Passive sensor imaging using ambient noise

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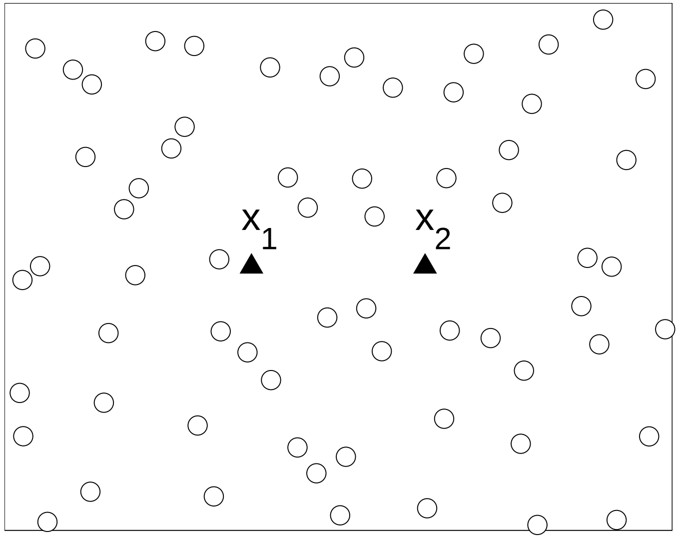
Classical problem in geophysics: Travel time estimation (for background velocity estimation).

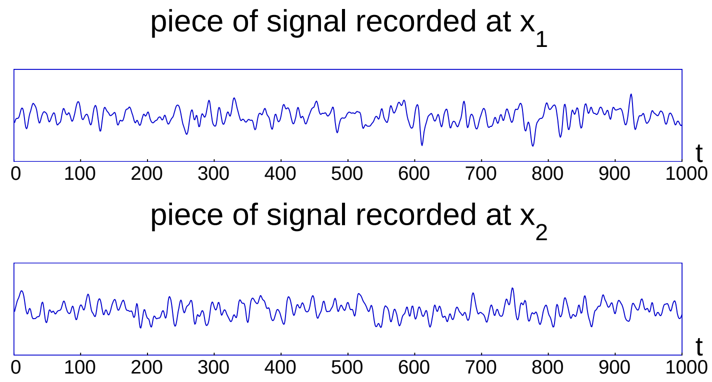
- Method 1: Use of earthquake signals.
- Method 2: Use of seismic noise and cross correlation techniques in order to estimate the Green's function.

In this talk:
Part I: Travel time estimation for background velocity estimation.
Part II: Passive sensor imaging.

Part I: Travel time estimation by cross correlation

- Ambient noise sources ($\circ$) emit stationary random signals.
- The waves propagate in the (inhomogeneous) medium.
- The signals $u(t, \boldsymbol{x}_1)$ and $u(t, \boldsymbol{x}_2)$ are recorded at two sensors $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$.





- What information (about the medium) can possibly be in these signals ?

Part I: Travel time estimation by cross correlation

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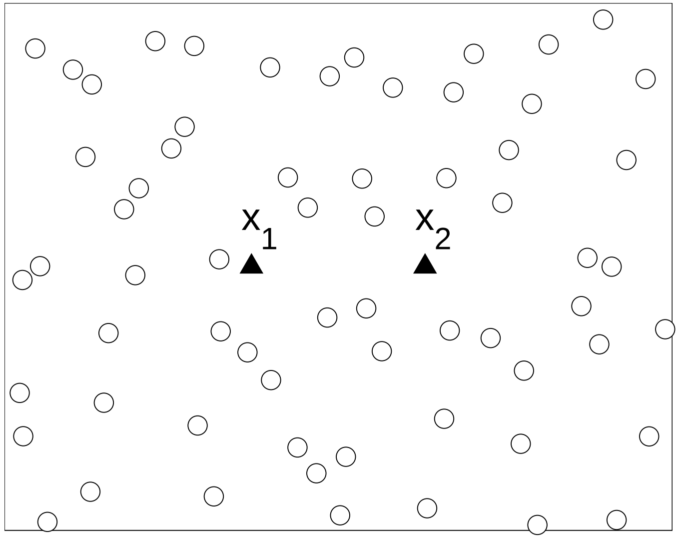
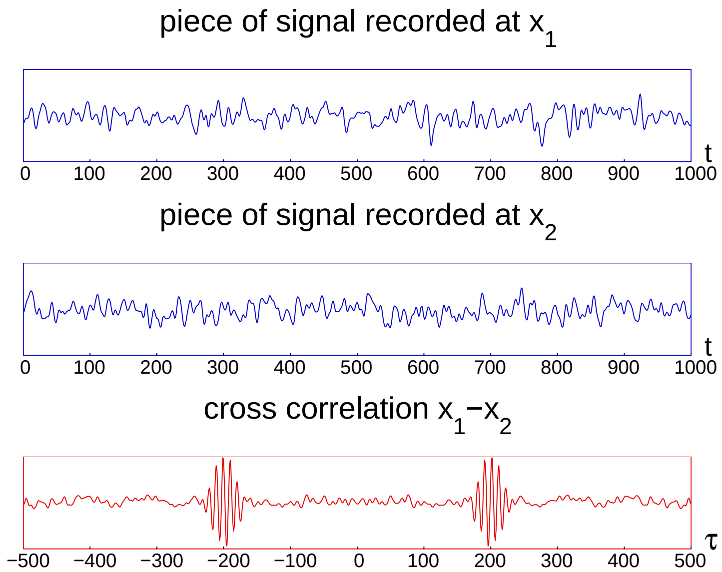


