GONZALO CONTRERAS

KUPKA-SMALE thm for geodesic flows:

① Can make generic the k-Jets of Poincaré maps of closed geodesics:

(a) Klingenberg - Takens for perturbations of <u>each</u> periodic orbit.

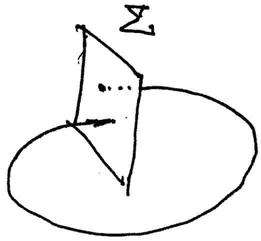
(b) Needs Anosov for the 1-Jet (to make periodic orbits of similar period <u>isolated</u>)

<u>Formally</u>:

$J_s^k(n) :=$ k-Jets of smooth symplectic maps $(\mathbb{R}^n, 0) \circlearrowleft$

$Q \subset J_s^k(n)$ is <u>invariant</u> iff

$\forall \sigma \in J_s^k(n), \quad \sigma Q \sigma^{-1} = Q$



<u>OBS</u>: Q invariant $\Rightarrow$ Property "Poincaré map $\in Q$" is independent of the section Σ.

<u>Theorem</u>:

If $Q \subset J_s^k(n)$ is open, dense & invariant.

$\Rightarrow \forall r \geq k+1 \ \exists \mathcal{G} \subset \mathcal{R}^k(M)$ residual s.t. $\forall g \in \mathcal{G}$.

① The Poincaré maps of all periodic orbits of ϕ^g are in Q.

② All heteroclinic intersections are transversal.

OBS:
(a) Also holds for Q residual and invariant
(b) Donnay for $n=2$, Petroll $n \geq 2$ show how to perturb a single non-transverse intersection. But perhaps this is not enough.

② Transversality of invariant manifolds

(a) Hamilton-Jacobi thm:

For a hamiltonian H if $\mathbb{L}$ is a Lagrangian submfld s.t. $H(\mathbb{L}) = \text{const.}$ $\Rightarrow$ $\mathbb{L}$ is invariant.

Γ. The ham. v. f. $X \in \mathbb{L}$ because if not it can be added to $T\mathbb{L}$ to make a bigger isotropic subspace $\not\Leftarrow$

$$\ell \in T\mathbb{L}, \quad \omega(X, \ell) = - dH(\ell) = 0 \quad /\!/\!/$$

- W^s is Lagrangian in (T^*M, ω_0)

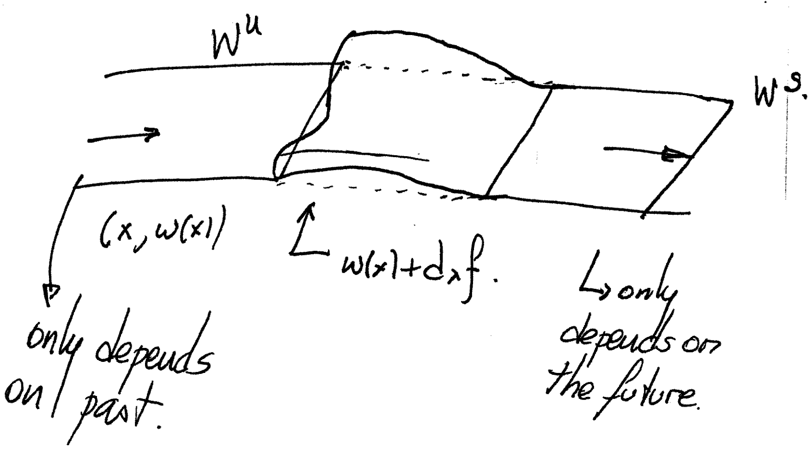
Γ $\omega_0(d\phi_s u, d\phi_s v) = \omega_0(u, v)$ $\therefore$ isotropic + dim. $\perp$

$\downarrow s \to +\infty$
0

-

- Choose a place where W^s is locally a lagrangian graph
  $\Rightarrow$ $W^s \approx \{(x, \eta(x)) \mid \eta$ closed 1-form on $U \subset M\}$.
- Deform to another lagrangian graph which is $\pitchfork W^u$ (by adding a $d_x f$).

$\omega_0 = dp \wedge dx$ fixed canonical symplectic form on T^*M.



- Change the metric s.t. $H(\text{new } W^s) = 1$

⌈ recall $H(x;p) = \frac{1}{2}\sum p_i\, g^{ij}(x)\, p_j$
$[g^{ij}(x)] = [g_{ij}(x)]^{-1}$ ↳ riem. metric on TM ⌋

$\Rightarrow$ new W^s is invariant