Low-complexity global optimization and its application to shape optimization: *From CAD to Level Set by CAD-Free*

Minimisation problem

Consider $\min_x J(x), \quad x \in \mathcal{O}_{ad} \subset IR^N$

Suppose the global minimum J_m is known

Aims :

- global optimization by solution of BVP
- shape parameterization
  - CAD
  - CAD-Free
  - Level Set
  - Regularity control

- Low-complexity sensitivity and incomplete gradients:
  - gradient modeling (as for the state)
  - $\tilde{J}'(n,h,x) \to J'(\infty,0,x) = J'(x)$ incomplete evaluation
  - Projection operator in admissible space for regularity control, preconditionning, smoothing

- Incomplete linesearch. Rotation and smoothing operator on the admissible unit sphere

Closure equation for x (deterministic)

Global optimization problem has a solution

if $\forall\epsilon > 0, \forall x_0 \in \mathcal{O}, \exists T < \infty$ s. t. $x(T) \in B_\epsilon(x_m)$

where $J(x_m) = J_m$ is the infimum and $x(t)$ is solution of:

$$\dot{x} = -M(x(t))d(x(t)), \quad x(0) = x_0,$$

The answer is usually no. Overdetermined ODE.

The answer might be yes if x_0 belongs to the attraction bassin of J_m.

Removing the over-determination

Consider $x_0 = v$ as a new variable

to be found minimizing $h(v) = J(x_v(T))$

where $x_v(T)$ is the solution found at $t = T$ starting from v.

2nd order continuous model

$\nu\ddot{x} + \dot{x} = -M(x(t))d(x(t)), \quad (*)$

$x(0) = x_0, \quad \dot{x}(0) = v$

Global optimization problem has a solution

if $\forall\epsilon > 0, \forall x_0 \in \mathcal{O}$, $\exists(T < \infty$ and $v \in \mathcal{O}')$ s. t. $x_v(T) \in B_\epsilon(x_m)$

where $x_v(t)$ is solution of (*) starting from v.

We never require $\dot{x}(T) = 0$.

Analogy with a Stefan problem

Water-ice interface

$$\begin{cases} \nu x_{\zeta\zeta} + x_\zeta = -M(x(\zeta))d(x(\zeta)), \\ x(0) = x_0, \quad J(x_m) = J_m \\ F(u(x)) = 0 \qquad \text{state equation} \end{cases} \tag{1}$$

The interface between ice and water (x_m) is unknown. We only know its temperature (J_m).

Links with Genetic Algorithms - 1

Population $X^0 = \{x_i^j, \quad i = 1,..,I, \quad j = 1,..,J\}$

$I =$ individual, $J =$ space dimension

sketch of a GA : 3 steps at each iteration.

X^n known, X^{n+1} evaluated by:

1. Selection (remove the weakest element(s))

$X^{n+1/3} = \mathcal{D}X^n$, $\mathcal{D}$ rectangular $(I-1, I)$.

2. Mutation

$X^{n+2/3} = \mathcal{E}X^{n+1/3}$, $\mathcal{E} = (I-1, I-1)$,

$\mathcal{E} = I + \epsilon^n \delta_{ij}$, $\sigma(\epsilon_i^n)_{(n\to\infty)} \to 0$

3. Cross-over

$X^{n+1} = \mathcal{C}X^{n+2/3}$, $\mathcal{C}$ rectangular $(I, I-1)$

s. t. $\sum_{j=1}^{I-1} \mathcal{C}_{i,j} = 1, \forall i = 1,..,I$

GA and dynamic systems

$$X^{n+1} = \mathcal{CED}X^n = X^n + (\mathcal{CED} - I)X^n$$

$$X^{n+1} - X^n = (\mathcal{CED} - I)X^n = \Lambda^n X^n$$

represents the discretization of I coupled 1st order stochastic ODEs :

$$\dot{X} = \Lambda X \text{ with } X(0) = X^0$$

Links with the level set method

$$\psi_\zeta + V\nabla\psi = 0$$

if $V = \nabla J \ . \ n$ where $n = \nabla\psi/|\nabla\psi|$

therefore, $\psi_\zeta = -\nabla J|\nabla\psi|$

We recover, $\nu = 0$, $M(x(\zeta)) = I$ and $d = \nabla J|\nabla\psi|$

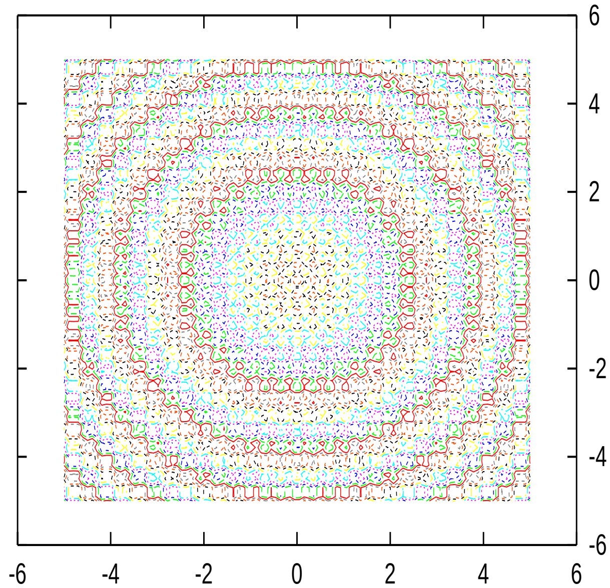
Coupling two generic state equations reads:

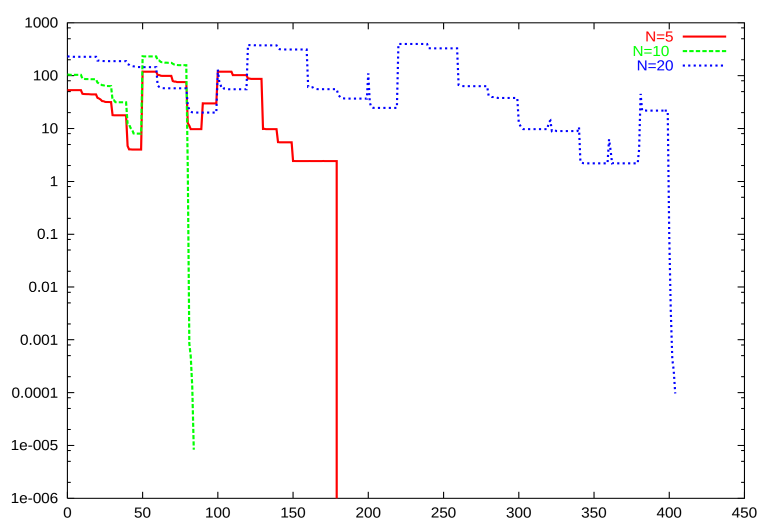
$$(f(u) + g_\Gamma(u)\delta_\Gamma)\chi(\psi) = (g(u) + f_\Gamma(u)\delta_\Gamma))(1 - \chi(\psi))$$

And shape optimization with ψ as design parameter.