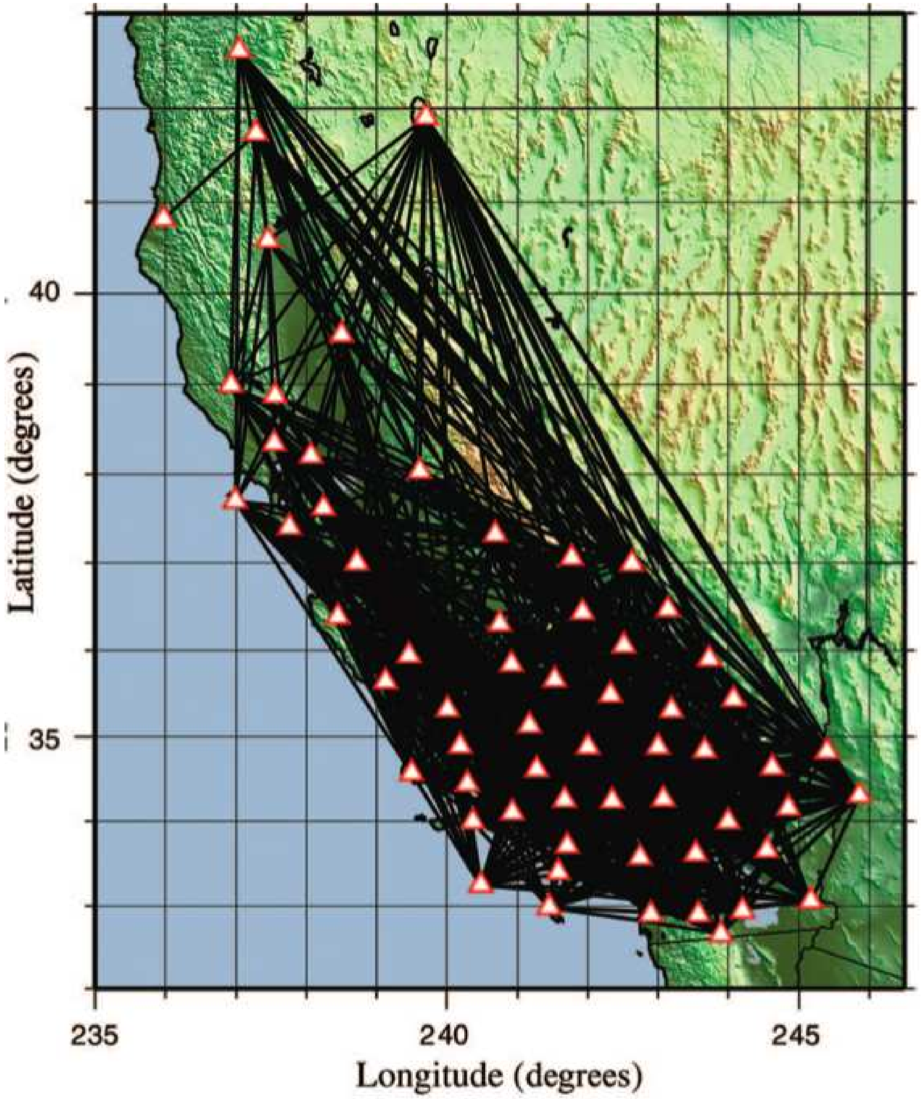


- Compute the empirical cross correlation:

$$C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{1}{T} \int_0^T u(t, \boldsymbol{x}_1) u(t+\tau, \boldsymbol{x}_2) dt$$

