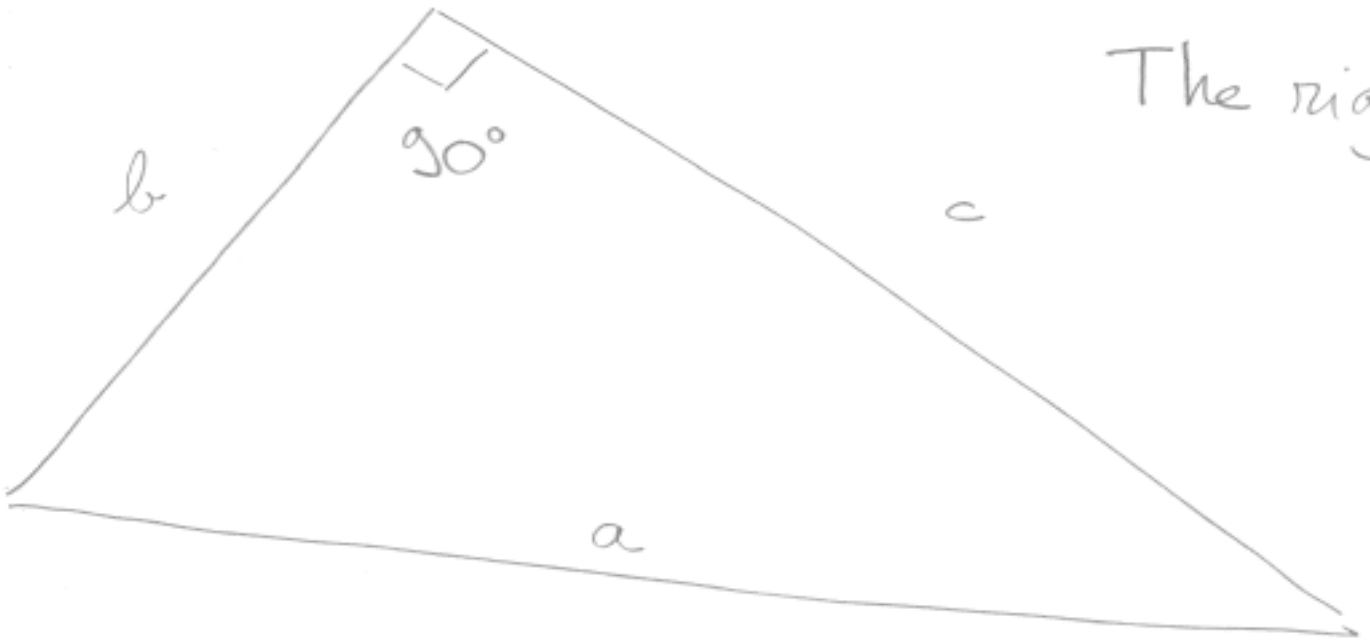
What is the Earth ?

The Earth is a disk

The world in 500 BCE

PYTHAGORAS
(570 - 495 BCE)



