# Homogenization of the dislocation dynamics


Nicolas Forcadel

ENPC, CERMICS

Joint works with: Cyril Imbert and Régis Monneau


**Phase-field models for the evolution of complex structures**

june 06, 2007

# Plan

1. Physical motivations
2. Homogenization of the dislocation dynamics
3. Qualitative properties of the effective Hamiltonian

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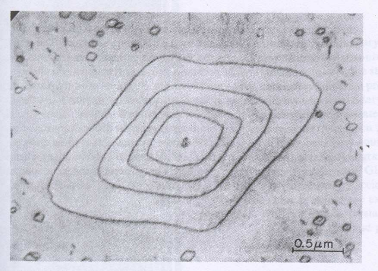
# Observation of dislocations




Definition: a dislocation is a line of crystal defects.

Goal : modeling of plastic behaviour of crystal.

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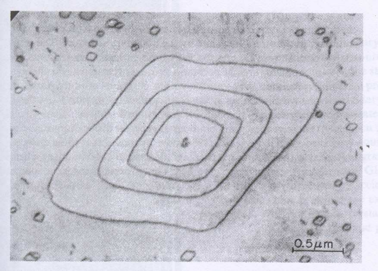


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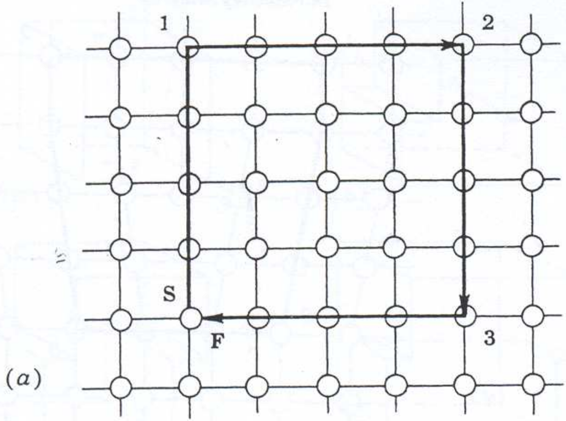
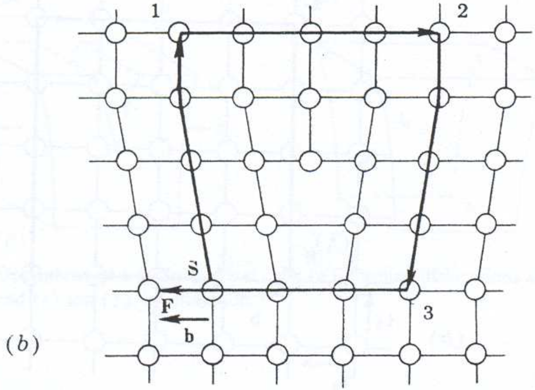
# Perfect crystal



# Singular deformation of the crystal




Topological defect = dislocation

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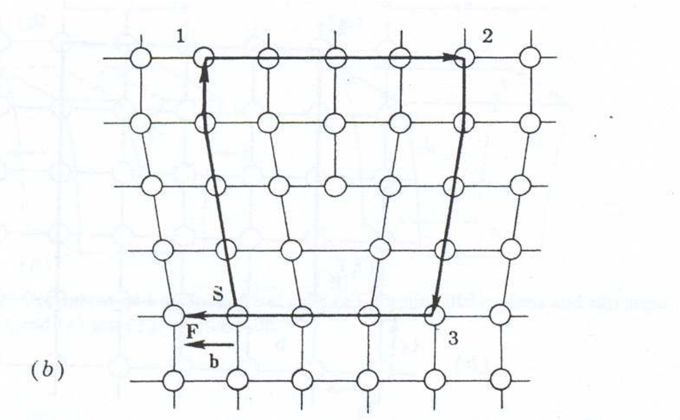
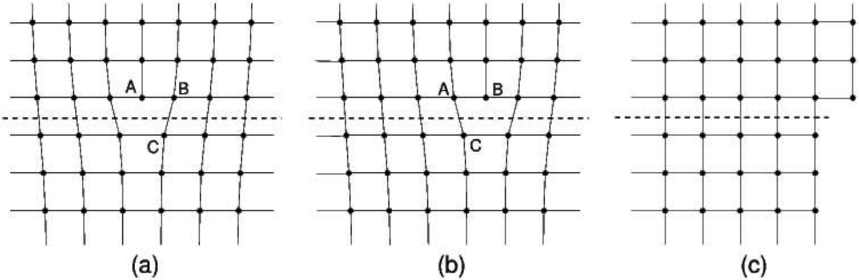
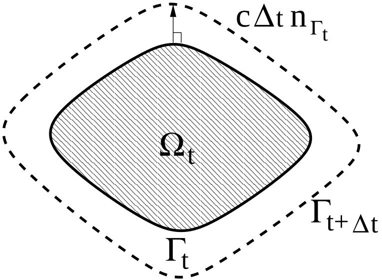



Topological defect = dislocation

# Displacement of a dislocation



# Dynamics of a dislocation




$$\frac{d\Gamma_t}{dt} = c\, n_{\Gamma_t} \quad \text{with} \quad c = c(\Gamma_t)$$