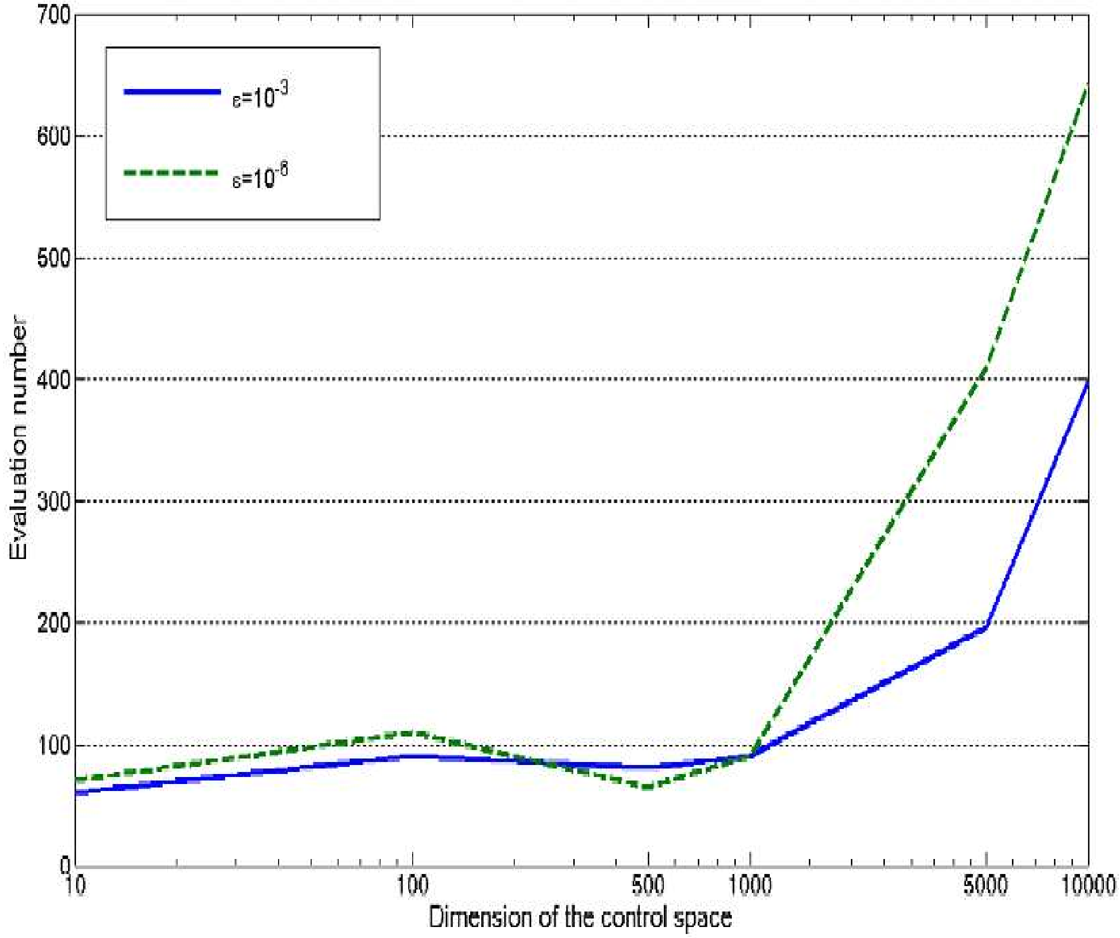
Generalized Rastrigin function

$$J(x) = N + \sum_{i=1}^{N}(x_i^2 - cos(18x_i))$$
$$x \in [-5, 5]^N$$





Complexity evolution



Shape parameterization

Direct simulation loop (D):

$$\Gamma_{\text{Param}} \to \Gamma_h \to \Omega_h \to q_h \to U_h \to J_h,$$

1. Param = CAD $(\#\Gamma_{\text{CAD}} << \#\Gamma_h)$

2. CAD-Free: param = Γ_h

- Needs regularity control: suppose

$$J(x) = (Ax - b)^2 \ ,$$

$$x \in H^1(\Omega),\ A \in H^{-1}(\Omega),\ b \in L^2(\Omega).$$

Therefore $\delta x = -\rho J'_x = -\rho(2(Ax - b)A) \in V$,

$H^{-1}(\Omega) \subset V \subset L^2(\Omega)$. We need to project into $H^1(\Omega)$.

3. Level set : param = signed distance to shape $(\#\Gamma_{\text{LS}} >> \#\Gamma_h)$

Needs regularity control: use same ingredients

Rmq: Small deformation prescribed by 1, 2 or 3 can be taken into account by equivalent b.c.

Smoothing - projection - preconditionning:

redefinition of the scalar product for the Hilbert space where optimization is performed.

$$J^{n+1} \leq J^n + (J_x'^n, \delta x)_0, \quad \delta x = \Pi(-\rho J_x'^n),$$

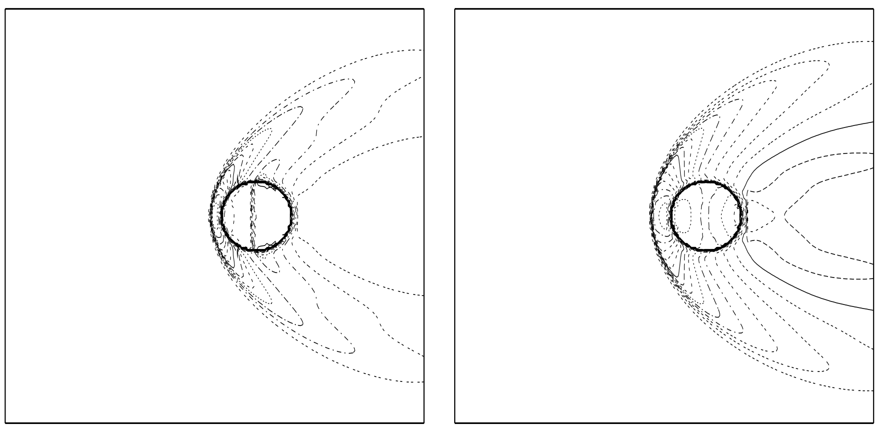
$$J^{n+1} \leq J^n + (J_x'^n, \Pi(-\rho J_x'^n))_0 \leq J^n,$$

$$J^{n+1} \leq J^n + (J_x'^n, \Pi(-\rho J_x'^n))_M$$

Rotation matrix for sensivitity calculation from IS on the admissible unit sphere for CAD-Free and LS param.