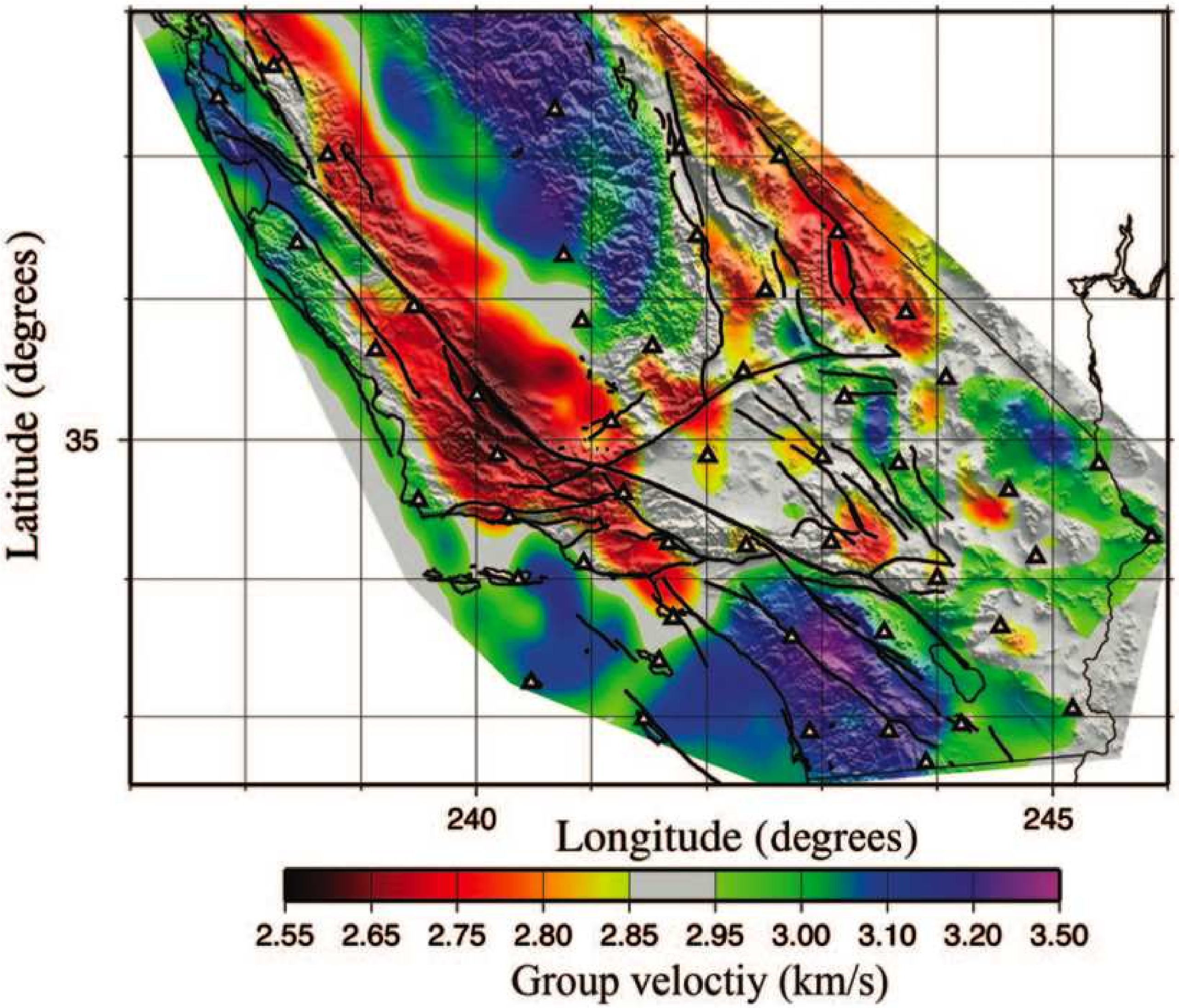
- $C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2)$ is related to the Green's function from $\boldsymbol{x}_1$ to $\boldsymbol{x}_2$!
- The singular component of the Green's function from $\boldsymbol{x}_1$ to $\boldsymbol{x}_2$ gives the travel time from $\boldsymbol{x}_1$ to $\boldsymbol{x}_2$.

Estimations of travel times between pairs of sensors



Surface (Rayleigh) waves [from Shapiro, Campillo, et al, Science 307 (2005), 1615]

Background velocity estimation from travel time estimations



[from Shapiro, Campillo, et al, Science 307 (2005), 1615]

Wave equation

$$\frac{1}{c^2(\boldsymbol{x})}\frac{\partial^2 u}{\partial t^2} - \Delta_{\boldsymbol{x}} u = n(t,\boldsymbol{x})$$

- Sources $n(t,\boldsymbol{x})$: Gaussian process, with mean zero, with covariance

$$\langle n(t_1,\boldsymbol{y}_1)n(t_2,\boldsymbol{y}_2)\rangle = F(t_2-t_1)\Gamma(\boldsymbol{y}_1,\boldsymbol{y}_2)$$

- Stationary in time: $\big(n(t,\boldsymbol{y})\big)_{t,\boldsymbol{y}}$ et $\big(n(t+h,\boldsymbol{y})\big)_{t,\boldsymbol{y}}$ have the same statistical distribution for any $h \Longrightarrow$ the time correlation function F depends only on $t_2 - t_1$.

- The spatial distribution of the sources is characterized by $\Gamma(\boldsymbol{y}_1,\boldsymbol{y}_2)$. To simplify:

$$\Gamma(\boldsymbol{y}_1,\boldsymbol{y}_2) = K(\boldsymbol{y}_1)\delta(\boldsymbol{y}_1-\boldsymbol{y}_2)$$

The function K characterizes the spatial support of the sources.
For more general Γ, use microlocal analysis [1].

[1] C. Bardos, J. Garnier, and G. Papanicolaou, *Inverse Problems* **24**, 015011 (2008).

Empirical cross correlation and statistical cross correlation

Empirical cross correlation:

$$C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{1}{T} \int_0^T u(t, \boldsymbol{x}_1) u(t + \tau, \boldsymbol{x}_2) dt$$

with $u(t, \boldsymbol{x}) = \iint G(s, \boldsymbol{x}, \boldsymbol{y}) n(t - s, \boldsymbol{y}) ds d\boldsymbol{y}$ and G = causal time-dependent Green's function.