The right triangle

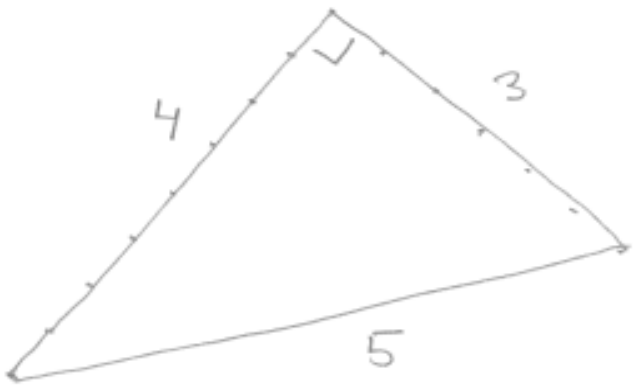
$$a^2 = b^2 + c^2$$

NUMBERS and GEOMETRY

How to construct a right angle

$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$



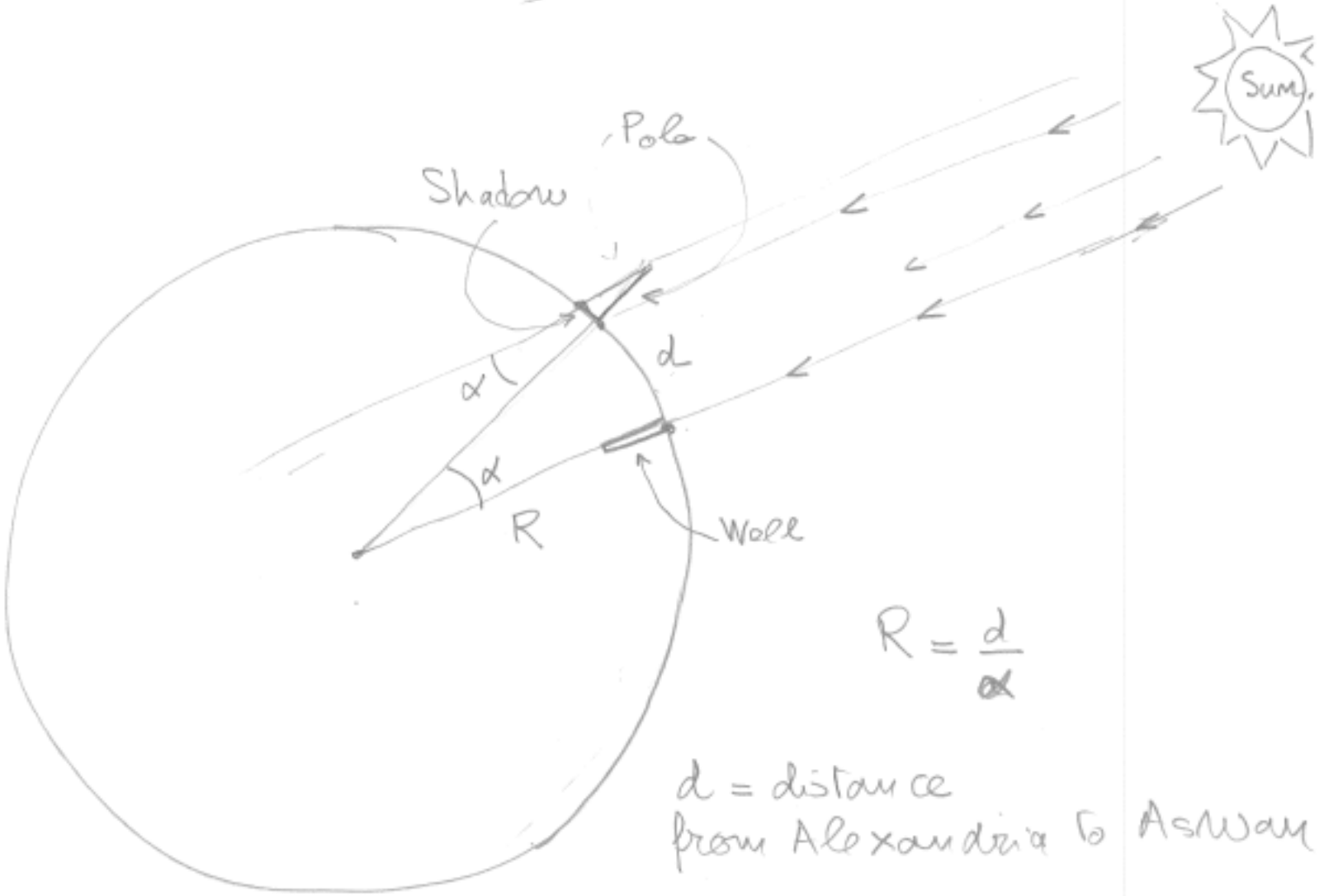
The Earth is a globe

Dürer, 1515


TRASKIA
SEPTENTRIO
AQVILO
CORVS
CAECIAS
FAVONIVS
SVBSOLANVS
AFRICVS
VVLTVRNVS
LIBONOTVS
EVROVSTER
AVSTER
EVROPA
ASIA
AFRICA
Indicum Mare
Editum ex Tabula ligned ab Alberto Durer incisa, quae in Bibliotheca Aug. Vindob. extat. M. DCC. LXXXI.