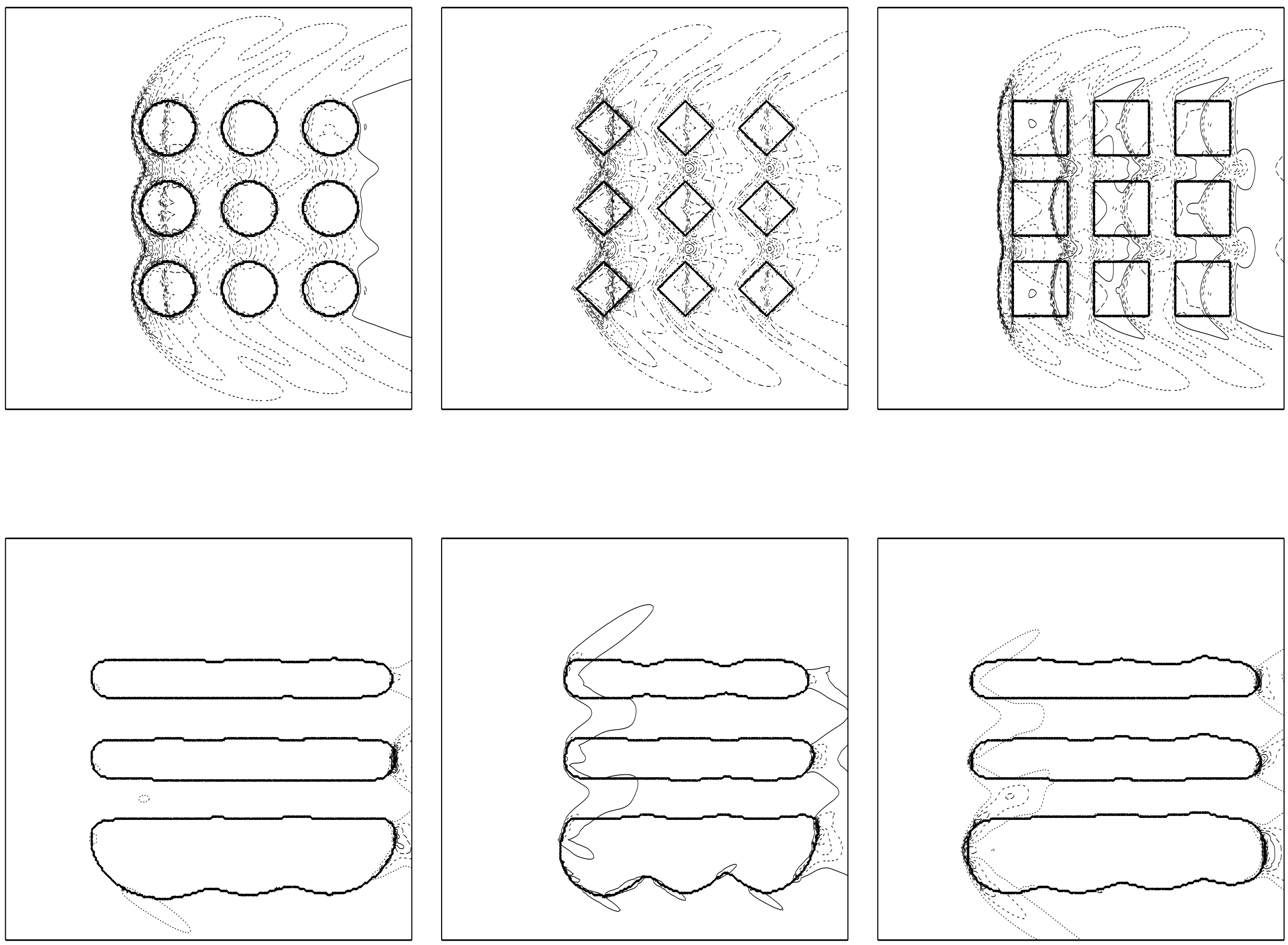
Level set method for complex flows

Euler vs. NS



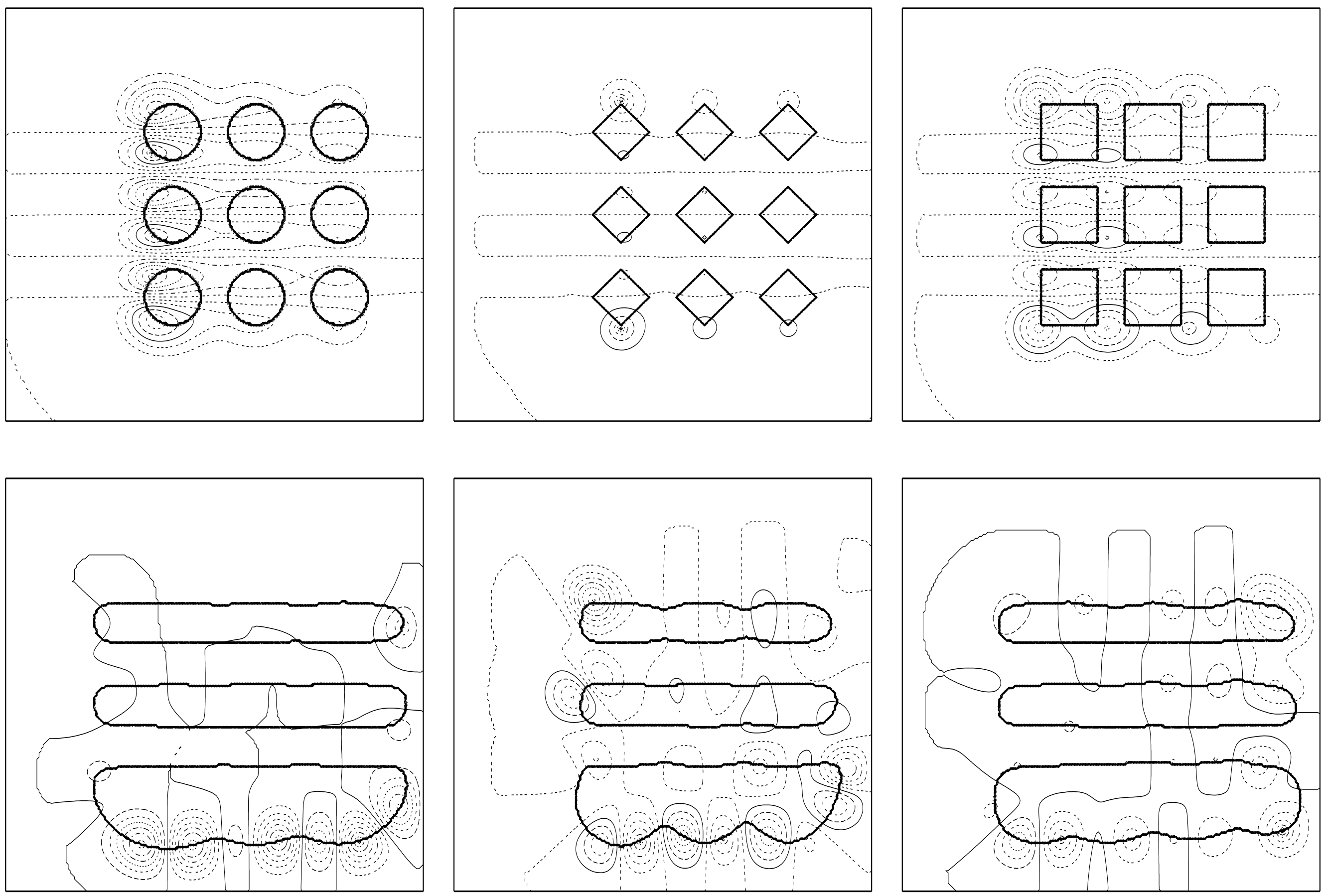
slip vs. non slip b.c.