1. The expectation of C_T (with respect to the distribution of the sources) is independent of the integration time T:

$$\langle C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) \rangle = C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2)$$

where the statistical cross correlation $C^{(1)}$ is given by

$$C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{1}{2\pi} \int d\boldsymbol{y} \int d\omega \overline{\hat{G}}(\omega, \boldsymbol{x}_1, \boldsymbol{y}) \hat{G}(\omega, \boldsymbol{x}_2, \boldsymbol{y}) K(\boldsymbol{y}) \hat{F}(\omega) e^{-i\omega\tau}$$

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2. The empirical cross correlation is a self-averaging quantity:

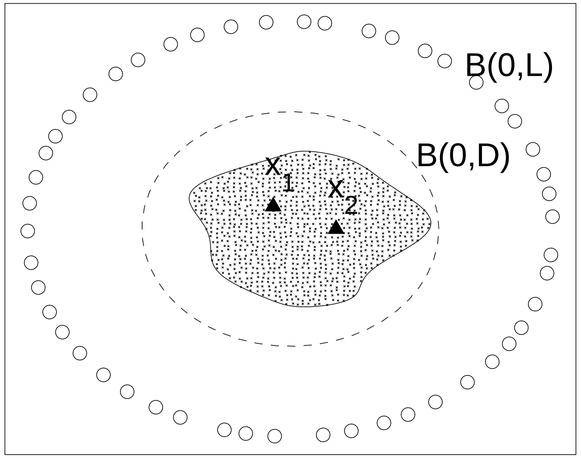
$$C_T(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) \stackrel{T\to\infty}{\longrightarrow} C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2)$$

in probability.
More precisely, the fluctuations of C_T around its expectation $C^{(1)}$ are of order $T^{-1/2}$.

The ideal situation

Cross correlation with noise sources distributed on a closed surface $\partial B(\mathbf{0}, L)$:

$$C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{1}{2\pi} \int d\omega \int_{\partial B(\mathbf{0},L)} dS(\boldsymbol{y}) \overline{\hat{G}}(\omega, \boldsymbol{x}_1, \boldsymbol{y}) \hat{G}(\omega, \boldsymbol{x}_2, \boldsymbol{y}) \hat{F}(\omega) e^{-i\omega\tau}$$



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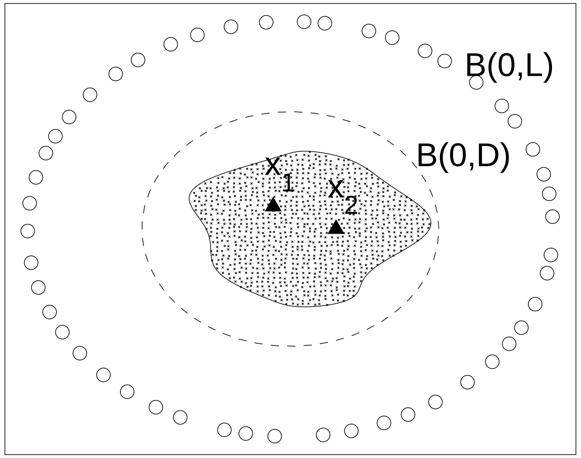

Helmholtz-Kirchhoff theorem:

$$\hat{G}(\omega, \boldsymbol{x}_1, \boldsymbol{x}_2) - \overline{\hat{G}}(\omega, \boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{2i\omega}{c_e} \int_{\partial B(\mathbf{0},L)} dS(\boldsymbol{y}) \overline{\hat{G}}(\omega, \boldsymbol{x}_1, \boldsymbol{y}) \hat{G}(\omega, \boldsymbol{x}_2, \boldsymbol{y})$$

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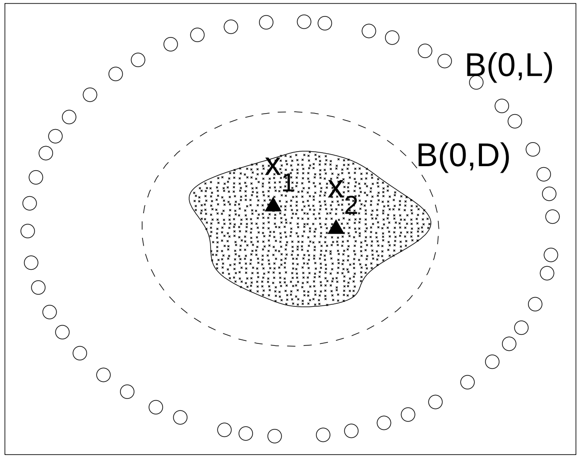

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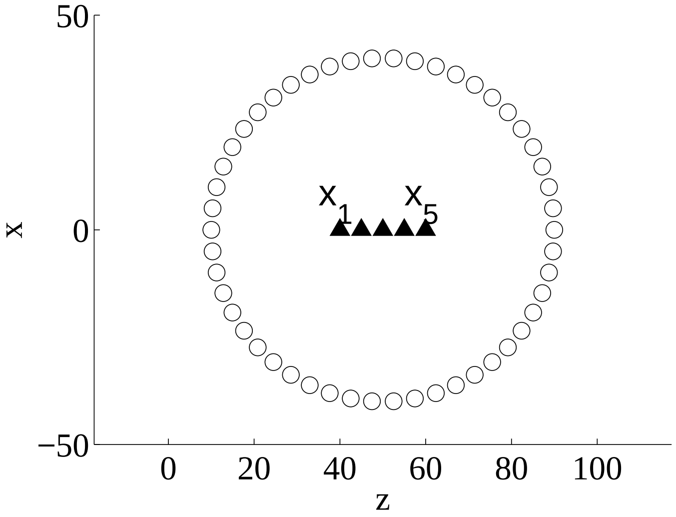
If 1) the medium is homogeneous outside $B(\mathbf{0}, D)$,

