Is there a pilot in the shuttle?!

No leading principle beyond the orbital decomposition

$$\begin{aligned}\int_\Gamma q_s(x)d\lambda(x) &= \int_{\Gamma/\mathcal{G}} \int_{\mathcal{G}} q_s(gx)d\nu(g)\,d\lambda(x)\\ &= \int_{\Gamma/\mathcal{G}} |\mathcal{G}|\,\bar{q}_s(x)d\lambda(x)\end{aligned}$$

$\boxed{\textbf{Is this at all related to Monte Carlo integration???}}$

A flawed generality

- Does not take into account the cost (a) of selecting the group $\mathcal{G}$ and (b) of computing the average $\bar{q}_s$
- The estimation problem, as stated, does not have an optimal solution
- Variance computations are not necessarily adequate
- No allusion to the more relevant $(n_1, \ldots, n_k)$ design

Confusion between decision theory and theory of algorithms

Obvious gain in replacing q_s with $\bar{q}_s$ has no lower bound:

Example – Evaluation of

$$c_\sigma = \int_\Gamma \frac{dx_1\, dx_2}{\{x_1^2 + (x_2+\sigma)^2\}^2} = \frac{\pi}{4\sigma^2}$$

or rather c_σ / c_1.

Reparameterisation on $[0,1]^2$:

$$c_\sigma = \int_{[0,1]^2} \frac{1}{\{x_1^2 + (x_2+\sigma)^2\}^2} \frac{dy_1\, dy_2}{y_1(1-y_1)(1-y_2)^2}$$

$$[x_1 = \log y_1/(1-y_1),\; x_2 = y_2/(1-y_2)]$$

The Jacobian matters in the symmetric representation!!!

The $\mathfrak{G}_n$ group

Action of $\mathfrak{G}_n$ on $(x_1, x_2) \in [0,1]^2$: change any (or none or all) of the first n bits of x_1 and x_2

$$[0,1]^2/\mathfrak{G}_n = ([0, 2^{-n}[)^2$$

[Size of $\mathfrak{G}_n$, 2^{2n}]