Level set method for complex flows - $J = \frac{C_d}{C_l V}$



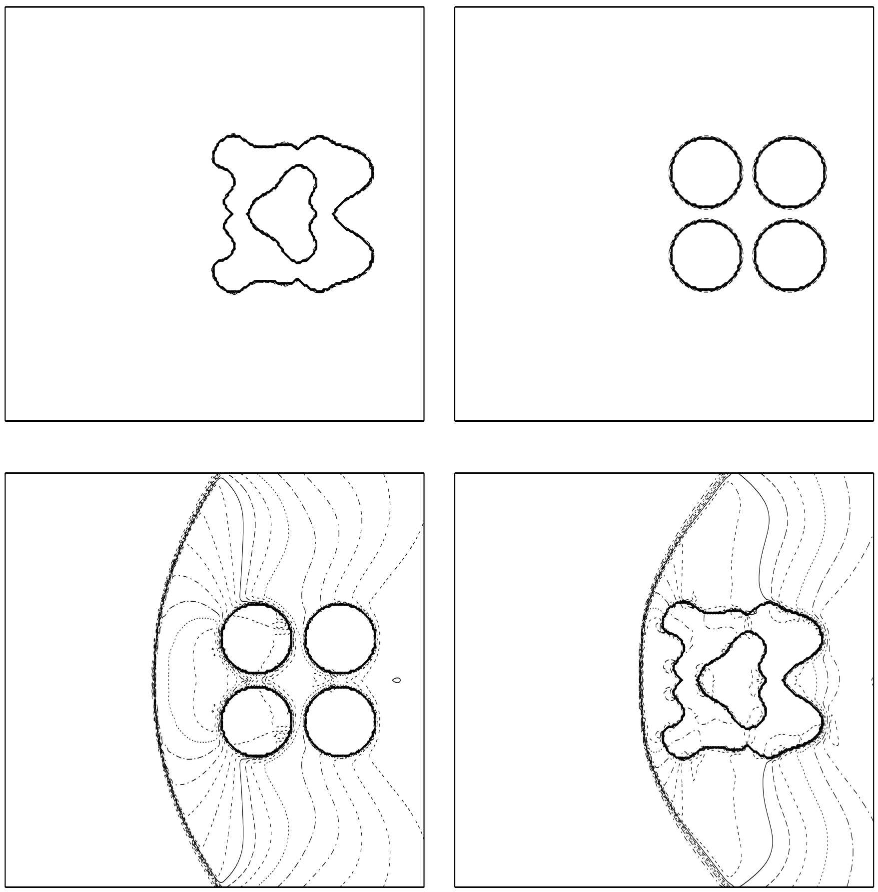
Level set method

Incomplete gradient comparison : regularity control as in CAD-Free approach using a 4th order smoother.



Level set method

Symmetry during optimization.



Incomplete sensitivity and low-complexity models

From:

$$x \to q(x) \to U(q(x)) \to J(x, q(x), U(q(x)))$$

To:

$$x \to q(x) \to \tilde{U}(q(x))(\frac{U}{\tilde{U}})$$

where $\tilde{U} \sim U$ is obtained using a simple model.

Incomplete sensitivity can be improved by:

$$\frac{dJ}{dx} \sim \frac{\partial J(U)}{\partial x} + \frac{\partial J(U)}{\partial q}\frac{\partial q}{\partial x} + \frac{\partial J(U)}{\partial U}\frac{\partial \tilde{U}}{\partial q}\frac{\partial q}{\partial x}\frac{U}{\tilde{U}}$$

$\tilde{U}$ never used, only $\partial \tilde{U}/\partial q$.

Free choice for low-complexity models. Wall functions are natural $\tilde{U}(y^+)$ candidates.

Efficiency improvement for a blade

$$\eta = \frac{q \triangle p}{\omega T_r}$$

constraint on q (inflow rate), ω, Δp (between in and outlet).

But Δp not in the validity domain of IS.

Momentum eq. $\int_\Gamma u(u.n) d\sigma + \int_\Gamma \tau n d\sigma = 0$,

Suppose $n_{i/o} = (\pm 1, 0, 0)$, neglecting viscous terms on in and outlet boundaries and using periodicity:

$$\int_{\Gamma_i} p + \frac{u^2}{2} - \int_{\Gamma_o} p + \frac{u^2}{2} = \int_{\Gamma_w} -p + \nu \frac{\partial u}{\partial n} = C_d$$

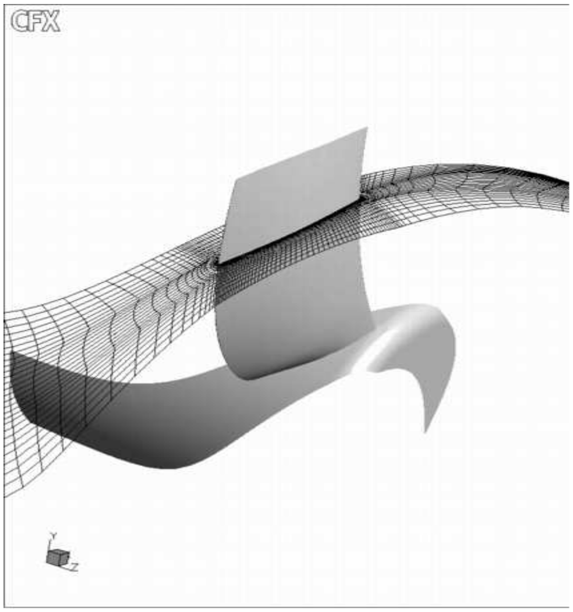
If outlet far enough s.t. $u_o > 0$, from $\nabla . u = 0$ we have:

