Storers, processors and speculators

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- What happens if one introduces a financial market alongside the physical market ?
- What happens if speculators (money managers) are allowed into that new market ?
- What happens if agents are risk-averse ?

The markets

One commodity to be traded between dates t and $t+1$.
There is a physical market and a futures market open at each date.

- The physical market is a spot market:
  - corn trades at price p_t for immediate delivery
  - all positions are long
- The futures market is a financial market
  - each contract is bought at price f_t, and sold at price p_{t+1}
  - short positions are allowed

Interest rate is $r \geq 0$

Information structure

Let z_t be the physical quantity available for trading at time t (once contracts from period $t-1$ have been settled)
At time t,

- z_t, p_t, f_t are common knowledge
- agents make their decisions conditional on z_t, p_t, f_t
- denote $\mathrm{E}_t\left[X_{t+1}\right] = \mathrm{E}_t\left[X_{t+1} \middle| z_t, p_t, f_t\right]$ and similarly for $\mathrm{Var}_t\left[X_{t+1}\right]$

Market participants

At time t

- Storers buy quantity x_t at time t (at price p_t) and sell it at time $t+1$ (at price p_{t+1}).
- Processors commit to buy y_t on the spot market at time $t+1$, process it and sell the finished product at price Q
- Storers, processors and speculators buy quantities q_t^I, q_t^P and q_t^S of contracts at price f_t

Demand

Market demand:

- Storers, processors and speculators are all mean-variance.
- Type i maximises $\mathrm{E}_t\left[X_{t+1}\right] - \alpha_i \mathrm{Var}_t\left[X_{t+1}\right]$ where X_{t+1} is the profit and $i = I, P, S$

Residual demand:

- There are other uses for the commodity, and traders coming from other markets will want to buy it. There is also free disposal, and if a threshold price at which there is an unlimited supply of a substitute commodity. We have $z = D(p)$, with:

$$D(p) = \begin{cases} [M,\ \infty) & \text{if} \quad p = 0 \\ M - mp & \text{if} \quad 0 \leq p \leq Mm^{-1} \\ (-\infty, 0] & \text{if} \quad p = Mm^{-1} \end{cases}$$

Individual behaviour

Speculators

$$q^{\star}_{S,t} = (1+r)\,\frac{\mathrm{E}_t[p_{t+1}] - (1+r)\,f_t}{\alpha_S \mathrm{Var}_t[p_{t+1}]}$$

Storers (cost of storage is $\frac{\beta}{2}x^2$)

$$x^{\star}_t = \frac{1}{\beta}\max\{f_t - p_t, 0\}, \quad q^{\star}_{I,t} = (1+r)\,\frac{\mathrm{E}_t[p_{t+1}] - (1+r)\,f_t}{\alpha_I \mathrm{Var}_t[p_{t+1}]} - x^{\star}_t$$

Processors (cost of production is $\frac{\delta}{2}y^2$)

$$y^{*}_t = \frac{1}{\delta}\max\{Q - f_t, 0\}, \quad q^{\star}_{P,t} = (1+r)\,\frac{\mathrm{E}_t[p_{t+1}] - (1+r)\,f_t}{\alpha_P \mathrm{Var}_t[p_{t+1}]} + y^{*}_t$$

Note that the physical position is fully hedged, and does not reflect the risk aversion ! Already noted by Anderson-Danthime

Spot market

Only storers and residual traders are active. We get, with $n_I := N_I \delta^{-1}$

$$z_t = n_I \max\{f_t - p_t, 0\} + M - mp_t \quad \text{if} \quad 0 \leq p_t \leq Mm^{-1}$$
$$z_t \geq n_I \max\{f_t - p_t, 0\} + M \quad \text{if} \quad p_t = 0$$
$$z_t \leq n_I \max\{f_t - p_t, 0\} \quad \text{if} \quad p_t = Mm^{-1}$$

Note that n_I is also the elasticity of total demand wrt f_t, which opens the way to callibration

Futures market

Storers, processors and speculators are active. The hedging pressure is, with $n_P := (\beta Q)^{-1}$:

$$h_t := n_I \max\{f_t - p_t, 0\} - n_P \max\{Q - f_t, 0\}$$

It turns out that the bias $(1+r)^{-1}\mathrm{E}_t[p_{t+1}] - f_t$ is proportional to the hedging pressure:

$$\frac{\mathrm{E}_t[p_{t+1}]}{1+r} - f_t = \frac{\alpha \mathrm{Var}_t[p_{t+1}]}{(1+r)^2} h_t$$

$$\alpha := \left(\frac{n_I}{\alpha_I} + \frac{n_P}{\alpha_P} + \frac{n_S}{\alpha_S}\right)^{-1}$$

- $(n_P - n_I)$ is the elasticity of the hedging pressure wrt f_t and $-n_I$ its elasticity wrt p_t,
- the presence of risk aversion creates a bias : $\mathrm{E}_t[p_{t+1}] \neq (1+r)\, f_t$