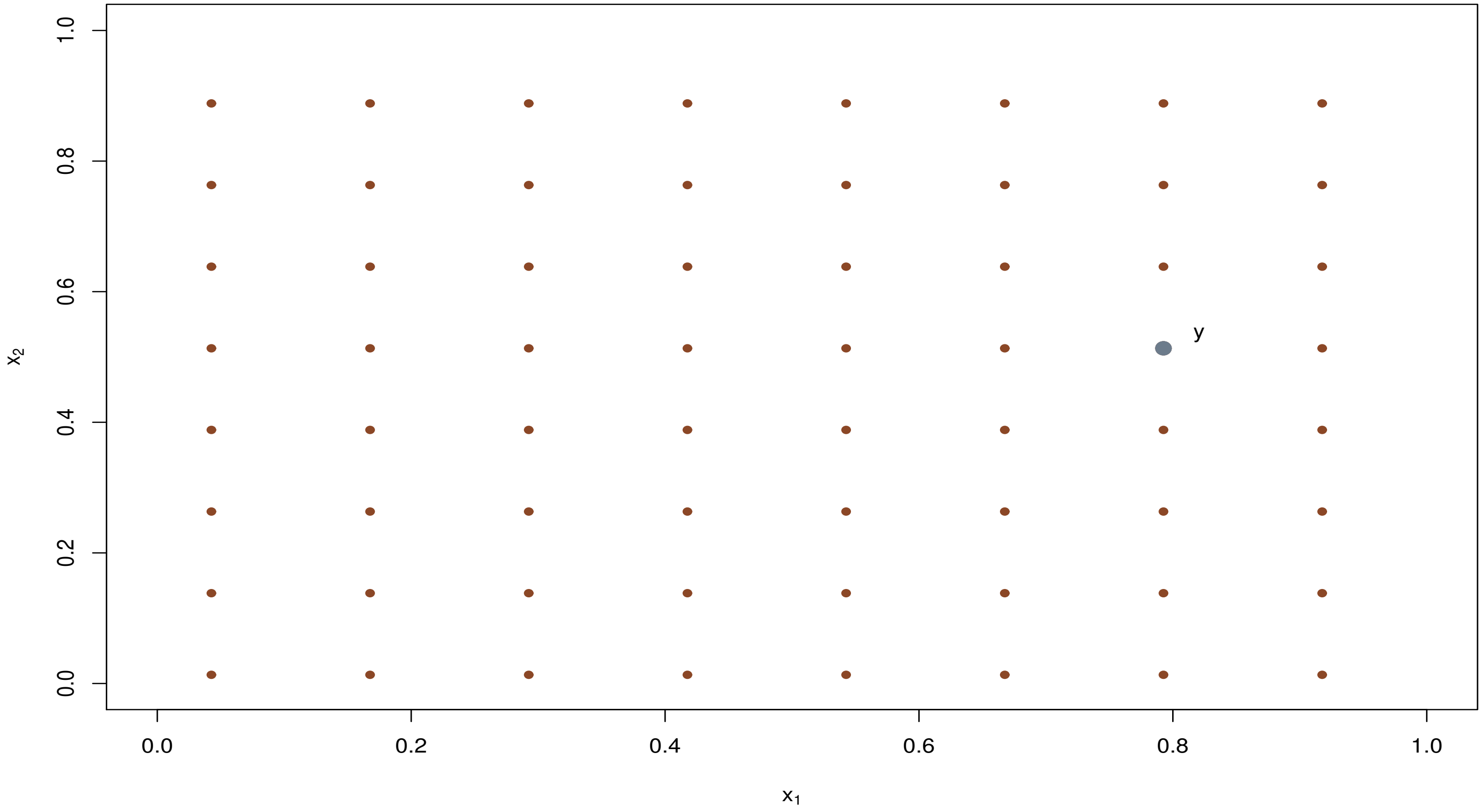

Figure 1: $\mathfrak{S}_3$ action on y

An ultimate boostrap???

- Works with a **single** simulation!
- Improvement increases with n
- Possible infinite improvement