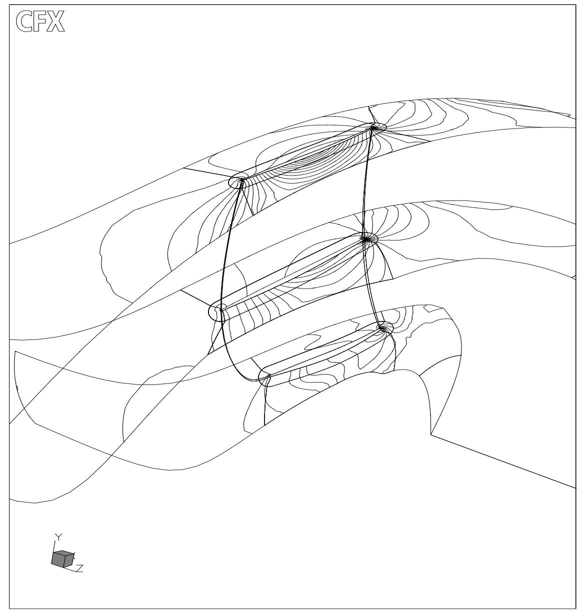
$$\triangle p = C_d$$

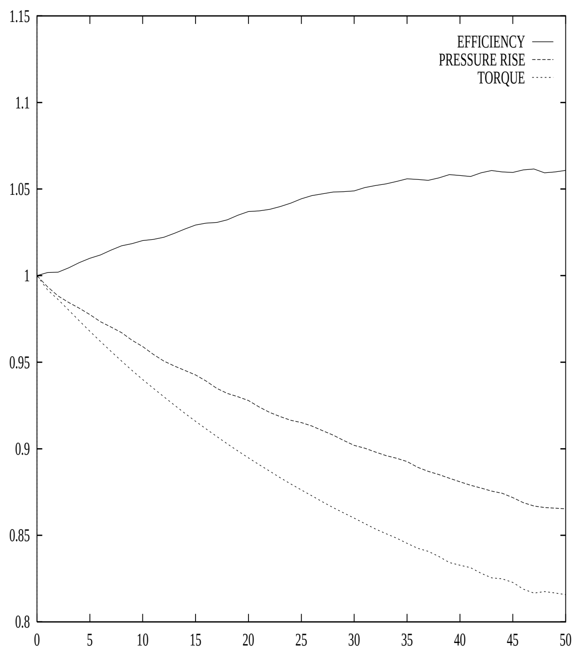
already linearized with wall functions.

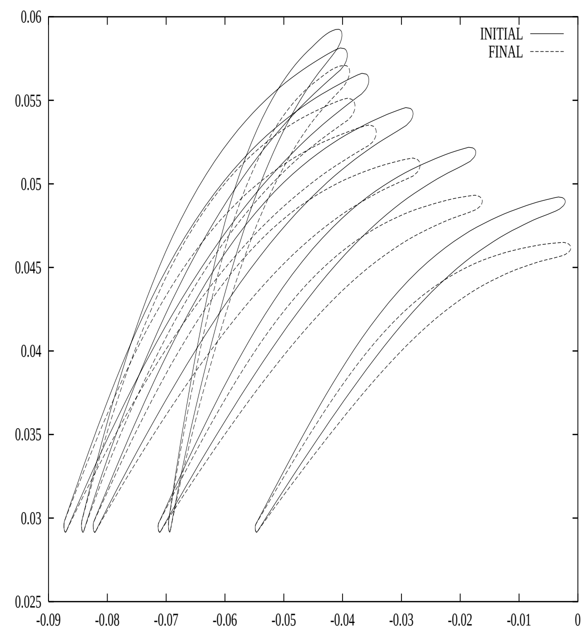
reduce torque @ given drag.

Efficiency improvement for a blade $\eta = \frac{q \triangle p}{\omega T_r}$









Application to active control using Hadamard b.c.

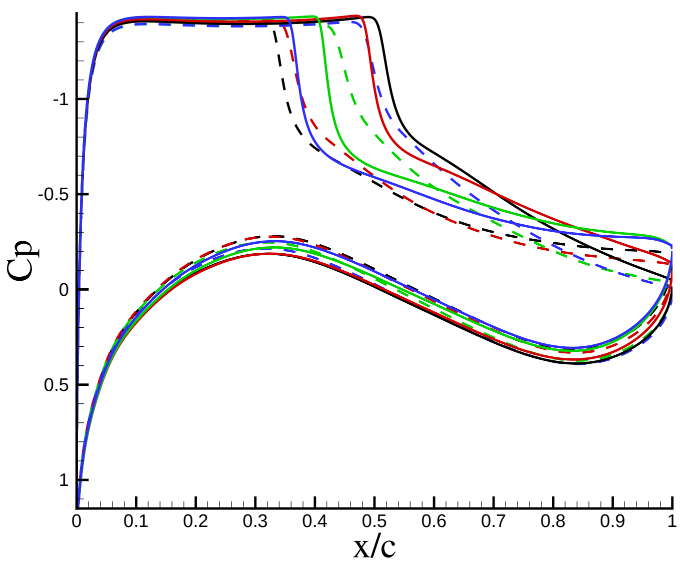
$$u.n(t) = -u(t)\delta n(t) + \delta x(t)/\delta t.n(t)$$