Two dates

$t = 1, 2$

Closing the model

There are three markets: spot at $t = 1$, spot at $t = 2$, and futures
At $t = 1$, the only input is the crop $z_1 = \omega_1$. The equilibrium equation becomes:

$$\omega_1 = n_I \max\{f - p_1, 0\} + M - mp_1 \tag{1}$$

At $t = 2$, we have $z_2 = \tilde{\omega}_2 + n_I x_1$. and storers no longer are in operation. The equilibrium equation becomes:

$$\tilde{\omega}_2 + n_I \max\{f - p_1, 0\} = M - m\tilde{p}_2 \tag{2}$$

In the futures market (take $r = 0$), the equilibrium equation is:

$$\frac{\mathrm{E}[\tilde{p}_2] - f}{\alpha \mathrm{Var}[\tilde{p}_2]} = n_I \max\{f - p_1, 0\} - n_P \max\{Q - f, 0\} \tag{3}$$

Three equations for p_1, f and $\tilde{p}_2$

Reducing the system

We can derive $\mathrm{E}[p_2]$ and $\mathrm{Var}[p_2]$ from the second equation. The other two become:

$$\begin{aligned} mp_1 - n_I \max\{f - p_1, 0\} &= M - \omega_1 \\ mf + \gamma\left(n_I \max\{f - p_1, 0\} - n_P \max\{Q - f, 0\}\right) &= M - \mathrm{E}[\tilde{\omega}_2] \end{aligned}$$

with

$$\gamma = 1 + \frac{1}{m} \frac{\mathrm{Var}[\tilde{\omega}_2]}{\frac{n_P}{\alpha_P} + \frac{n_I}{\alpha_I} + \frac{n_S}{\alpha_S}}$$

We have two (nonlinear) equations for two unknown scalars p_1 and f

The four regions

The right-hand side is always the same:

$$M - \omega_1$$
$$M - \mathrm{E}[\tilde{\omega}_2]$$

The left-hand side is a piecewise linear function:

- The plane (p_1, f) is devided into four regions

	$f - p_1 \geq 0$	$f - p_1 \leq 0$
$Q - f \geq 0$	(1)	(3)
$Q - f \leq 0$	(2)	(4)

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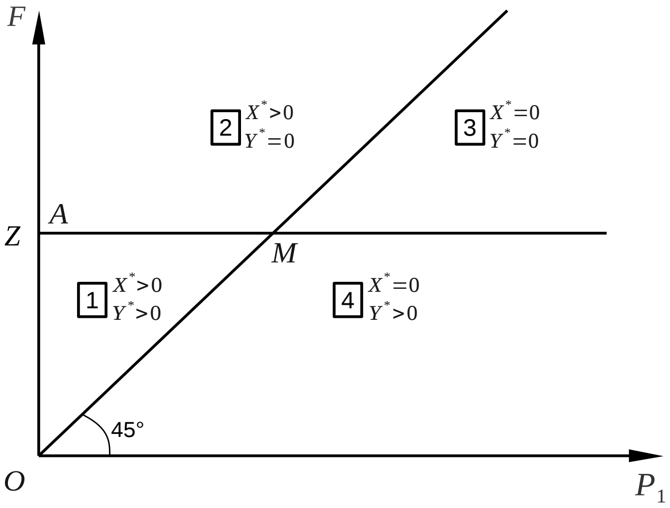
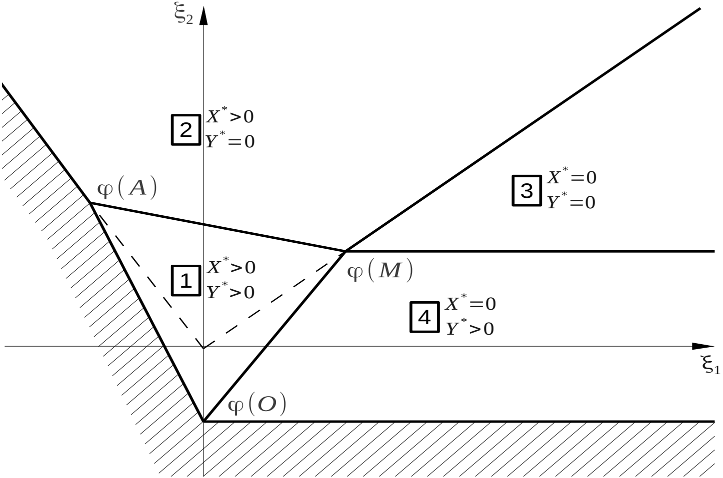
$$\begin{array}{ccc} & f - p_1 \geq 0 & f - p_1 \leq 0 \\ Q - f \geq 0 & (1) & (3) \\ Q - f \leq 0 & (2) & (4) \end{array}$$