# Rigourous result

- Well-posed dynamics for short time: [Alvarez, Hoch, Le Bouar, Monneau], [Forcadel], [Forcadel, Monteillet]
- Well-posed dynamics for long time: [Alvarez, Cardaliaguet, Monneau], [Barles, Ley], [Barles, Cardaliaguet, Ley, Monneau], [Forcadel, Monteillet]
- Convergence of numerical scheme: [Alvarez, Carlini, Monneau, Rouy], [Carlini, Falcone, Forcadel, Monneau], [Forcadel]
- Approximation of dislocation dynamics by mean curvature motion: [Garroni, Müller], [Da Lio, Forcadel, Monneau], [Forcadel]
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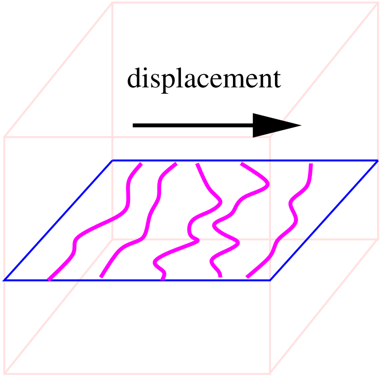
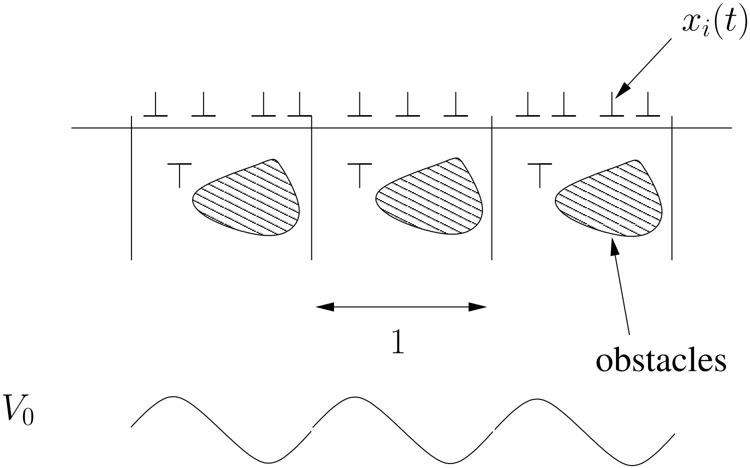
# Plan

# Homogenization



# 1D Homogenization



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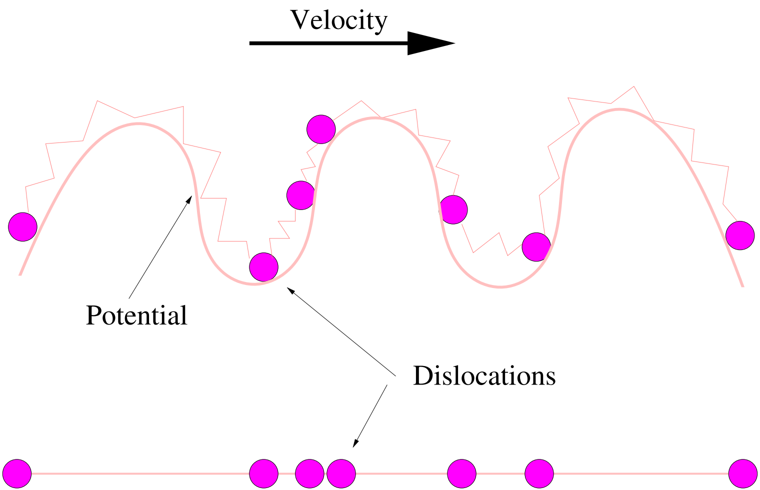

# Modeling

$$\dot{x}_i = -\nabla_{x_i} E$$

with

$$E = \sum_i V_0(x_i) + \sum_{i<j} V(x_i - x_j)$$

and

$$V_0(x+1) = V_0(x)$$

$$V(x) = \ln |x|$$

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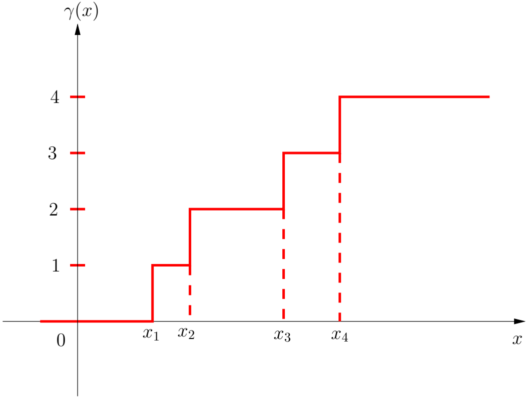
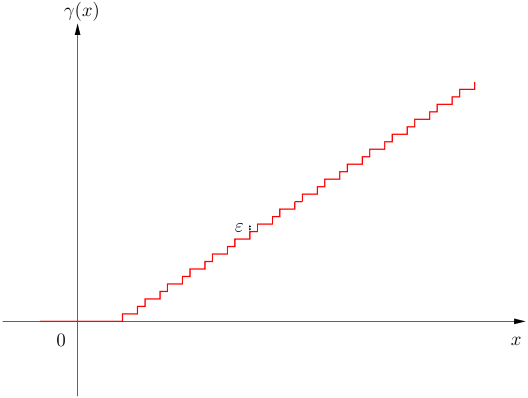