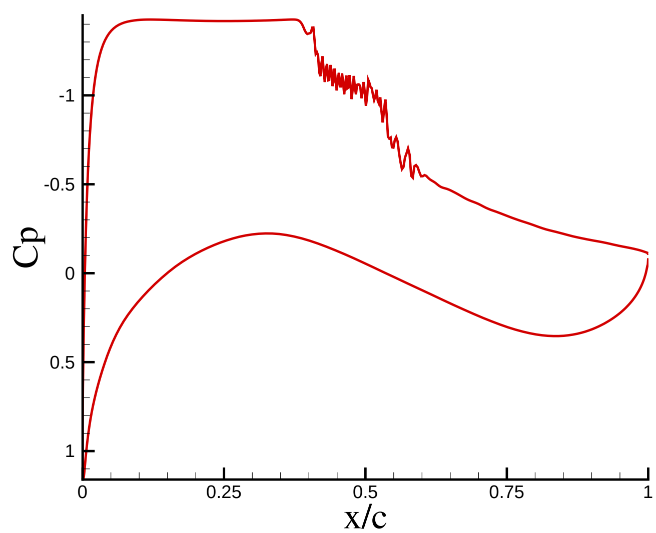
$$\delta x(t) = -\rho \nabla J(t)$$

OAT15A profile:
$Ma = 0.736, Re/m = 4.3610^6, \alpha = 4^o$

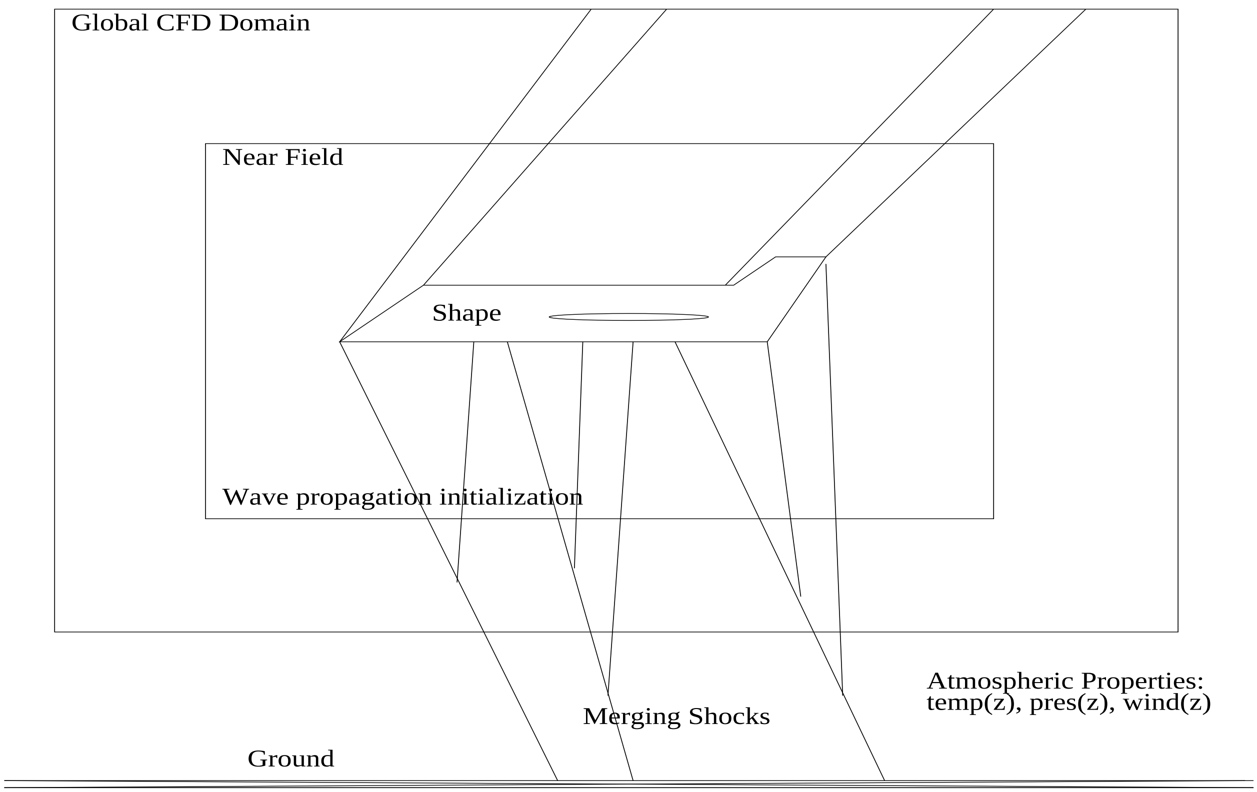
Buffeting control: location of actuators, control frequency

Instantaneous pressure coefficients without and after the control is turned on.





Optimization of aerodynamic and acoustic performances of supersonic civil transports



Near-field : 3D Euler system,

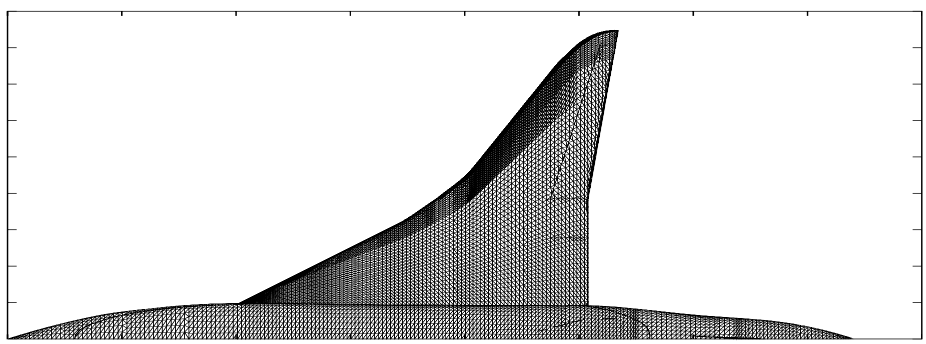
Far field : waveform propagation method using CFD predictions.

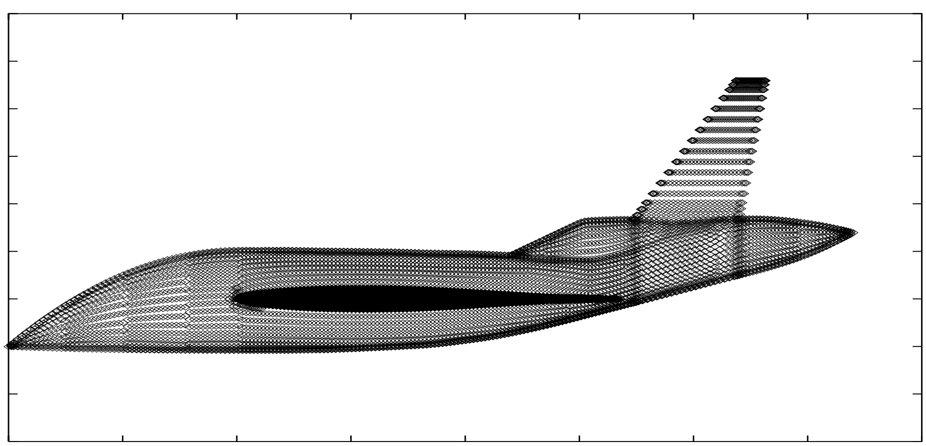
CAD-Free control space parameterization

Input in CAD

Optimization in CAD-Free

Output in CAD.



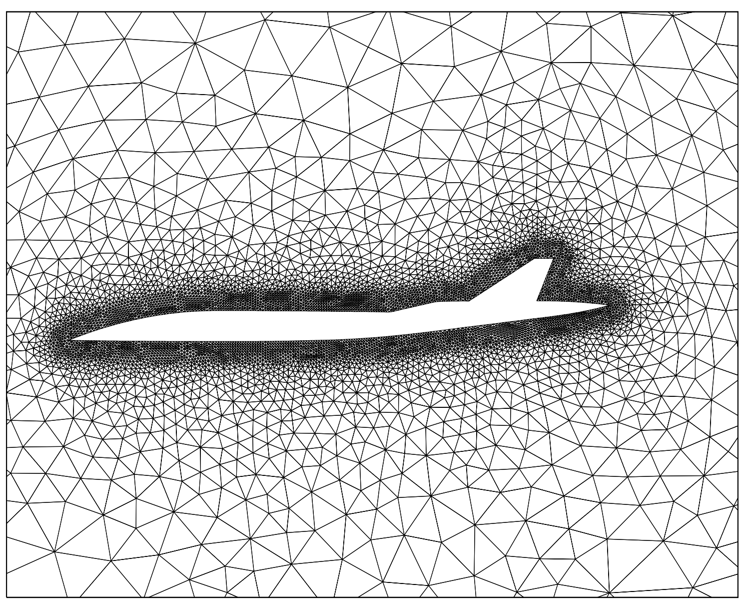


Functional definition

$$J(x) = C_d \quad + \quad (C_l^0 - C_l)_+$$

$$+(\mathsf{V}^0 - \mathsf{V})_+ \quad + \quad \int_{\mathsf{shape}} |d - d_0| d\gamma$$

$$+\int_{\mathsf{ground}} |\Delta p_g| d\gamma$$



Functional reformulation

(Bow shocks <=> less boom)
=> smooth leading edges

C_d^{loc} from $LE = \{x, \vec{n}.\vec{u}_\infty < 0\}$ to remain unchanged or to increase and C_d to decrease.

