuscript for publication, asks for clarifications. Poincaré answers in December

- Poincaré pulls back his manuscript and sends in a new one with opposite conclusions: the system is unstable

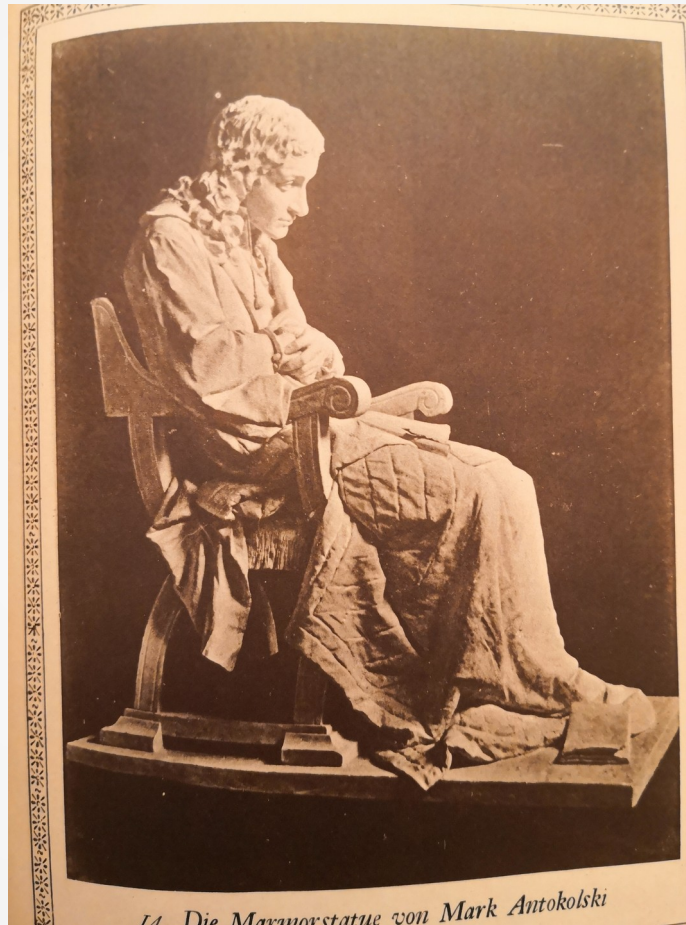- This is the birth of <mark>chaos theory</mark>

Famous mistakes : Lebesgue

- In 1905 Lebesgue proves that all subsets of the real line are Borelian. This proves the <mark>continuum hypothesis</mark> : there is no intermediate infinite between N and R

- En 1916, Suslin (1894-1916), a student of Luzin, finds a mistake in Lebesgue's paper and creates the modern descriptive theory of sets :

    - The <mark>continuum hypothesis</mark> is undecidable

What is truth ?

- Chaos theory has some relevance to the physical reality
  - Laskar has shown that the solar system is unstable : anything can happen after 400 My
  - Ethica I , Axiomata VI *« Idea vera debet cum suo ideato convenire »*
- Descriptive set theory has none :
  - Banach-Tarsky : we can cut a sphere in two pieces and reassemble them into a sphere of smaller volume
  - Ethica II Def.4 *« Per ideam adaequatam intelligo ideam, quae, quatenus in se sine relatione ad objectum consideratur, omnes verae ideae proprietates, sive denominationes intresecas habet »*
- Truth in multiple worlds : Spinoza's teaching on parallelism

From mathematics to ethics : Spinoza

14. Die Marmorstatue von Mark Antokolski

(atrr. To Spinoza) « Ich sehe an der menschlichen Bösheit vorbei, denn sie stört mich im Dienste der Gottesidee »

From mathematics to ethics : Spinoza

- Can one extend the mathematical experience to the whole of human experience ?

