Multi-Resolution algorithms for Active Contour Models

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Summary. Multi-resolution methods applied to active contour models can speed up processes and improve results. In order to estimate those improvements, we describe and compare in this paper two models using such algorithms. First we propose a multi-resolution algorithm of an improved snake model, the balloon model. Convergence is achieved on an image pyramid and parameters are automatically modified so that, at each scale, the maximal length of the curve is proportional to the image size. This algorithm leads to an important saving in computational time without decreasing the accuracy of the result at the full scale. Then we present a multi-resolution parametrically deformable model using Fourier descriptors in which the curve is first described by a single harmonic; then harmonics of higher frequencies are used so that precision increases with the resolution. We show that boundary finding using this multi-resolution algorithm leads to more stability. These models illustrate two different ways of using multi-resolution methods: the first one uses multi-resolution data, the second one applies multi-resolution to the model itself.

1. Introduction

We propose two applications of multi-resolution methods for active contour models. In section 2 a first model which consists in a multi-resolution approach of the balloon model, introduced by Cohen [2], is described. Its aim is to speed-up the process while allowing a constant accuracy of the result. Then section 3 presents a parametric model based on Fourier descriptors showing that a multi-resolution algorithm can increase the model stability. These algorithms are applied for facial features extraction. Their respective advantages are discussed in the conclusion.

2. A multi-resolution balloon model

2.1 The balloon model

The use of energy–minimizing curves, known as "snakes", to extract features of interest in an image has been introduced by Kass, Witkin and Terzopoulos [6]. Further improvements to this model were successively developed by many other authors [1, 2, 3, 8].

The contour model, as introduced in [6], is a curve $v(s) = (x(s), y(s))$ that minimizes an energy functional of the following form:

$$E(v) = \int_{\Omega} \alpha \|v'(s)\|^2 + \beta \|v''(s)\|^2 + P(v(s)) ds \tag{2.1}$$

where P is the potential associated to the image I. Usually P is equal to the opposite of the square of the image gradient norm:

$$P = -|\nabla I|^2$$

If v is a local minimum for E, it satisfies the associated Euler-Lagrange equation:

$$\begin{cases} -(\alpha v')' + (\beta v'')'' = F(v) = -\nabla P(v) \\ + \text{ Cyclic Boundary conditions} \end{cases} \tag{2.2}$$

After discretizing equation (2.2) by finite differences we obtain a linear system:

$$AV = F,$$

where A is a pentadiagonal matrix, $V = (v_i)_i$ is the vector of positions $v_i = v(ih)$, F represents the forces at these points and h is the space discretization step.

The associated evolution equation (see [2]) may be defined, after temporal discretization, by:

$$V^t = (Id + \tau A)^{-1}(V^{t-1} + \tau F(V^{t-1})) \tag{2.3}$$

where τ is the time step, Id denotes the identity matrix, t is the time parameterization index and V^t describes the curve position at step time t.

The balloon model [2] is an improvement of this classical snake model that modifies the force F by normalizing the associated potential force and by adding an internal pressure force:

$$F = -k \frac{\nabla P}{||\nabla P||} + \gamma \overrightarrow{n}(s) \tag{2.4}$$

where $\overrightarrow{n}(s)$ is the vector normal to the curve. The first term of equation (2.4) corresponds to the normalized image force. The second term is the internal pressure with amplitude γ.

2.2 Multi-resolution algorithm

A multi-resolution approach for the balloon model consists in solving iteratively the problem at successive scales. First, the active contour solution is searched and found at a coarse scale, needing few discretization nodes and solved by a small linear system on a small image. Then the solution curve at this coarse scale is used as initialization at a finer one. This process is iterative. A similar algorithm is presented in [4] for different scales of blurring of the potential image.

Giving α, β and γ as parameters, the initial curve is projected on a coarse image of size $2^{N-S} \times 2^{N-S}$ and discretized with 2^{M-S} nodes ; where $2^N \times 2^N$ is the size of the original image, 2^M is the number of discretization nodes at the finest scale and S is the coarsest scale. When the convergence at scale S is achieved, the same process is applied at a finer scale $(S-1)$ using the solution curve obtained at scale S as initializing curve. By propagating this result from the coarser to the finer scale, we obtain a result on the initial full scale image without loss of precision. since the convergence process ends at scale 0 which corresponds to the original image.