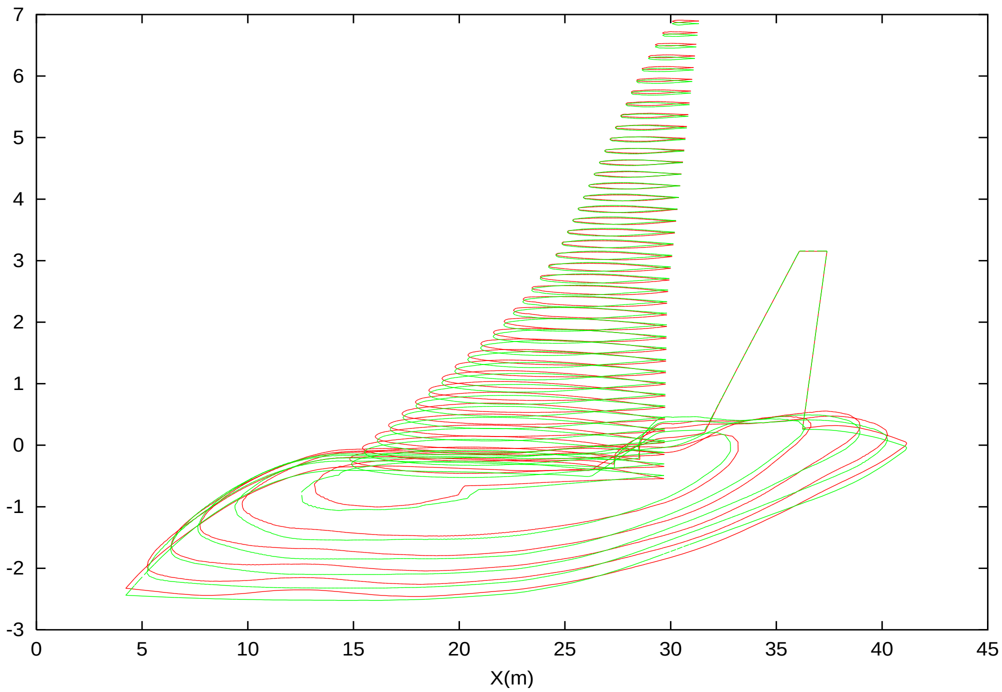
$$C_d^{loc} = \frac{2}{\rho_\infty |\vec{u}_\infty|^2} \int_{LE} p.\vec{n}.\vec{u}_\infty$$

New cost function

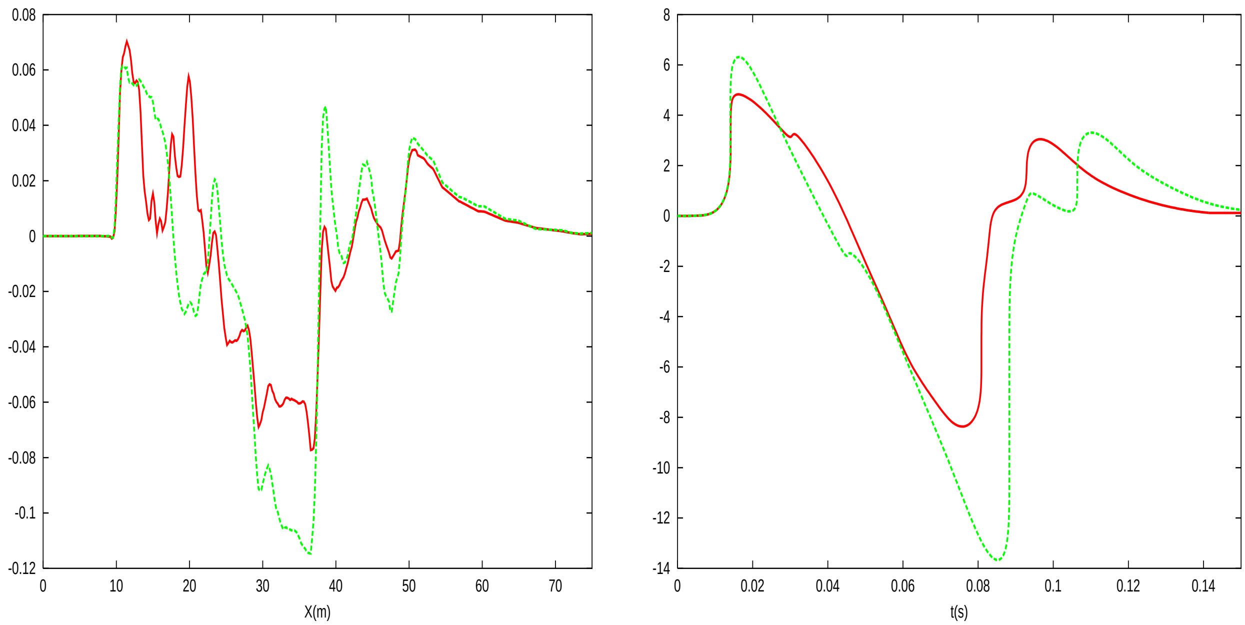
$$J(x) = C_d \quad + \quad (C_l^0 - C_l)_+$$

$$+(\mathsf{V}^0 - \mathsf{V})_+ \quad + \quad \int_{\mathsf{shape}} |d - d_0| d\gamma$$