- Truth exists, and is unique

  - Ethica I, App « *Unde per certo statuerunt, Deorum judicia humanum captum longe longissime superare : quae sane unica fuisset causa, ut veritas humanum genus in aeternum laeteret ; nisi Mathesis, quae non circa fines, sed tantum circa figurarum essentias, & proprietates versatur, aliam veritatis normam hominibus ostendisset... »*

- The truth of an idea depends on formal considerations within a system of true ideas (*idea adaequata*)

The theory underlying the praxis

- The conditions for truth are auto-referencing and depend on an outside guarantor
  - In mathematical practice, one never writes a complete proof relating to the axioms (Russell-Whitehead)
  - The system cannot be proved to be consistent from within the system (Gödel)
- Spinoza's doctrine on <mark>the uniqueness of substance</mark> :
  - Ethica I , Prop. XIV *« Praeter Deum nulla dari, neque concipi potest, substantia»*
  - Ethica I , Prop. XV *« Quicquid est, in Deo est, & nihil sine Deo esse, neque concipi potest»*
- *Deus sive Natura* is a system : everything depends on everything else and follows from logical rules
  - At the basis of the system there may be infinitely many axioms

The praxis : freedom

- There is no natural boundary to human understanding
  - Jules Lagneau : « *Tout est plat chez Spinoza* »
  - Nothing is too difficult or too deep to understand
- The human condition is to seek knowledge, and to check the truth
  - The argument of authority (superior expertise) is not acceptable, and there is no excuse for bowing to it
  - In matters of importance (economics, social justice, climate change), one must make up one's own mind
  - Current economic theory, for instance, is a social construct with strong support in academic, business and political circles but with very little basis in reality
- Freedom is freedom of the mind

Relations with other humans

- We recognize each other in the quest for truth

  - Ethica IV, Prop. 18, Scholium « *Homini igitur nihil homine utilius* »

  - Ethica IV, Prop. 35*, « Quatenus homines ex ductu rationis vivunt, eatenus tantum naturâ semper necessariò conveniunt »*

- A basis for political theory

  - Ethica IV, Prop. 36 « *Summum bonum eorum, qui virtutem sectantur, omnibus commune est, eoque omnes aeque gaudere possunt* »

  - Tractatus Theologico-Politicus Caput XX, « *ergo sit, ut illud imperium violentum habeatur, quod in animos est et … quando unicuique praescribere vult, quid tamquam verum amplecti et tanquam falsum rejicere* »

Relation with Nature

- There is no duality body/mind humans/nature in Spinoza
  - Tout est plat chez Spinoza
  - There is only one substance, Deus sive Natura, and we are part of it
  - *« Quicquid est, in Deo est, & nihil sine Deo esse, neque concipi potest»*
- There is no hierarchy either, no special place for humans
  - Tout est plat chez Spinoza,
  - Nature does not serve humans, nor any particular aim
  - Ethica I, App *« si Deus propter finem agit, aliquid necessariò appetit, quo caret»*
  - Ethica I, App *« rerum perfection ex ex solâ earum naturâ, & potentiâ est aestimanda, nec ideò res magis, aut minus perfectae sunt, propterea quod hominum sensum delectant vel offendunt, quod humanae naturae conducunt, vel quod eidem repugnant »*

Spinoza today

- The criterion of a good life is joy and friendship
  - Ethica V , Prop 42, « *Beatitudo non est praemium virtutis, sed ipsa virtus»*
- The relation with Nature is one of inclusion and love

  - Ethica V Prop. 36, « *Mentis Amor Intellectualis erga Deum est ipse Dei Amor, quo Deus se ipsum amat, non quatenus infinitus est, sed quatenus per essentiam humanae Mentis, sub specie aeternitatis consideratam, explicari potest, hoc est, Mentis erga Deum Amor Intellectualis pars est infiniti amoris, quo Deus se ipsum amat»*

- Spinoza is a philosopher for our time (Naess and deep ecology°

Spinoza tomorrow

- The end of civilization (nuclear war, climate change, chemical impregnation, loss of biodiversity)

  - Guicciardini, Ricordi, 189 *« Tutte le città, tutti li stati, tutti i regni sono mortali ; ogni cosa o per natura o per accidente termine e finisce qualche volta. Pero uno cittadino che si truove alla fine della sua patria, non puo tanto dolersi della disgrazia di quella e chiamarla mal fortunata, quanto della sua propria : perche alla patria e accaduto quelle che a ogni modo aveva a accadere, ma disgrazia è stata di colui abattersi a nascere a quella età che aveva a essere tale infortunio »*

- Is it still worth doing mathematics ? Or philosophy ?

- What would Spinoza have said