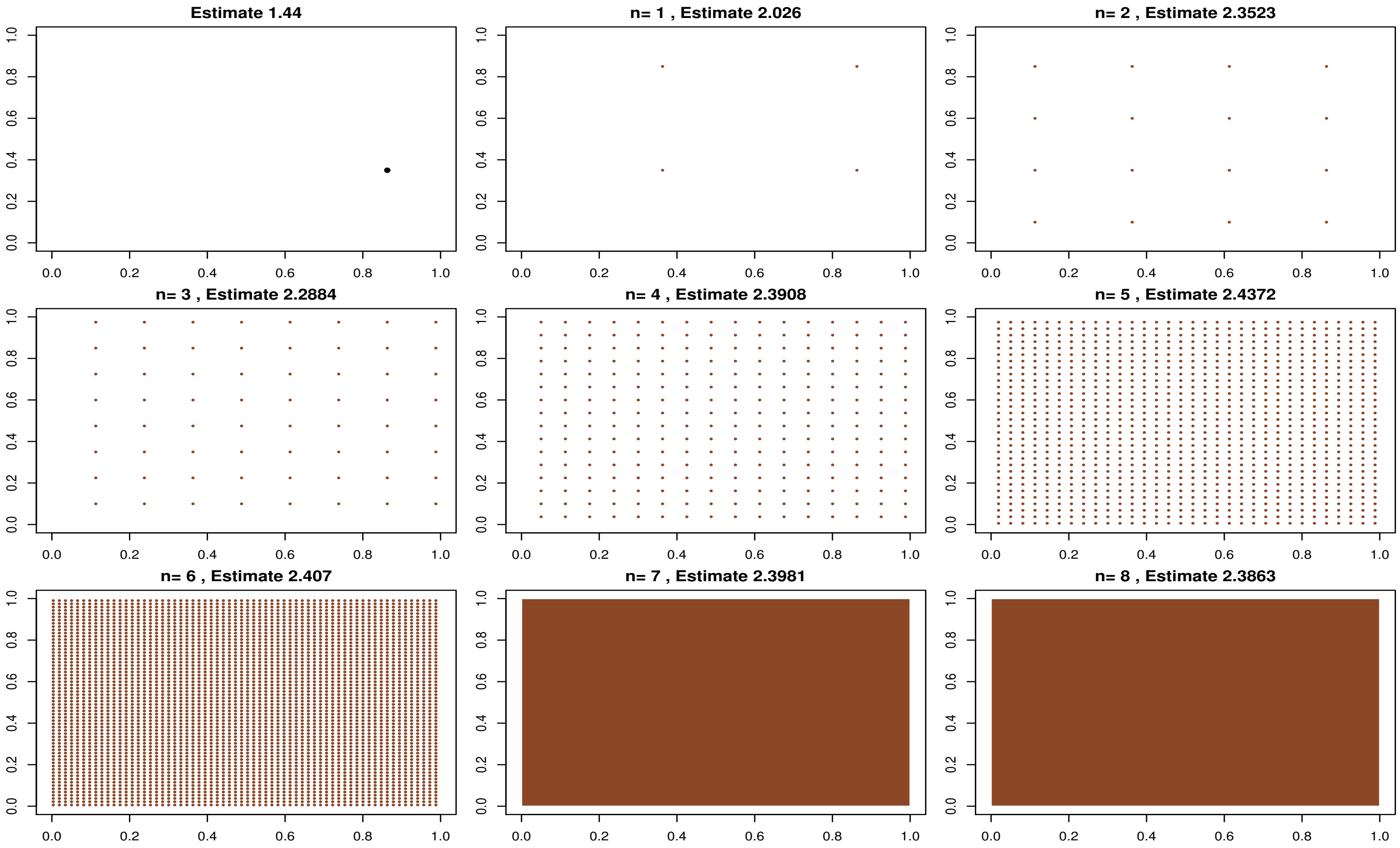

Figure 2: Successive actions of $\mathfrak{G}_n$ and estimates of $1/\sigma^2 = 2.38$

Is there a result in the paper?!

- Finite support reasoning for continuous support settings
- If the group action is unrelated to the problem/integrand, it may bring no visible improvement
- No strong validity of the Fisher variance approximation

$$\hat{c} \sim \mathcal{N}(c, \hat{I}(c))$$

- E.g., valid for $\log c_\sigma$'s but not for c_σ's, since most IS estimators have infinite variance (Slutsky's lemma?)

If $q_s(gx) = 0$ for most g's in $\mathcal{G}$, little gain and large expense!

Example – Evaluation of

$$c_\sigma = \int \exp\{-[(x_1 - 5)^2 + (x_2 + 10)^2]/2\sigma^2\} dx_1 \, dx_2 = 2\pi \, \sigma^2$$