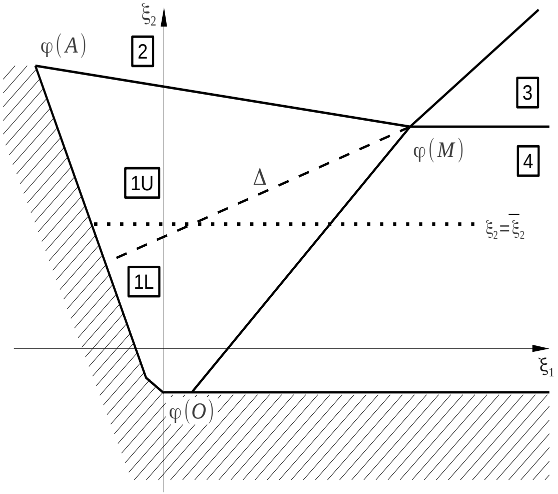
- In each of the region, the left-hand side is linear. In region 1 for instance, it is the linear map

$$\begin{pmatrix} p_1 \\ f \end{pmatrix} \to \begin{pmatrix} mp_1 - n_I (f - p_1) \\ mf + \gamma (n_I (f - p_1) - n_P (Q - f)) \end{pmatrix}$$


F
2 $X^*>0$ $Y^*=0$
3 $X^*=0$ $Y^*=0$
A
Z
M
1 $X^*>0$ $Y^*>0$
4 $X^*=0$ $Y^*>0$
45°
O
P_1



ξ2
2
X*>0
Y*=0
3
X*=0
Y*=0
φ(A)
1
X*>0
Y*>0
φ(M)
4
X*=0
Y*>0
ξ1
φ(O)


ξ_2
$\varphi(A)$
2
3
$\varphi(M)$
4
1U
Δ
$\xi_2 = \bar{\xi}_2$
1L
ξ_1
$\varphi(O)$

2	$P_1 < F$	$F < E[\tilde{P}_2]$	$F > Z$
	$X^\star > 0$	$f_S > 0$	$Y^\star = 0$
1U	$P_1 < F$	$F < E[\tilde{P}_2]$	$F < Z$
	$X^\star > 0$	$f_S > 0$	$Y^\star > 0$
Δ	$P_1 < F$	$F = E[\tilde{P}_2]$	$F < Z$
	$X^\star > 0$	$f_S = 0$	$Y^\star > 0$
1L	$P_1 < F$	$F > E[\tilde{P}_2]$	$F < Z$
	$X^\star > 0$	$f_S < 0$	$Y^\star > 0$
4	$P_1 > F$	$F > E[\tilde{P}_2]$	$F < Z$
	$X^\star = 0$	$f_S < 0$	$Y^\star > 0$
3	$P_1 > F$	$F = E[\tilde{P}_2]$	$F > Z$
	$X^\star = 0$	$f_S = 0$	$Y^\star = 0$

Table 1: Relations between prices and physical and financial positions. In reference to figures, regions are listed counter-clockwise.

	$\lvert E[\tilde{P}_2] - \tilde{F} \rvert$	$\tilde{F}$	$\tilde{X}^\star$	$\tilde{Y}^\star$	$\tilde{P}_1$	$\tilde{P}_2$	$\text{Var}[\tilde{F}]$	$\text{Var}[\tilde{P}_1]$	$\text{Var}[\tilde{P}_2]$	
2	$\searrow$	$\nearrow$	$\nearrow$	0	$\nearrow$	$\searrow$	$\searrow$	$\searrow$	$\nearrow$	$E[\tilde{P}_2] - \tilde{F} > 0$
1U	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$	$\searrow$	$\searrow$	$\nearrow$	
1L	$\searrow$	$\searrow$	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\tilde{F} - E[\tilde{P}_2] > 0$
4	$\searrow$	$\searrow$	0	$\nearrow$	$\longleftrightarrow$	$\nearrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	
3	$\longleftrightarrow$	$\longleftrightarrow$	0	0	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\tilde{F} = E[\tilde{P}_2]$

Table 2: Impact of speculators on prices and quantities. Legend: $\nearrow$ variable increases; $\searrow$ variable decreases; 0 variable is null; $\longleftrightarrow$ no impact on variable.

Particular cases

- if one or more of the participants is risk-neutral , so that $\alpha_i = 0$ for some i, then $\gamma = 1$
- if there is no storage cost , so that $\delta = 0$ and $n_I = \infty$, then $\gamma = 1$
- if there is no production cost , so that $\gamma = 0$ and $n_P = \infty$, then $\gamma = 1$
- if there is no financial market , the physical positions become:

$$x^{\star}_{\mathrm{NF}} = \left(\delta + \alpha_I \frac{\mathrm{Var}[\tilde{\omega}_2]}{m^2}\right)^{-1} \max\{\mathrm{E}[\tilde{p}_2] - p_1, 0\}$$
$$y^{\star}_{\mathrm{NF}} = \left(\beta Q + \alpha_P \frac{\mathrm{Var}[\tilde{\omega}_2]}{m^2}\right)^{-1} \max\{Q - \mathrm{E}[\tilde{p}_2], 0\}.$$

Contrary to what happens in the presence of a financial market, the physical positions now reflect the risk aversion of the participants

- nevertheless, in all the preceding cases we still get the four regions

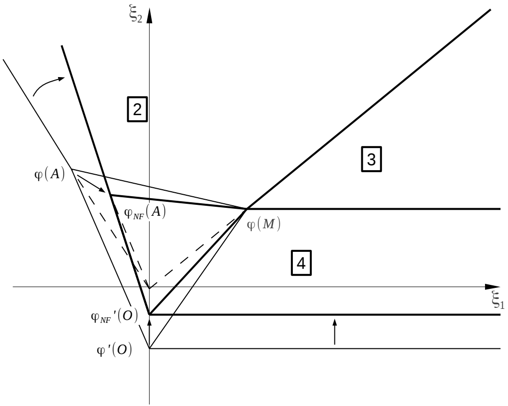


Figure 9: Existence conditions: comparison between with and without futures market.