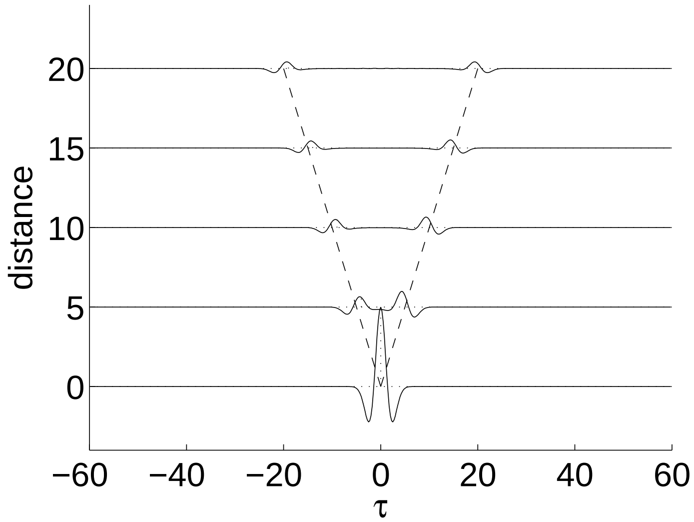
2) the sources are distributed uniformly on the sphere $\partial B(\mathbf{0}, L)$, with $L \gg D$.

Then for any $\boldsymbol{x}_1, \boldsymbol{x}_2 \in B(\mathbf{0}, D)$ we have (up to a multiplicative constant):

$$\frac{\partial}{\partial \tau} C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = F *_\tau G(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) - F *_\tau G(-\tau, \boldsymbol{x}_1, \boldsymbol{x}_2)$$

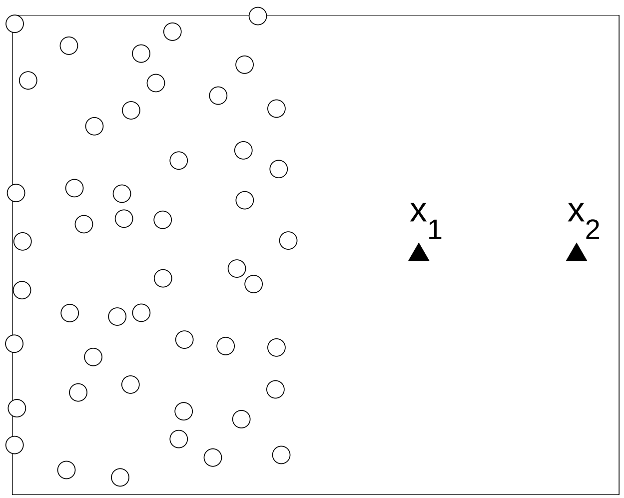
Ideal situation:

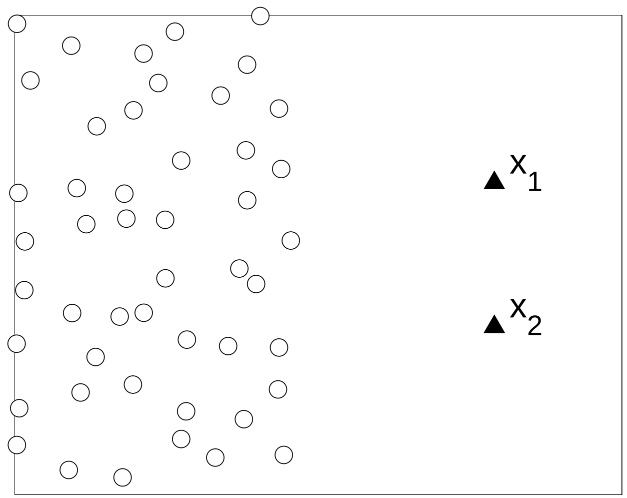




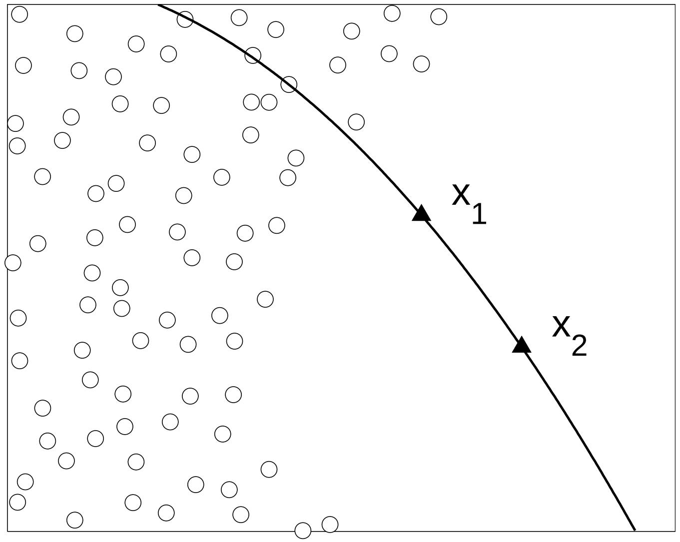
Configuration $\qquad$ $C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_j)$

What about situations such as

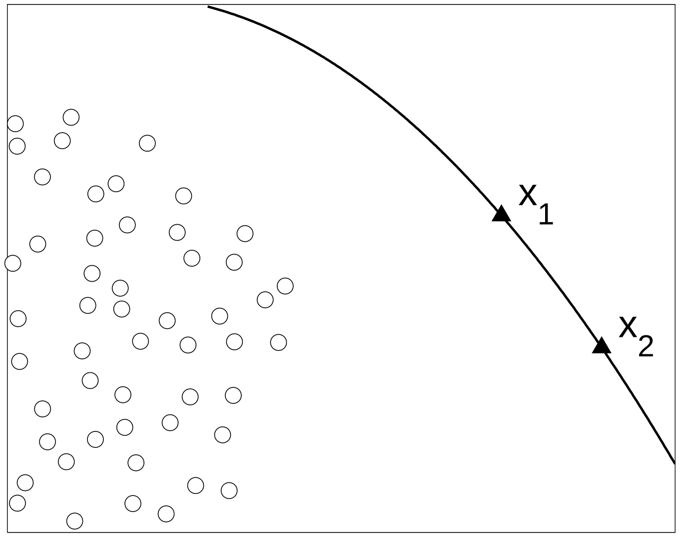




?