Use of the $\mathfrak{S}_n$ group after reparameterisation to $[0, 1]^2$

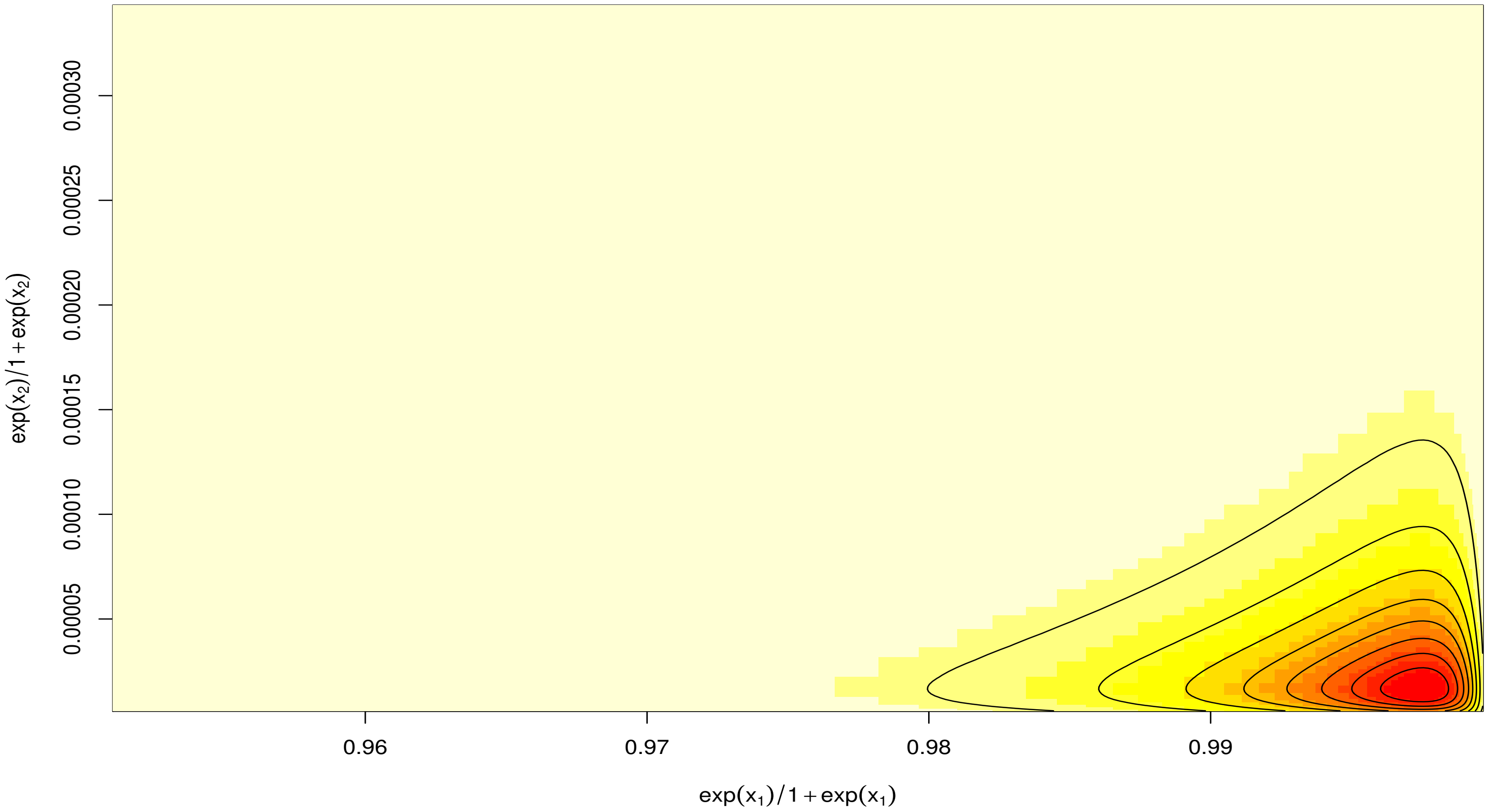

Figure 3: Level set representation of the transform of $\mathcal{N}_1((5,-10), I_2)$ on $[0,1]$