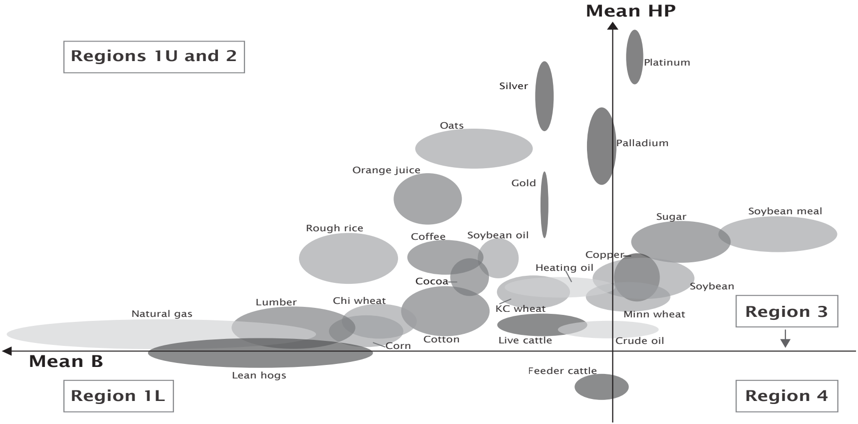


Figure 7: Map of commodity markets, according to Kang et al. (2014) and to our analysis.

Extension 1 (M. Isleimeyyeh)

Two physical markets connected by a financial market.

- there is a complete (physical+financial) market for commodity a and a complete market for commodity b, with specialized storers, processors and speculators.
- speculators become unspecialized: they trades futures on both commodities

Speculator's optimal position becomes:

$$f_S^{a*} = \left(\frac{1}{\left(1 - corr^2\left(\tilde{P}_2^a, \tilde{P}_2^b\right)\right)} \right) \left[f_{SELV}^{a*} - corr\left(\tilde{P}_2^a, \tilde{P}_2^b\right) \frac{\sigma_2^b}{\sigma_2^a} f_{SELV}^{b*} \right]$$

Extension 2: (with E.Jaeck)

HIgh Frequency Trading The financial market opens at the intermediate time $t = 1.5$, and everyone can access it

- If the all agents share the same horizon, i.e. they want to optimize their profit at time $t = 2$, then no trade occurs at the intermediate time and the prices at time $t = 1$ are unaffected by the new possibility
- If the speculators have a shorter horizon, i.e. if the speculators present at $t = 1$ leave at $t = 1.5$ and are replaced by new ones, who will leave at $t = 2$, then physical positions and spot prices are unaffected, but financial positions and futures prices are affected

Extension 3 (with M. Isleimeyyeh)

Speculators are hedgers too.

The futures contracts speculators hold are part of a larger portfolio. Their total profit from investing k in a financial index and f in the commodity futures is:

$$k\left(V_2 - V_1\right) + f\left(P_2 - P_1\right)$$

The optimal position of the speculator on the futures market becomes:

$$f_S^* = \frac{E(\tilde{P}_2) - F}{\alpha_S \sigma_P^2}\left(\frac{1}{1-\rho^2}\right) - \rho\frac{E(\tilde{V}_2) - V_1}{\sigma_P \sigma_V \alpha_S (1-\rho^2)} \tag{4}$$

Markov strategies

$t = 1, 2, ...$

(with E. JAECK)

Operating indefinitely

The markets are open for all $t \geq 1$, so that available quantity is:

$$z_t = \omega_t + h_{t-1}$$

and the operators optimize their short-term profit (from t to $t+1$). The equations are:

$$z_t = n_I \max\{f_t - p_t, 0\} + M - mp_t$$

$$\frac{\mathrm{E}_t[p_{t+1}] - f_t}{\alpha \mathrm{Var}_t[p_{t+1}]} = n_I \max\{f_t - p_t, 0\} - n_P \max\{Q - f_t, 0\}$$

Note the presence of the expectations $\mathrm{E}_t[p_{t+1}]$ and $\mathrm{Var}_t[p_{t+1}]$. Agents have short-term objectives but are sophisticated: they have to factor in their own behaviour at time $t+1$. To decide what to do today I need to know what I will do tomorrow

Equilibrium equations

Replace $\mathrm{E}_t[p_{t+1}]$ by e and $\mathrm{Var}_t[p_{t+1}] \geq 0$. Replace also f_t by f and p_t by p. The equations become:

$$z = n_I \max\{f - p, 0\} + M - mp_t$$
$$e - f = \alpha v \left[n_I \max\{f - p, 0\} - n_P \max\{Q - f, 0\}\right]$$