Singular component at $\mathcal{T}(\boldsymbol{x}_1, \boldsymbol{x}_2)$



No singular component

High-frequency analysis (geometric optics regime): The cross correlation $C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2)$ has singular components iff the ray joining $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ reaches into the source region (i.e. the support of K). Then there are one or two singular components at $\tau = \pm\mathcal{T}(\boldsymbol{x}_1, \boldsymbol{x}_2)$.

[More exactly:

the rays $\boldsymbol{y} \to \boldsymbol{x}_1 \to \boldsymbol{x}_2$ contribute to the singular component at $\tau = \mathcal{T}(\boldsymbol{x}_1, \boldsymbol{x}_2)$,

the rays $\boldsymbol{y} \to \boldsymbol{x}_2 \to \boldsymbol{x}_1$ contribute to the singular component at $\tau = -\mathcal{T}(\boldsymbol{x}_1, \boldsymbol{x}_2)$.]

Sketch of proof: WKB approximation + stationary phase analysis

Time correlation function of the sources $F^\varepsilon(t) = F(\frac{t}{\varepsilon}) \Longrightarrow \hat{F}^\varepsilon(\omega) = \varepsilon \hat{F}(\varepsilon\omega)$.

$$
\begin{aligned}
C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) &= \frac{1}{2\pi} \int d\boldsymbol{y} \int d\omega \, \overline{\hat{G}}(\omega, \boldsymbol{x}_1, \boldsymbol{y}) \hat{G}(\omega, \boldsymbol{x}_2, \boldsymbol{y}) K(\boldsymbol{y}) \varepsilon \hat{F}(\varepsilon\omega) e^{-i\omega\tau} \\
&\overset{\omega \to \frac{\omega}{\varepsilon}}{=} \frac{1}{2\pi} \int d\boldsymbol{y} \int d\omega \, \overline{\hat{G}}\Big(\frac{\omega}{\varepsilon}, \boldsymbol{x}_1, \boldsymbol{y}\Big) \hat{G}\Big(\frac{\omega}{\varepsilon}, \boldsymbol{x}_2, \boldsymbol{y}\Big) K(\boldsymbol{y}) \hat{F}(\omega) e^{-i\frac{\omega}{\varepsilon}\tau}
\end{aligned}
$$

WKB approximation for $\hat{G}\big(\frac{\omega}{\varepsilon}, \boldsymbol{x}, \boldsymbol{y}\big) \sim a(\boldsymbol{x}, \boldsymbol{y}) e^{i\frac{\omega}{\varepsilon}\mathcal{T}(\boldsymbol{x}, \boldsymbol{y})}$:

$$
C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{1}{2\pi} \int d\boldsymbol{y} \int d\omega \, a(\boldsymbol{x}_1, \boldsymbol{y}) a(\boldsymbol{x}_2, \boldsymbol{y}) K(\boldsymbol{y}) \hat{F}(\omega) e^{i\frac{\omega}{\varepsilon}T(\boldsymbol{y})}
$$

with the rapid phase

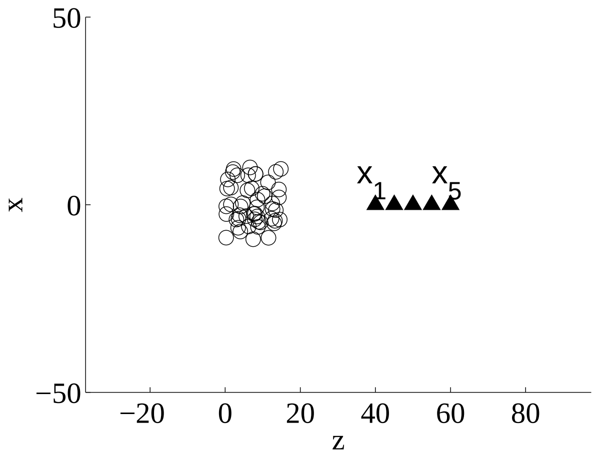
$$
\omega T(\boldsymbol{y}) = \omega[\mathcal{T}(\boldsymbol{x}_2, \boldsymbol{y}) - \mathcal{T}(\boldsymbol{x}_1, \boldsymbol{y}) - \tau]
$$