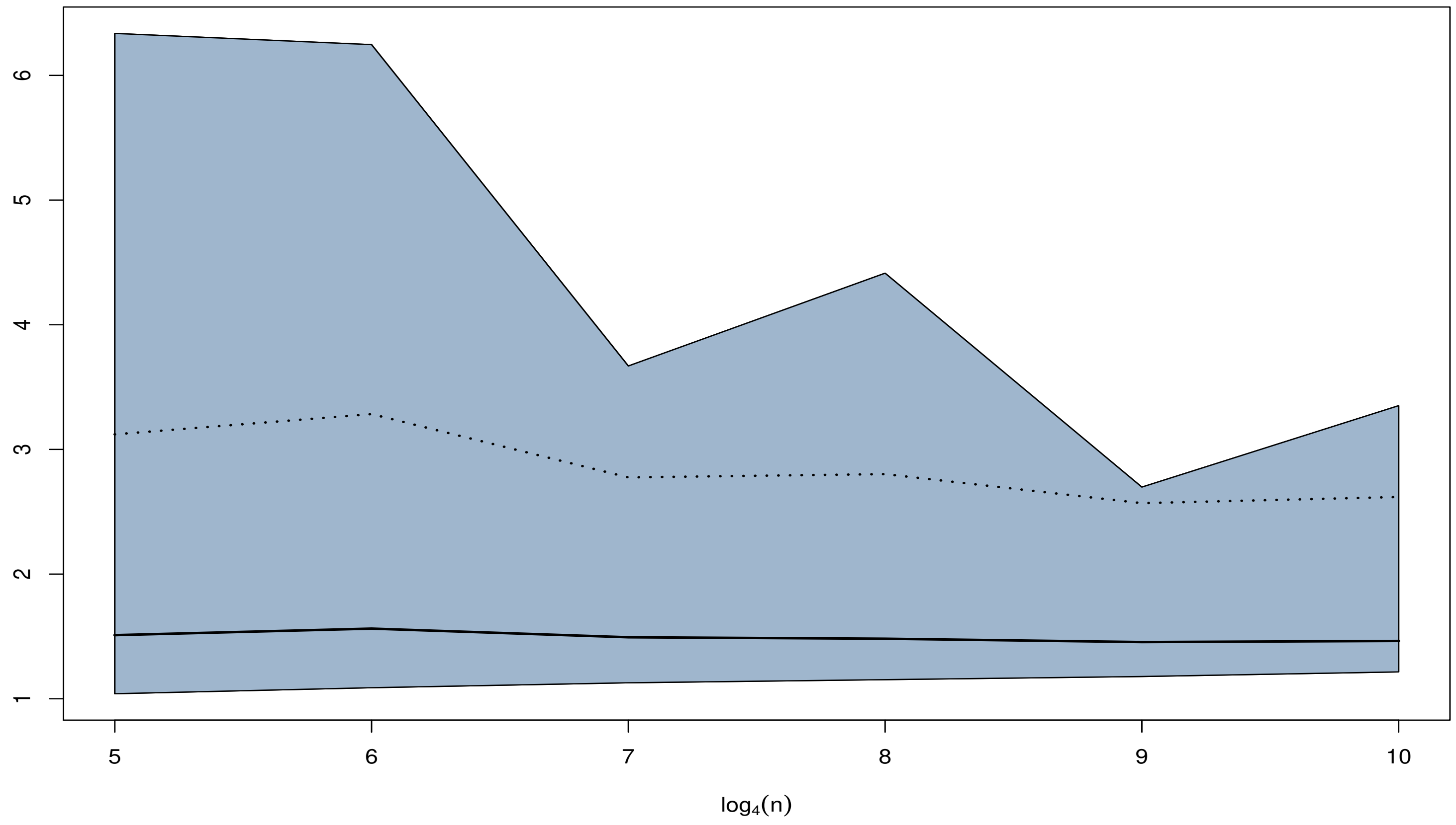

Figure 4: Estimates of $\sigma^2 = 1.5$ under actions of $\mathfrak{G}_n$: average over 100 replications, upper normal approximation 95% bound and observed range

The Ostrich principle

(i)

(ii)

(iii)

(i) Hide your head in the sand [by pretending that the measure μ is unknown],

(ii) take a hit at “orthodox Bayesians” [by pretending this is a real problem],

and

(iii) run [your program/for your life!]

(Drawings by Rachel Plessis–Robert)