There is a single solution for p and f, given in terms of the coefficients (z, e, v) by functions P and F which can be computed explicitly. The hedging pressure can be computed from P and F:

$$p = P(z, e, v), \quad f = F(z, e, v)$$
$$h = n_I \max\{f - p, 0\} - n_P \max\{Q - f, 0\} = H(z, e, v)$$

By definition, we have:

$$p_t = P\left(z_t, \mathrm{E}\left[p_{t+1}\right], \mathrm{Var}\left[p_{t+1}\right]\right)$$

Price today is a (known) function of the (unknown) anticipations

Finding the functions P and H

- As in the two-dates case, we find there are four regions

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- They are separated by four half-lines emanating from the point $\begin{pmatrix} M - mQ \\ (1+r)\, Q \end{pmatrix}$

	slope	direction
$\mathcal{D}_{12}$	$\frac{\alpha v}{1+r} \frac{n_I}{m+n_I}$	N-E
$\mathcal{D}_{23}$	$-\frac{1+r}{m}$	N-W
$\mathcal{D}_{34}$	0	W
$\mathcal{D}_{41}$	$-\frac{1}{m}\left(1+r+\frac{\alpha v}{1+r} n_P\right)$	S-E

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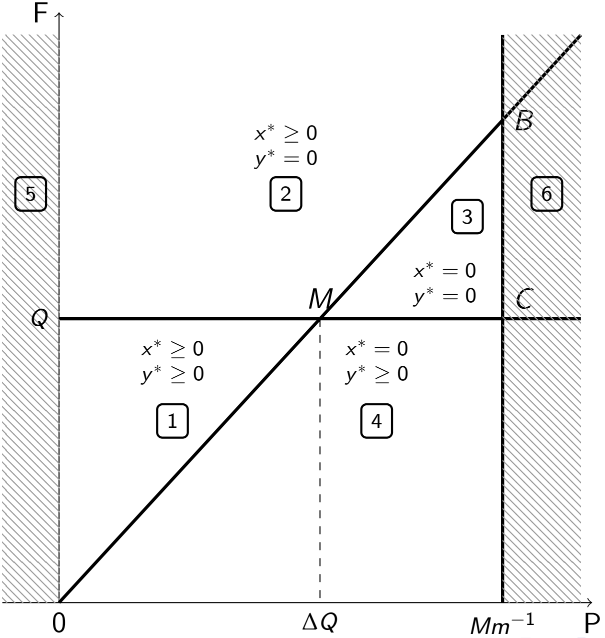
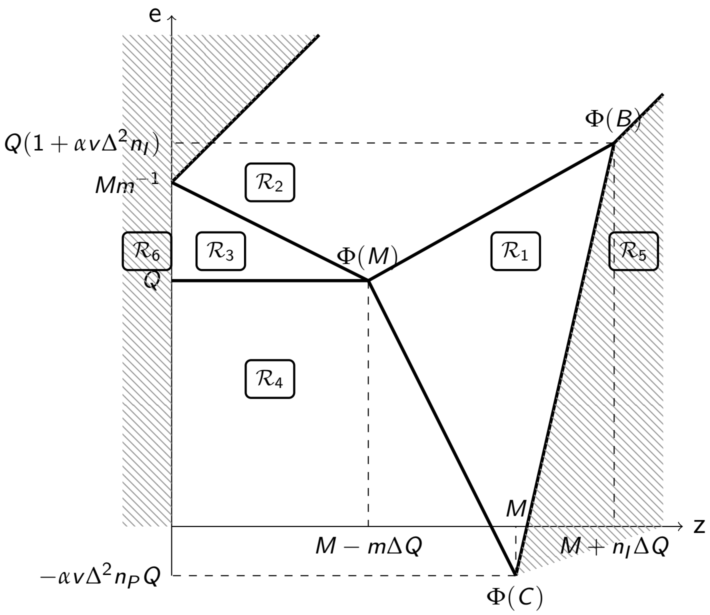
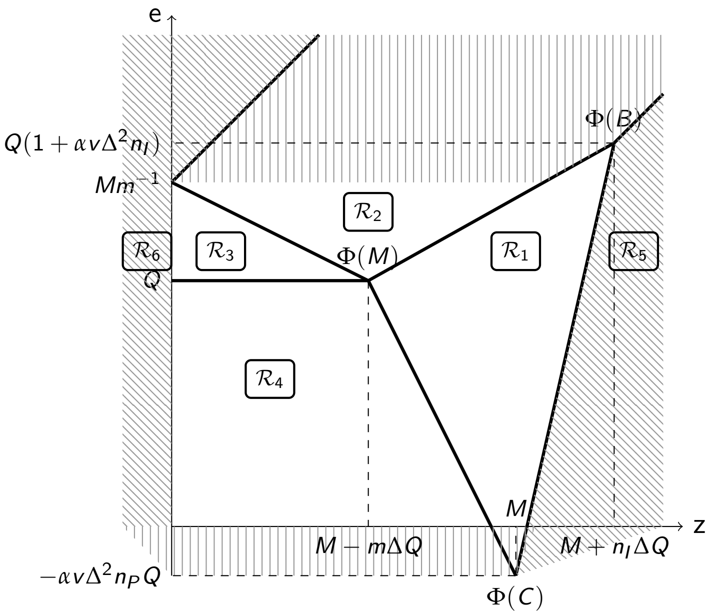
- Two more regions are added to take into account the constraints $P \geq 0$ and $P \leq Mm^{-1}$


