Lectures on Doubles Resonances

Vadim Kaloshin & Ke Zhang

Institut Henri Poincaré, Paris, December 12-16, 2016

Lecture 1: Single and double resonances, Invariant cylinders, Aubry-Mather type sets, Bernards forcing We state the main result about existence of diffusion along a predetermined diffusion path Γ and decompose Γ into two regimes: single and double resonant. At each regime the proof will consist of three parts:

- Construct 2-dimensional (normally hyperbolic) invariant cylinders C_js along the path.
- Show that each Γ_j carries a family of "minimal invariant sets of Aubry-Mather type.
- Prove that one can shadow these sets using Mather variational method and Bernards forcing relation.

Lecture 2: Invariant cylinders near double resonance, Kissing property. An heuristic mechanism of diffusion Dynamics at a double resonance is well approximated by a mechanical system of two degrees of freedom. We distinguish two regimes: low and high energy. Given a resonance we determine an integer homology h and show existence of a collection of (normally hyperbolic) invariant cylinders Λ_ks. Based on existence of these invariant cylinders we show one scenario of how diffusion in a double resonance can occur.

Lecture 3: Bernards forcing relation, the jump lemma, diffusion mechanism We introduce dynamics in the space of pseudo graphs proposed by Fathi and forcing relation between cohomology classes proposed by Bernard. We prove a jump lemma allows to switch form one resonance to another in "one jump. Time permitting we state discuss a genericity condition by Mather-Cheng-Yan, crucial to construct diffusion along single resonance and high energy double resonance.

Lecture 4: Diffusion along a single normally hyperbolic invariant cylinder We discuss the mechanism for diffusion along a single normally hyperbolic invariant cylinder, and more importantly, verify that the conditions imposed holds for generic Hamiltonians. We choose to present the ideas of the proof using the single resonance setting, but will point out the part of the proof which allows generalization.

Lecture 5: Genericity conditions We finish the discussion in the previous lecture in the single resonance setting. We then make the necessary adaptations to allow the same mechanism to work for double resonance, when the energy of the slow system is high.