The blue marble

The Earth according to Eratosthenes



$$R = \frac{d}{\alpha}$$

d = distance from Alexandria to Aswan

α = angle of shadow

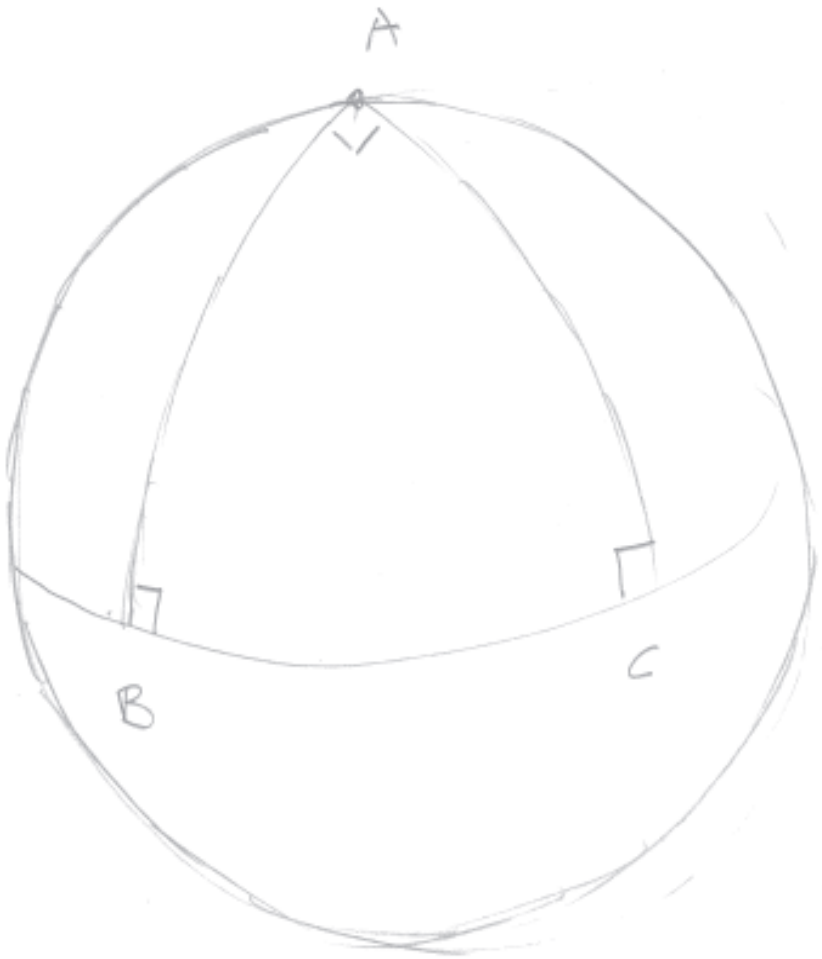
What is a model ?

- A model is a partial representation of reality, devised to a certain end, and capturing only the features that are relevant to this end
- A model can be material (wood, metal) or not: Eratosthenes' model of the Earth is purely mathematical
- A good model is versatile, and can be put to other ends than the one it was invented for

Mathematics of the globe: geodesy

- Non-euclidian geometry
  - The sum of the angles of a triangle is greater than 180 degrees
- The shape of the Earth
  - Triangulating the globe
- Gauss, the prince of mathematicians
  - The bell curve, the law of large numbers and the central limit theorem

Non-Euclidean geometry



A triangle with three right angles

$\hat{A} + \hat{B} + \hat{C} = 270^\circ$

The Earth is a planet

The journey to the Moon

A roaming planet

- From a finite world to an infinite universe
  - Ptolemy's start-studded room
  - Galileo's infinite universe
- The mathematics of space-time
  - The pendulum and the monocular
  - Separation of space and time
  - Newton's law of motion: $F = m\gamma$

Determinism

- Newton and the power of genius
- Descartes, Leibniz, Lagrange and the power of computations
- Predicting eclipses, discovering planets. The return of Halley's comet
- The emergence of determinism:
  - The present contains all the information about the future
  - The future can be predicted by applying physical laws and computing

Chaos

- Is the solar system stable ?
- Poincaré (1890) shows that determinism fails: the answer cannot be given by computations
- The reason is that the relevant information cannot be extracted from the present state of the world
- This is the butterfly effect