$$+((C_d^{loc})^0 - C_d^{loc})_+$$

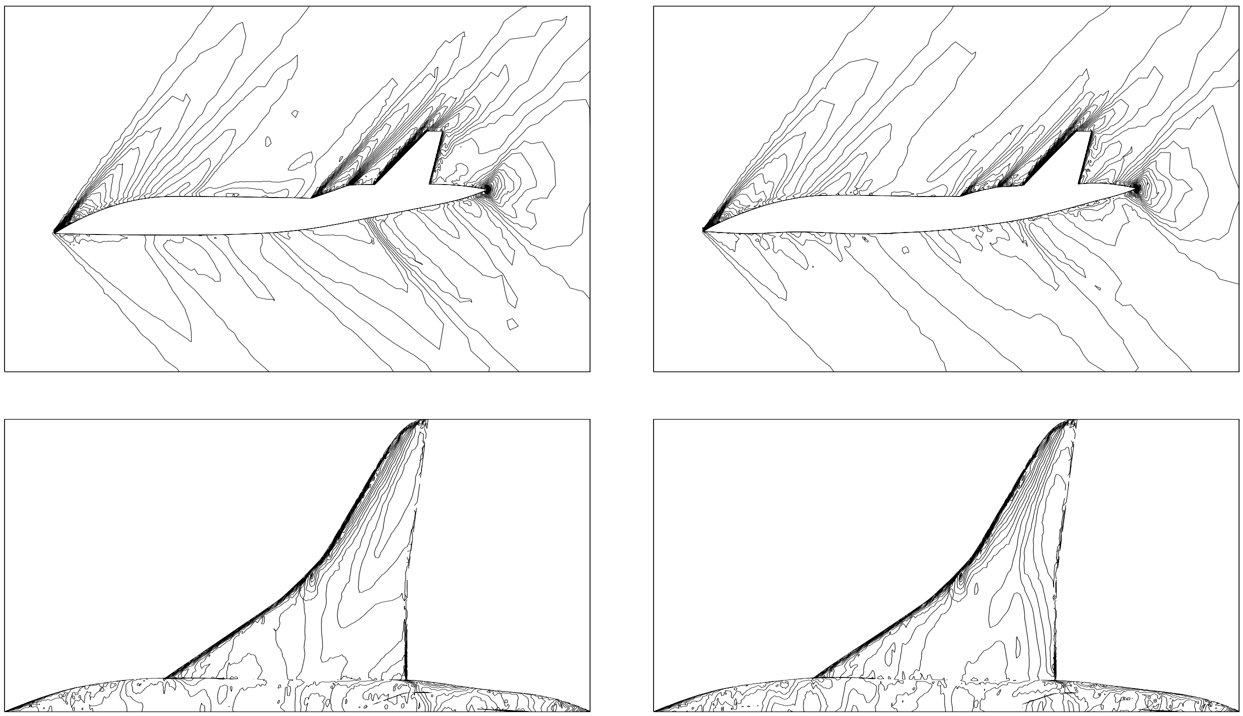


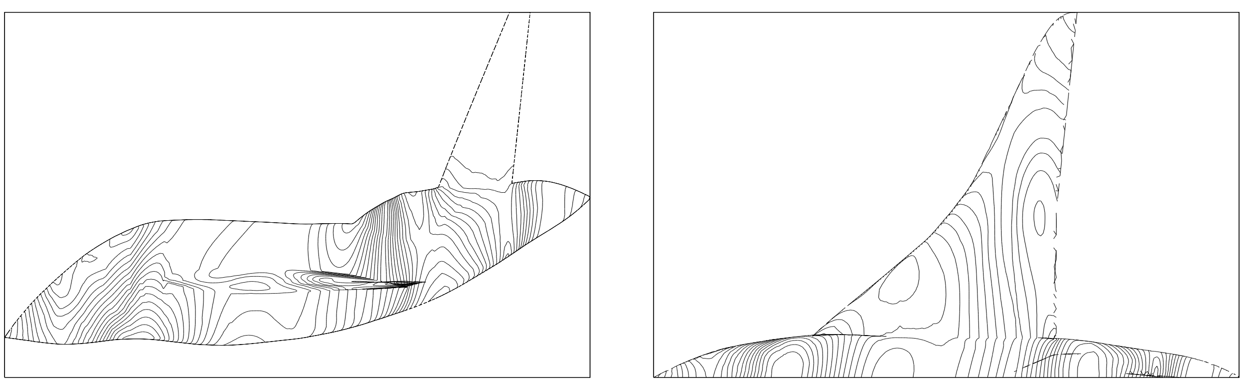
Near-field and ground pressure signatures



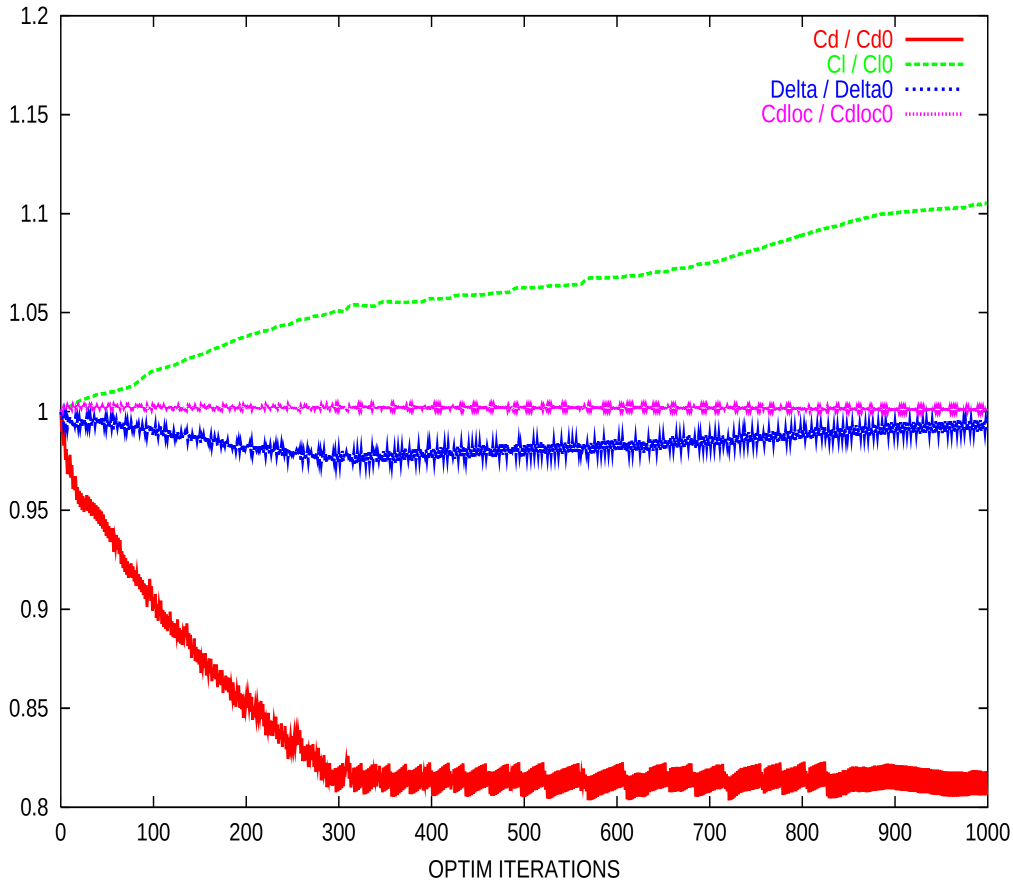
($\frac{p-p_\infty}{p_\infty}$) and ground pressure (25 % noise reduction) for the initial and optimized aircraft.

Iso-Mach contours for initial and final shapes





Iso-normal deformation



20% drag reduction

25% noise reduction

10% lift increase

Volume and maximum by-section thickness conserved.

Concluding remarks

Low complexity global optimization algorithm based on solution of a BVP.

Parameterization and regularity control in design: LS and CAD-Free

Incomplete sensitivity

Other applications: micro-fluidic devices optimization, pollution control in flames with complex chemistry, unsteady flow control.