F
P
0
ΔQ
Mm^{-1}
Q
M
B
C
x* ≥ 0
y* = 0
2
x* = 0
y* = 0
3
x* ≥ 0
y* ≥ 0
1
x* = 0
y* ≥ 0
4
5
6



e
z
$Q(1+\alpha v\Delta^2 n_I)$
Mm^{-1}
Q
$-\alpha v\Delta^2 n_P Q$
$\mathcal{R}_1$
$\mathcal{R}_2$
$\mathcal{R}_3$
$\mathcal{R}_4$
$\mathcal{R}_5$
$\mathcal{R}_6$
$\Phi(B)$
$\Phi(M)$
$\Phi(C)$
M
$M - m\Delta Q$
$M + n_I\Delta Q$


e
z
$Q(1+\alpha v\Delta^2 n_I)$
$\Phi(B)$
Mm^{-1}
$\mathcal{R}_2$
$\mathcal{R}_6$
$\mathcal{R}_3$
$\Phi(M)$
$\mathcal{R}_1$
$\mathcal{R}_5$
Q
$\mathcal{R}_4$
M
$M - m\Delta Q$
$M + n_I\Delta Q$
$-\alpha v\Delta^2 n_P Q$
$\Phi(C)$

Finding the functions P and H

In Region 1, where everyone is active, we have:

$$P(z,e,v) = \frac{\left[1+(n_I+n_P)\frac{\alpha v}{(1+r)^2}\right](M-z)+n_I\left[\frac{e}{1+r}+\frac{\alpha v}{(1+r)^2}n_P Q\right]}{m+n_I+\frac{\alpha v}{(1+r)^2}(mn_I+n_In_P+mn_P)}$$

$$H(z,e,v) = \frac{(mn_I+mn_P+n_In_P)\frac{e}{1+r}-n_P(m+n_I)Q-n_I(M-z)}{(m+n_I)+\frac{\alpha v}{(1+r)^2}(n_Pm+n_Pn_I+n_Im)}$$

Particular cases

If one of the participants is risk-neutral, if there is no cost of storage or if there is no cost of production, then $\alpha = 0$ and the equations simplify:

	$H(z,e,v)$	$P(z,e,v)$
$\mathcal{R}_1$	$\frac{(mn_I+mn_P+n_In_P)\frac{e}{1+r}-n_P(m+n_I)Q-n_I(M-z)}{m+n_I}$	$\frac{(M-z)+n_I\frac{e}{1+r}}{m+n_I}$
$\mathcal{R}_2$	$\frac{zn_I+\frac{e}{1+r}mn_I-Mn_I}{m+n_I}$	$\frac{(M-z)+n_I\frac{e}{1+r}}{m+n_I}$
$\mathcal{R}_3$	0	$\frac{M-z}{m}$
$\mathcal{R}_4$	$n_P\left(\frac{e}{1+r}-Q\right)$	$\frac{M-z}{m}$

Rational anticipations

We are interested in solving

$$p_t = P\left(z_t, \mathrm{E}\left[p_{t+1}\right], \mathrm{Var}\left[p_{t+1}\right]\right)$$
$$z_{t+1} = \omega_{t+1} + H\left(z_t, \mathrm{E}\left[p_{t+1}\right], \mathrm{Var}\left[p_{t+1}\right]\right)$$

To do that, we will seek the anticipations as Markovian functions of the available supply

$$\mathrm{E}\left[p_{t+1}\right] = E\left(z_t\right) \text{ and } \mathrm{Var}\left[p_{t+1}\right] = V\left(z_t\right)$$

Note that we then get a Markovian strategy for p_t:

$$p_t = P\left(z_t, E\left(z_t\right), V\left(z_t\right)\right) = \sigma\left(z_t\right)$$

Rational expectations

We need the expectations to be coherent with the strategy:

$$
\begin{aligned}
E(z) &= \mathrm{E}\left[P\left(z', E(z'), V(z')\right)\right] \\
V(z) &= \mathrm{Var}\left[P\left(z', E(z'), V(z')\right)\right] \\
z' &= H\left(z, E(z), V(z)\right) + \omega
\end{aligned}
$$

This defines the functions $E(z)$ and $V(z)$ as the solutions of a fixed-point problem

Theoretical results

- For every $r \geq 0$ there is a fixed point with $E(z)$ and $V(z)$ measurable and bounded
- For $r \geq 0$ small enough, there is a unique fixed point with $E(z)$ and $V(z)$ continuous and bounded
- Numerically, we have yet to find a case when $E(z)$ and $V(z)$ are not continuous