Therefore the multi-resolution algorithm may be summarized in the following way : Given the initial guess at scale S, denoted V^*_{S+1}, the iterative scheme has the following form at scale p, decreasing from S to 0, while the image size increases from 2^{N-S} to 2^N:

$$\begin{aligned} 1.V_p^0 &= \Pi_p(V^*_{p+1}) \\ 2.V_p^t &= (Id + \tau A_p)^{-1}(V_p^{t-1} + \tau F_p(V_p^{t-1})) \\ 3.V_p^* & \quad \text{is the solution of equation (2.3) at convergence at scale p} \end{aligned} \tag{2.5}$$

where:

- V_p is the vector of size 2^{M-p} representing the discrete curve at scale p;
- Π_p is the projection from scale $p+1$ to scale p;
- F_p is the force vector at scale p;
- A_p the stiffness matrix of size $2^{M-p} \times 2^{M-p}$ at scale p.

Since the size of the shape to be detected decreases when scale p increase, the parameter affected to the expansion is calculated at each scale so that the limit size of the curve is constant among scales. This is achieved by using a force defined as:

$$F = \frac{\gamma}{2^p}\overrightarrow{n}(s) - k\frac{\nabla P}{||\nabla P||}. \tag{2.6}$$

When there is no image force ($k=0$), it can be shown that the limit curve is a circle whose size is chosen as a characteristic to be invariant with respect to scale. To obtain a limit perimeter equal to L, the expansion parameter must be set to:

$$\gamma = 4\pi L(\alpha + 4\pi^2\beta) \tag{2.7}$$

In order to have a limit length at scale p equal to $\frac{L_0}{2^p}$ the expansion value is equal must be set to $\frac{\gamma}{2^p}$.

Therefore the multi-resolution algorithm may be summarized in the following way:

1. Build the pyramid of images from scale 0 (the original image) to scale S.
2. Given an initial curve V_{init} at scale 0, construct the curve V^*_{S+1} by projecting V_{init} in the image at scale S and reducing the number of discretization points to 2^{M-S}.
3. Calculate the solution V^*_S of the iterative scheme (2.5) at scale S using V^*_{S+1} as initialization.
4. For p decreasing from $(S-1)$ to 0, calculate, at scale p, the curve V^*_p (discretized by 2^{M-p} points) using the projection of V^*_{p+1} as initialization.

The main advantage of this method, as expected with multi-resolution algorithms [10], is to reduce computing costs:

– Initial convergence leads to a rough estimation of the boundary and is achieved at a coarse scale by solving a small linear system. At a finer scale the initial curve is already close to the boundary. The number of iterations to achieve convergence when dealing a large system to solve is thus smaller than the standard method.
– Since the number of discretization points decreases at the coarser scales, computation cost at each step of convergence is also smaller than with the standard method.

Figure 2.1 shows the results of mouth extraction using three consecutive scales (for representation images have been normalized to the same size). The computation time needed for convergence has been 55% shorter with the multi-resolution balloon model than with the standard balloon model.

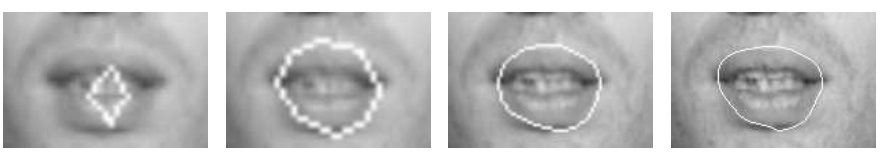

Fig. 2.1. Extraction of the mouth. From left to right: initialization at scale 2, results obtained at scales 2 then 1 and final result at scale 0.

3. A multi-resolution parametrically deformable model

3.1 Fourier descriptors active contour model

In this section, we present a parametric model based on the elliptic Fourier description. While in the snake model, the constraints on the global regularity of the contour are included in the internal energy function, these constraints may now be included in the parametrical model itself. It is possible to apply a multi-resolution algorithm to such a model by defining the scale as the number of harmonics used to describe the curve.

The use of Fourier descriptors for active contour model has been introduced by Staib and Duncan [9] in order to extract an object boundary. Their method is based on the use of probability distributions on the parameters.

In order to be less sensitive to the initial parameter value, we propose a variational approach similar to the method used in the snake model [7].

An elliptic Fourier representation of a closed curve is a parametrical curve v defined by:

$$v(\theta) = \left(\begin{array}{c} x(\theta) \\ y(\theta) \end{array} \right) = \sum_{k=0}^{N} A_k \left(\begin{array}{c} \cos(\theta k) \\ \sin(\theta k) \end{array} \right), \tag{3.1}$$

where A_k is a 2×2 matrix, N the number of harmonics used to describe the curve and θ the angular parameterization index.