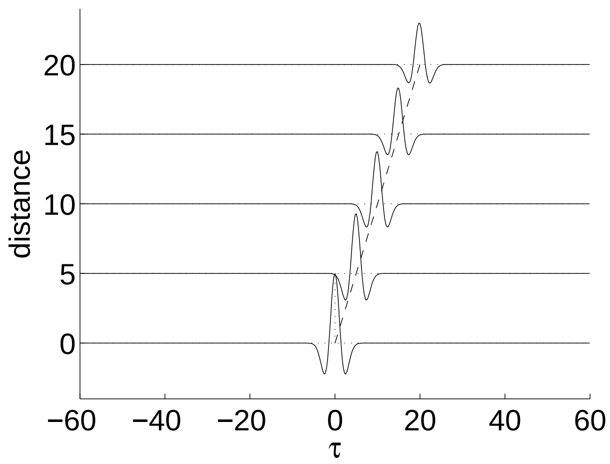
Use of the stationary phase theorem. The dominant contribution comes from the stationary points $(\omega, \boldsymbol{y})$ satisfying:

$$
\nabla_{\boldsymbol{y}} \Big(\omega T(\boldsymbol{y})\Big) = \boldsymbol{0}, \quad \partial_\omega \Big(\omega T(\boldsymbol{y})\Big) = 0
$$

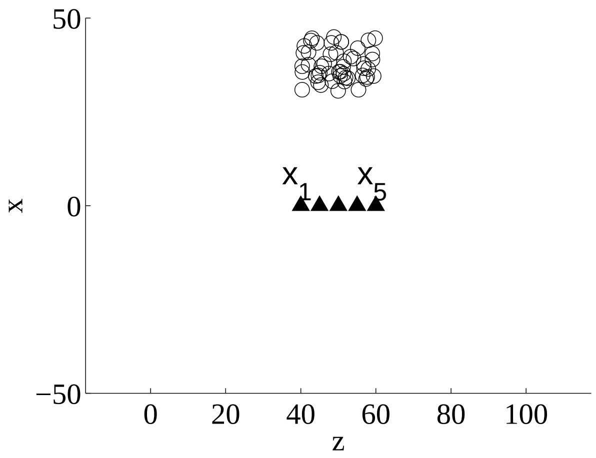
$\hookrightarrow$ two conditions:

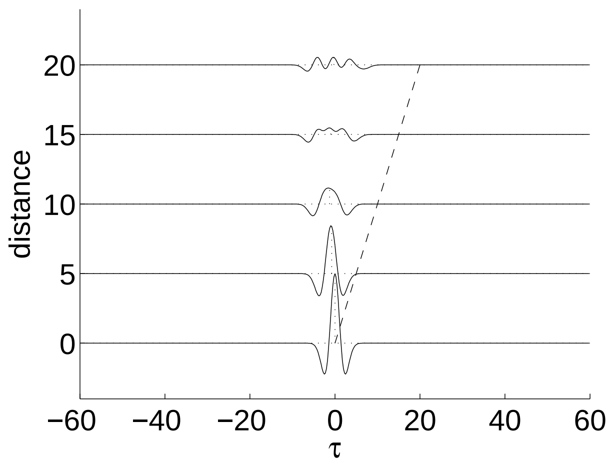
$$
\nabla_{\boldsymbol{y}} \mathcal{T}(\boldsymbol{y}, \boldsymbol{x}_2) = \nabla_{\boldsymbol{y}} \mathcal{T}(\boldsymbol{y}, \boldsymbol{x}_1), \quad \mathcal{T}(\boldsymbol{x}_2, \boldsymbol{y}) - \mathcal{T}(\boldsymbol{x}_1, \boldsymbol{y}) = \tau
$$