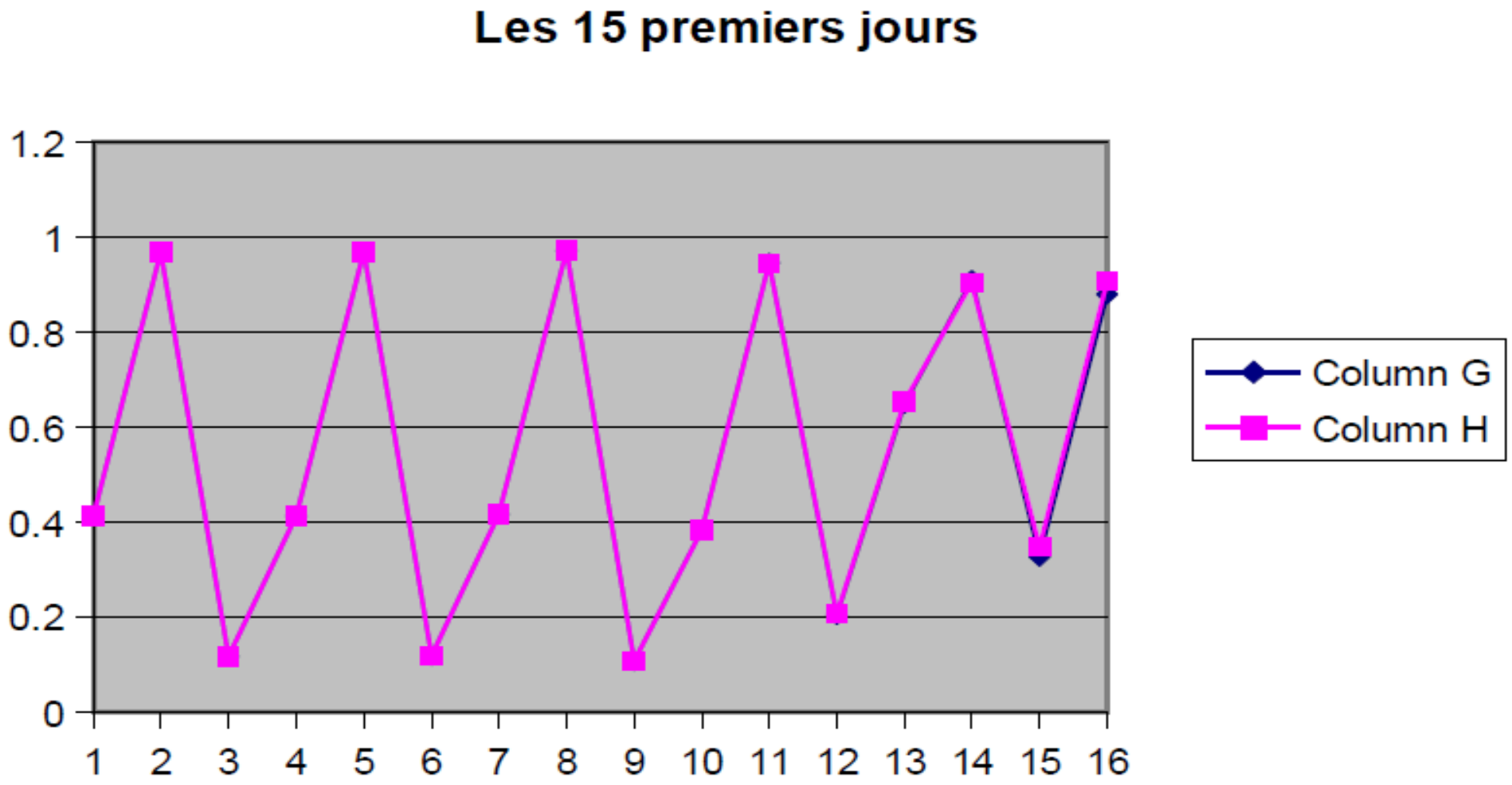
$$x_{n+1} = 4x_n(1-x_n)$$

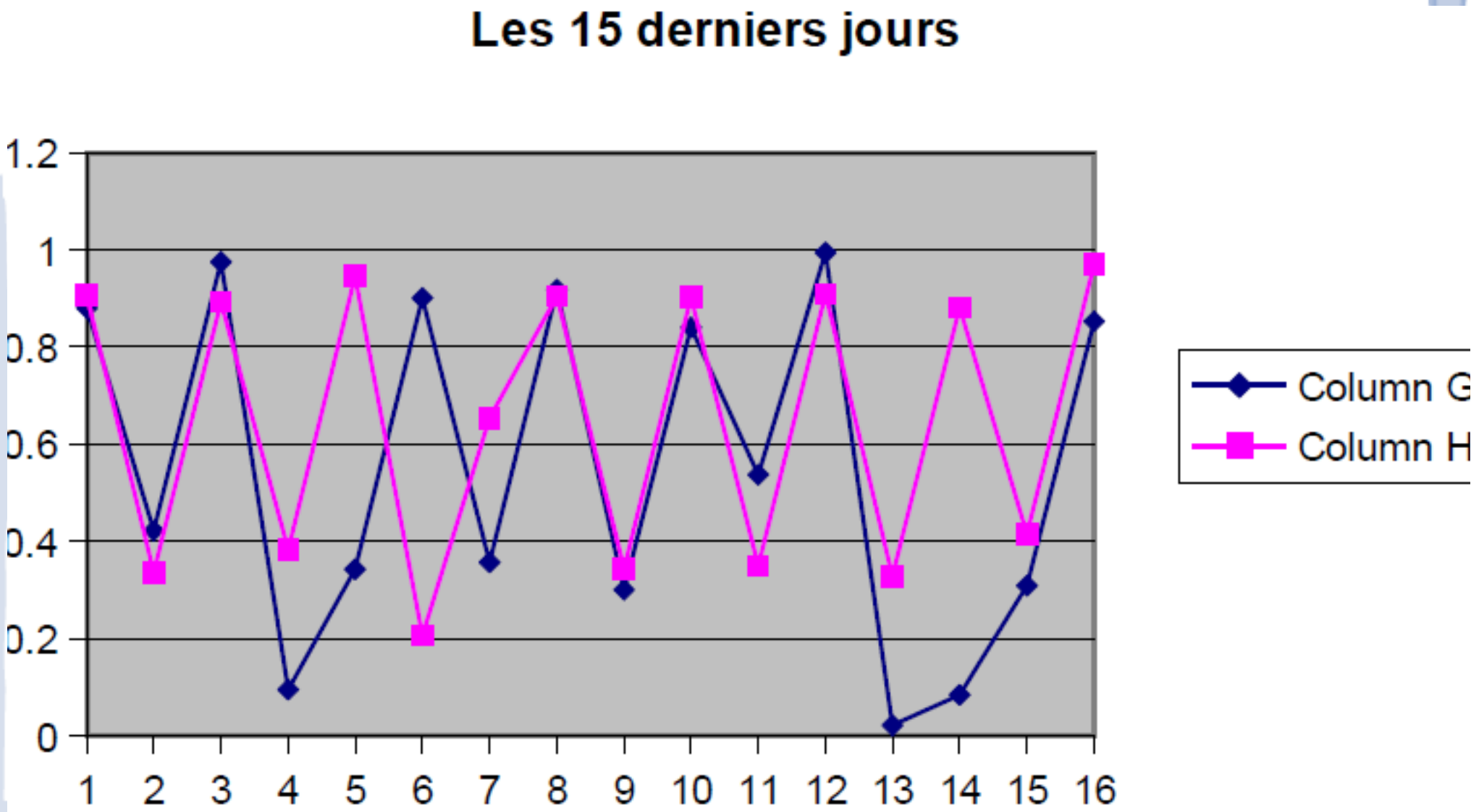
- This is a simple model for a physical law
- Knowing x_n one can compute $x_{n+1,}$ then $x_{n+2,}$ and so forth.
- However, small discrepancies (rounding errors, measurement accuracy) are multiplied by 10 between n and n+4
- Between n and n+42 they are multiplied by 10^{12}
- If initial information consists of 12 digits, it is lost after 42 iterations

Effect of a roundoff error:

X= 0.116996564757943

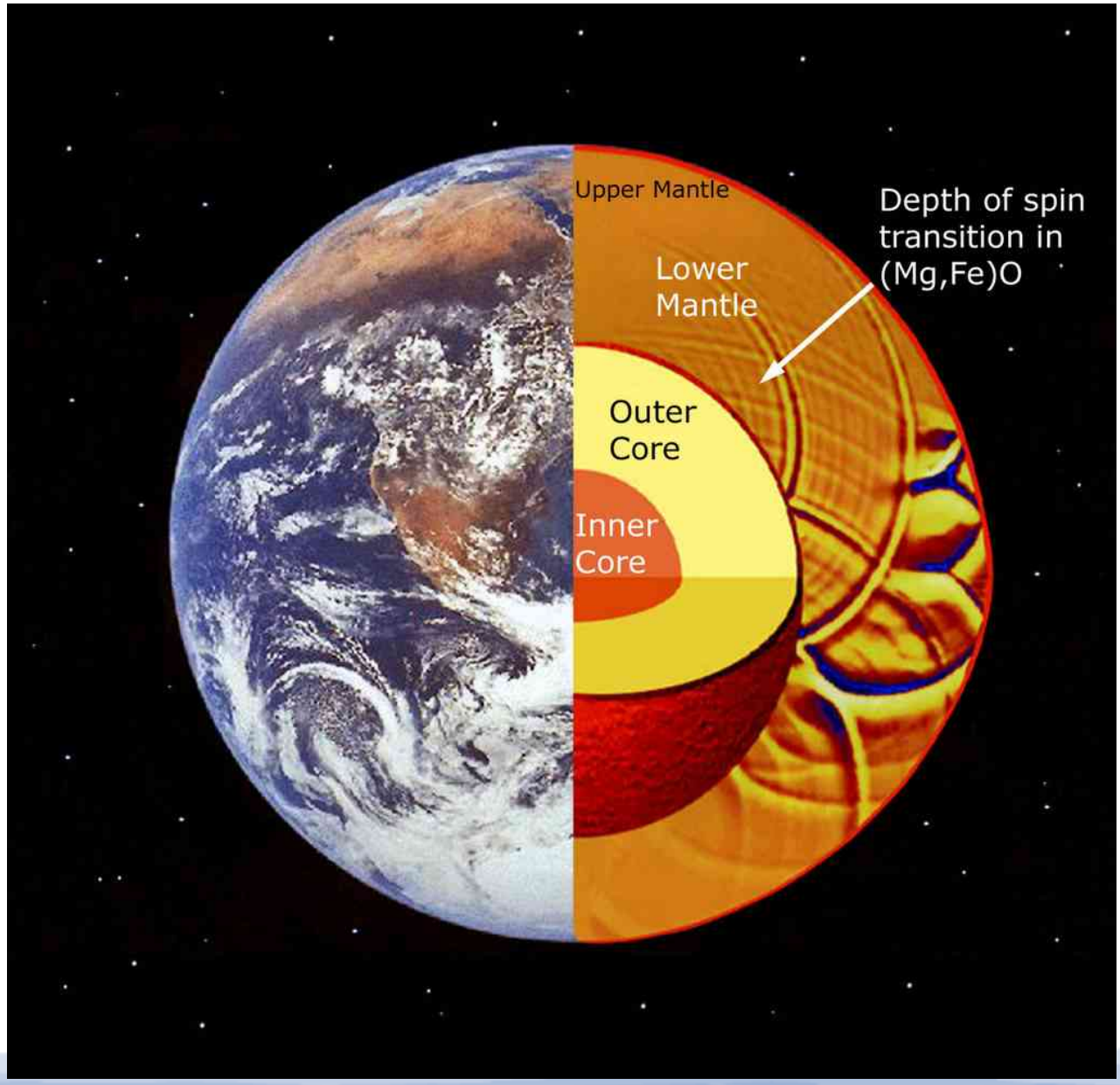
or X= 0.116997 (30ppm)




Les 15 derniers jours
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Column G
Column H

The Earth is a rotating fluid

From the kernel to the atmosphere



The lower layers

- The discovery of deep time in geology (Hutton, 1785) and biology (Darwin, 1859)
- At these scales, the Earth is a stratified fluid sphere (kernel, mantle, crust), heated by radioactivity and driven by currents (convection, rotation)
- The continental drift,
- Extreme events: volcanism and earthquakes

The upper layers

- The atmosphere (100 km, but ¾ in the first 10)
- The ocean (70% of the surface, depth 3 km)
- These layers are moving on shorter time scales than the lower ones
- Ranging from the day (clouds) to millenia (ice caps, Gulf Stream)

Fluid mechanics

- The mathematical tools: systems of partial differential equations
  - Euler (perfect fluid)
  - Navier-Stokes (viscous fluid)
  - Boltzmann (rarefied gases)
  - Incredible variety of situations according to physical properties of fluids (compressibility, mixtures, chemical reactions) and external forces (electromagnetism, heating)

The age of the computer

- Computations are beyond human capability
  - A (very)simple model of the atmosphere would require a grid of 100 km on the surface: that is 51,000 squares
  - Adding three points on the vertical, that is 150,000 cubes on which you have to solve systems of coupled PDES
  - And incorporate new data as it comes in
- From flops to petaflops (10^{15})

Limits of prediction

- The problem is no longer with computing power, it is with data acquisition:
  - In some systems (weather) the state is complex (many variables) and there are many equations: the curse of dimensionality
  - In other systems (celestial mechanics) the state is simple and there are few equations: the butterly curse
  - Sometimes both difficulties combine (weather)

Living with the curses

- One runs the same program many times with slight changes in the inputs
  - the scenario approach
  - the probabilistic approach
- Celestial mechanics:
  - the orbit of Venus may cross the orbit of Earth
  - the solar system is likely to become unstable after 400 million years
- Climate change
  - The Gulf Stream may disappear
  - The average temperature is likely to increase by 6 degrees by 2100 (International Energy Agency)

Thank you
To be continued !