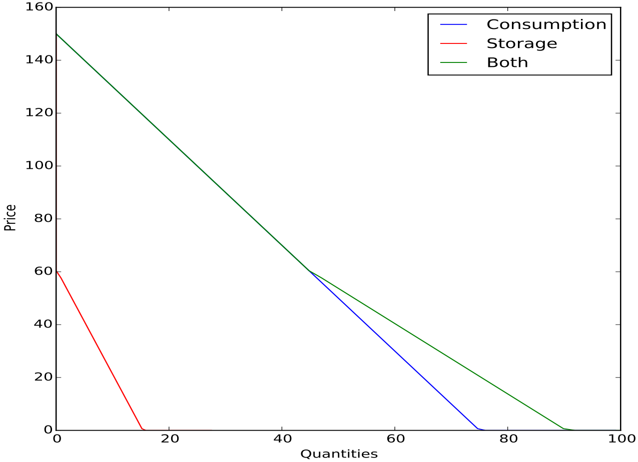
Numerical results

The algorithm is as follows:

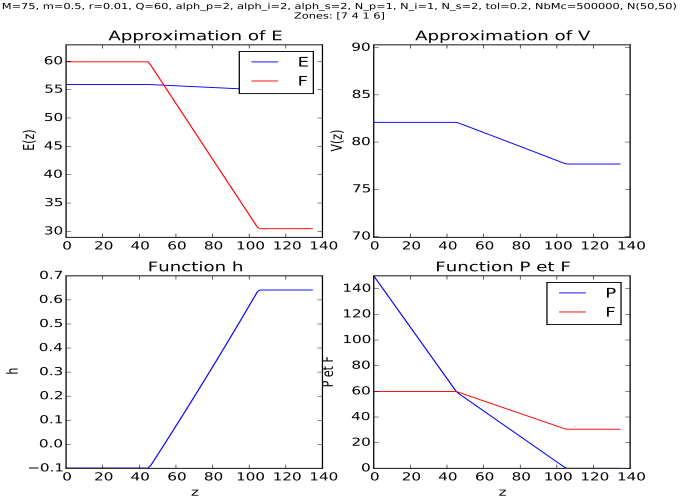
$$E_{n+1}(z) = \mathrm{E}\left[P\left(z', E_n(z'), V_n(z')\right)\right]$$
$$V_{n+1}(z) = \mathrm{Var}\left[P\left(z', E_n(z'), V_n(z')\right)\right]$$
$$z' = H\left(z, E_n(z), V_n(z)\right) + \omega$$

The law of ω is Gaussian. The law of z' is then a translate of the law of ω, and the expectation and variance of the random variable $P\left(z', E_n(z'), V_n(z')\right)$ is easily computed. The algorithm turns out to be very stable and to converge for all $r \geq 0$