The curve modeling the boundary of the object is obtained by minimizing an energy functional similar to the snake energy (2.1):

$$E(v) = \int_0^{2\pi} P(v(\theta)) + \lambda \frac{\partial v(\theta)}{\partial \theta}^2 d\theta, \tag{3.2}$$
$$\text{where } \lambda \in \Re^+.$$

The first term P is an image potential equal to the opposite of the square of the image gradient ($P = -|\nabla I|^2$) and the second one is an elasticity term associated to the curve tension. The energy gradient with respect to the parameters of the model is a vector of size $4N$, whose components are the partial derivatives $\frac{\partial E}{\partial a}$ with regard to each of the four elements of the N matrices A_k:

$$\frac{\partial E}{\partial a} = \int_0^{2\pi} \nabla P . \frac{\partial v(\theta)}{\partial a} + 2\lambda \frac{\partial v(\theta)}{\partial \theta \partial a} \frac{\partial v(\theta)}{\partial \theta} d\theta, \tag{3.3}$$

Given an initial set of parameters, the curve v^* that is the closest local minimum of E is obtained by applying a Newton minimization. This leads to several instability problems when high frequency harmonics are used because the curve has weak regularity constraints and can be attracted by noise points. It is thus interesting to apply this model with a multi-resolution algorithm in order to reduce the possibilities that the optimization process tends to a weak local minimum that is not meaningful.

3.2 Multi-resolution algorithm

On the contrary to the snake model, there is no geometrical constraint term in the energy functional because it is included within the model. Thus we restrict the space of admissible curves by defining the number of harmonics used to describe the curve. The aim of this multi-resolution algorithm is to obtain a better stability by increasing progressively the number of harmonics. Using only the first term of the Fourier decomposition defines an ellipse; this was used in [5]. The iterative algorithm is as follows:

1. Describe the object shape with an ellipse curve v_1.
2. Find a curve v_1^* described by a single harmonic which is a minimum of E using v_1 as initialization.
3. For p increasing from 2 to N, using the curve v_{p-1}^* as initialization, find the curve v_p^* described by p harmonics.

Figure 3.1 shows the convergence process of the multi-resolution algorithm applied to an image of the mouth. The curve is first described by a single harmonic and evolves from an initial position to a coarse approximation of the mouth. As the number harmonics increases, the regularity constraints are smaller and the mouth boundary can be described more precisely. In order to illustrate the stability gain due to multi-resolution algorithm, figure 3.2 shows several stages of the convergence process when using standard convergence method.

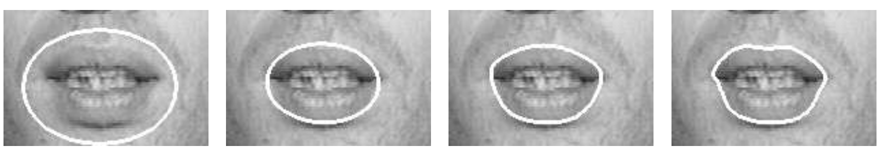

Fig. 3.1. Extraction of the mouth using the multi-resolution algorithm. From left to right: initialization and results obtain with 1, 4 then 9 harmonics.

4. Conclusion

Two different applications of multi-resolution algorithms for active contour models have been presented in this paper. For the balloon model, the multi-resolution method is applied to data. Since a large part of the convergence process is achieved at coarse scales on small images, this algorithm leads to saving in computational time. Although the expansion parameter is modified

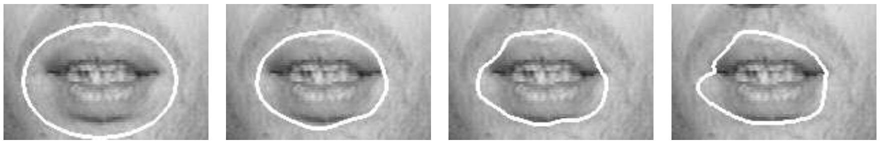

Fig. 3.2. Extraction of the mouth without the multi-resolution algorithm using 9 harmonics . From left to right: initialization and results obtained at several stages of the convergence. The right image presents the final result.

so that the maximal length of the curve is proportional to the size of the image, the other parameters bound to the regularity constraints are not modified when scale changes. With parametrically deformable model using Fourier descriptors the constraints on the global regularity of the contour are included in the parametrical model itself. Thus, by applying multi-resolution to the parameters and defining the scale as the size of the Fourier decomposition, the regularity constraints are determined by the scale. Such an algorithm can improve the model stability and allows the use of higher frequency harmonics to extract irregular object boundary. However, when extracting highly irregular boundary, a large number of harmonics are needed and it may be preferable to use the balloon model which can handle a large spectrum of shapes with limited number of parameters.

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