Plastic deformation

$$\gamma(x,t) = \sum_i H(x - x_i(t))$$

# Rescalling




Plastic deformation

$$\gamma^\varepsilon(x,t) = \varepsilon\gamma\left(\frac{x}{\varepsilon}, \frac{t}{\varepsilon}\right)$$

# Limit $\varepsilon = 0$

**Theorem ( F.,Imbert, Monneau)**

*We have*

$$\gamma^\varepsilon(x,t) \to \gamma^0(x,t)$$

*and the general plastic law*

$$\gamma_t^0 = \overline{H}(\sigma, \gamma_x^0)$$

$\gamma^0$: plastic strain

$\gamma_t^0$: plastic strain velocity

$\gamma_x^0$: dislocation density

$\sigma$: internal stress created by the density of dislocation $\gamma_x^0$

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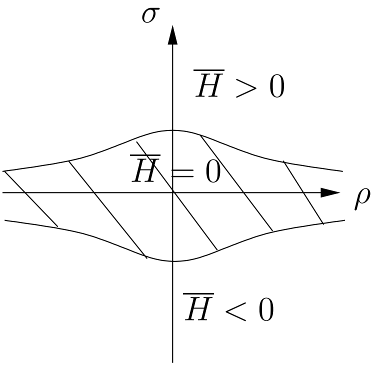
# A simple case: Orowan law

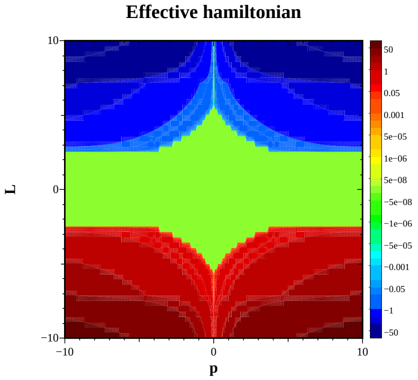
If $V_0 \equiv 0$, then $\overline{H}(\sigma, \rho) = \sigma \cdot \rho$, *i.e.*:

$$\gamma_t^0 = \sigma \cdot \gamma_x^0 \quad : \quad \text{transport with velocity } \sigma$$

# Schematic representation of the effective Hamiltonian



# Simulation of the effective Hamiltonian



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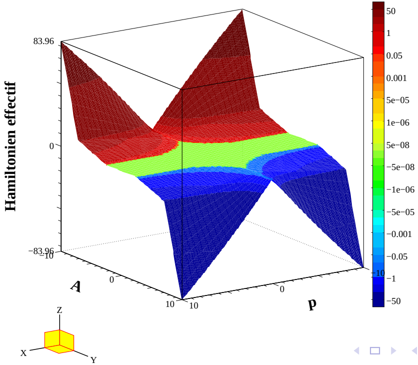

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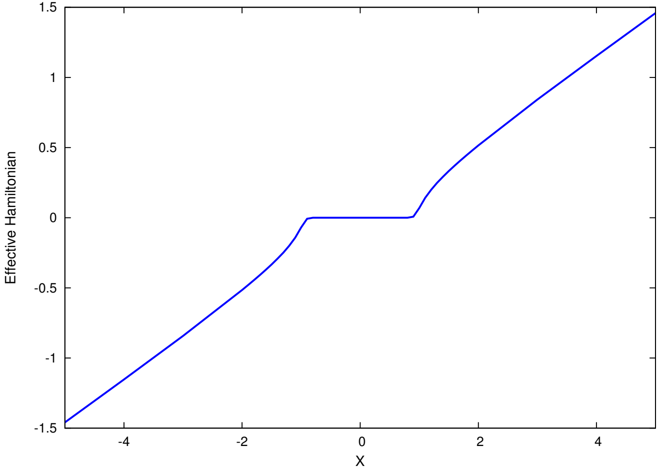
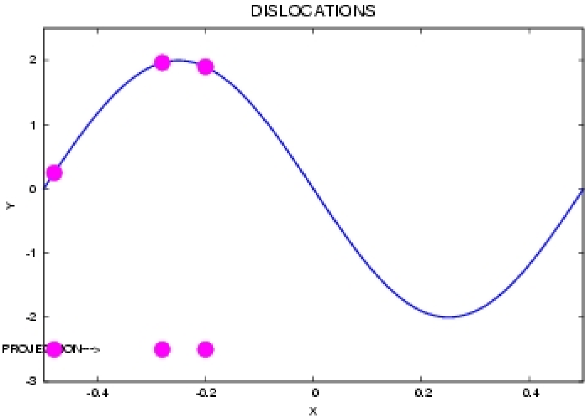


# Simulations of dislocation dynamics




Animation

# Perspectives and open problems

- Homogenization of the Frenkel-Kontorova model (Joint work with C. Imbert and R. Monneau)
- Homogenization of more realistic models