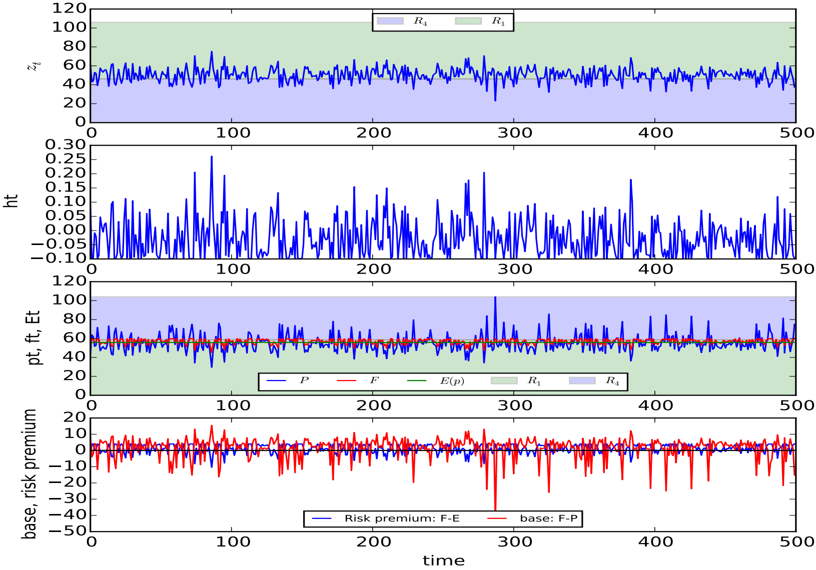
Demand for consumption and for storage



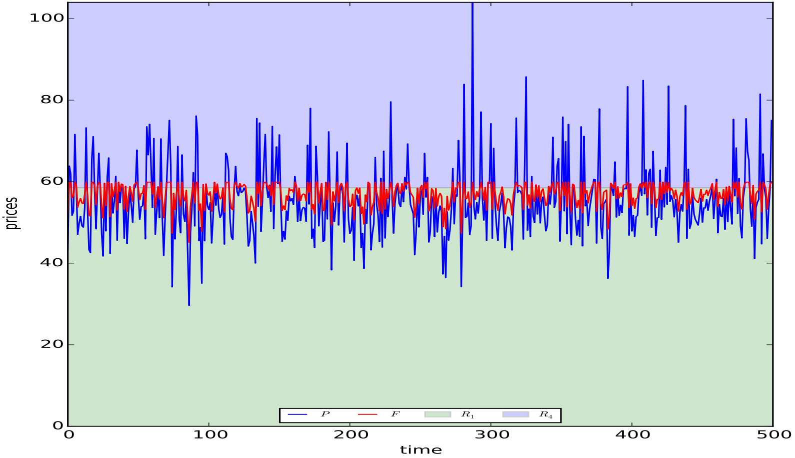
Functions at the equilibrium



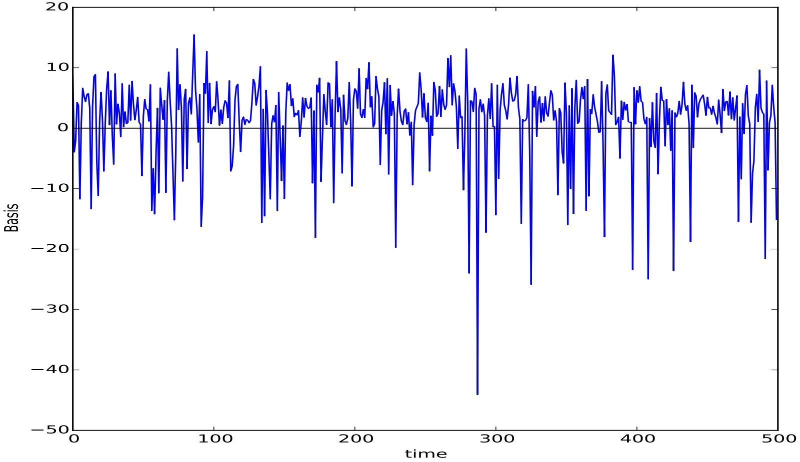
One simulated path



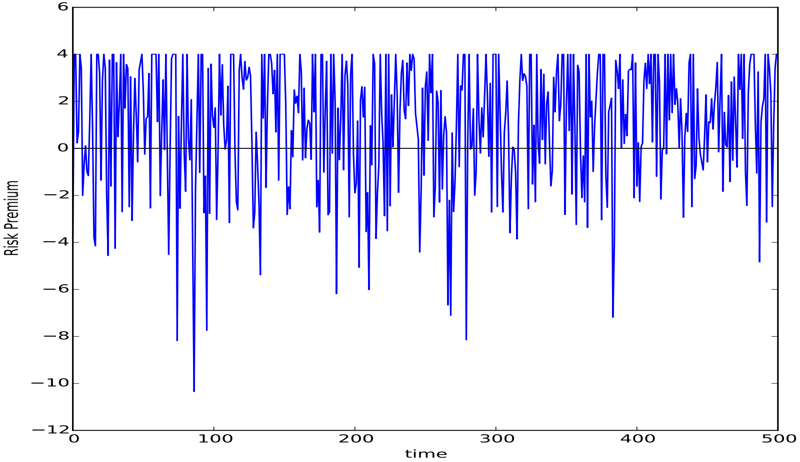
One simulated path: Prices



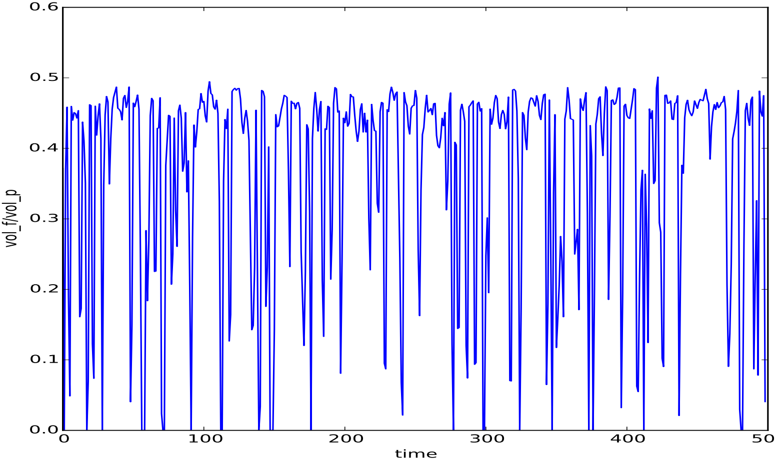
One simulated path: Basis



One simulated path: Risk premium



One simulated path: Ratio of volatilities



Conclusion

- We have a simple model which reproduces certain stylised facts
  - the Samuelson effect: futures prices are less volatile than spot prices
  - the presence of stocks dampens price movements
- It leads to certain conclusions:
  - there is a bias $f_t - E\left[p_{t+1}\right]$ which is proportional to the hedging pressure
  - in the presence of a financial market, the physical positions do not reflect the risk aversion of participants
  - the presence of speculators is beneficial to the dominant positions in the physical markets and detrimental to the others
- It contains as a particular case the Deaton-Laroque, Scheinkman-Schechtman and de Roon models
- It is versatile enough to accomodate more complex situations, where markets influence each other
- It is open to calibration