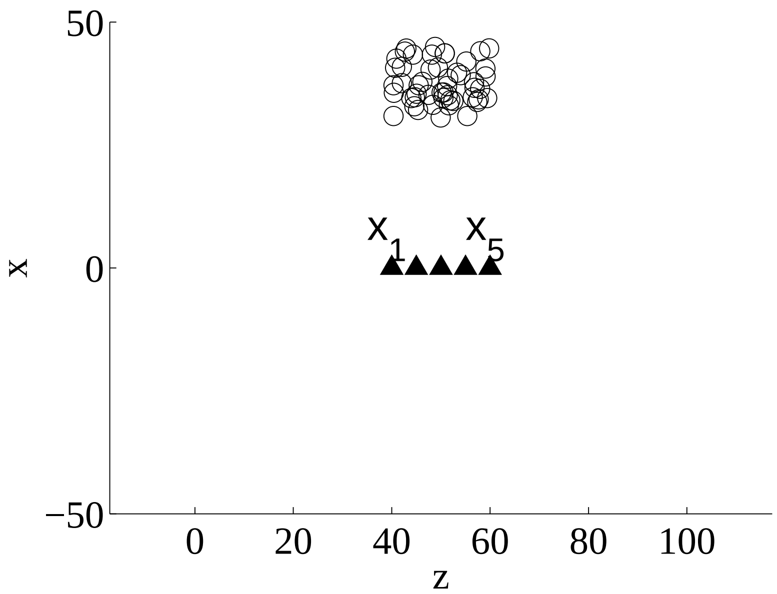


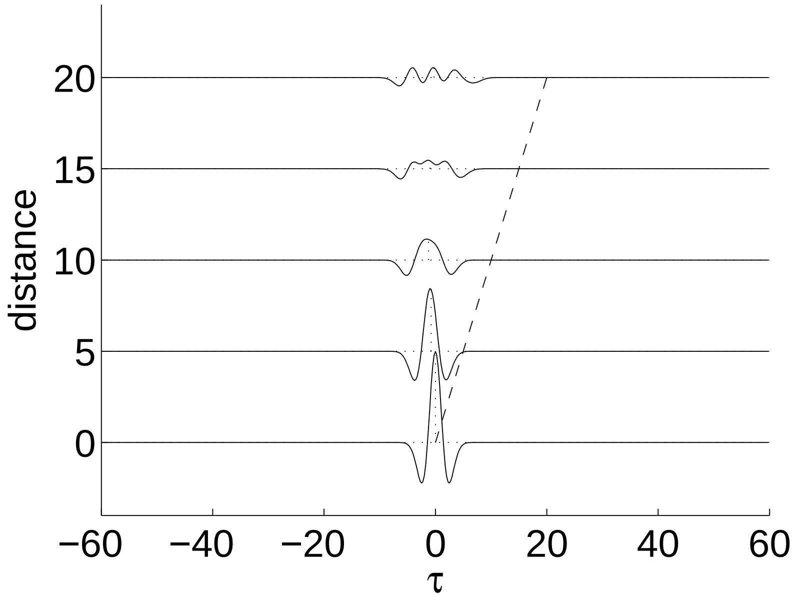
a) Favorable configuration **b)** $C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_j)$



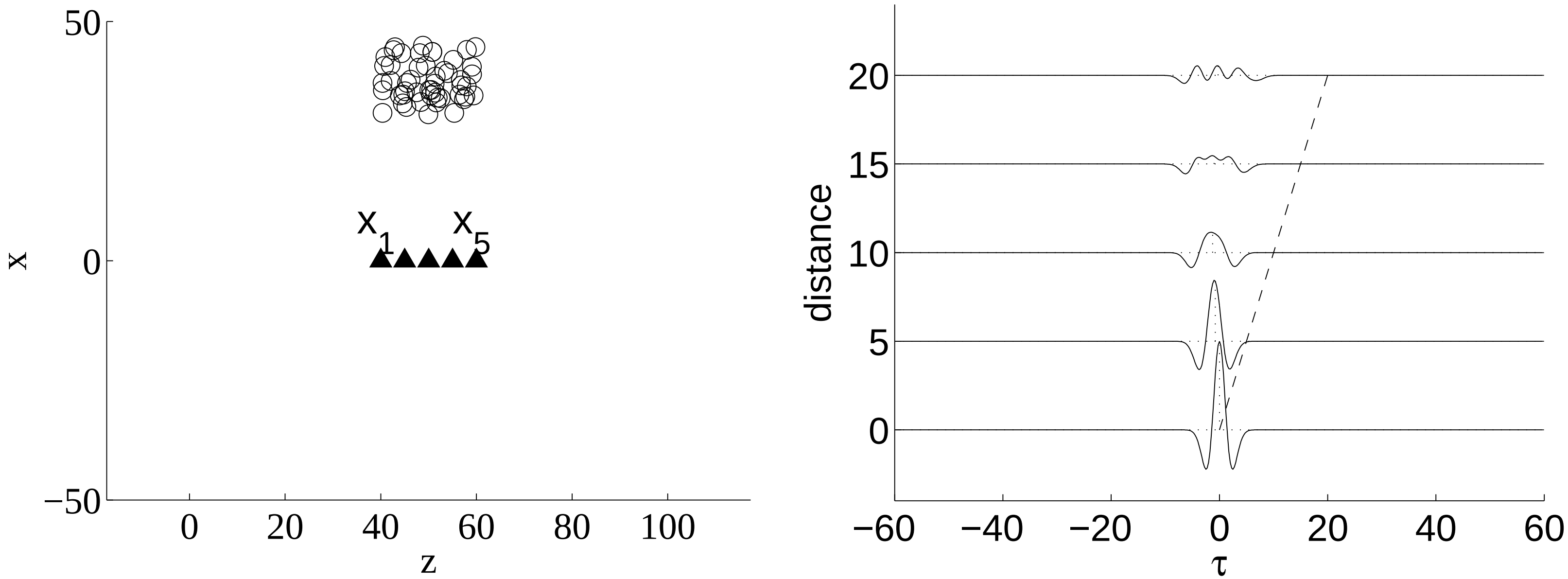


c) Unfavorable configuration **d)** $C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_j)$





• Here, the cross correlation method does not allow for travel time estimation, because there is not enough “directional diversity”.

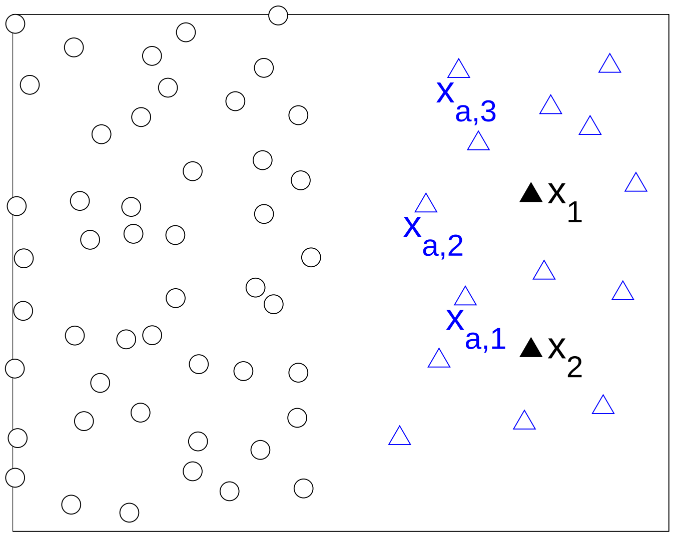


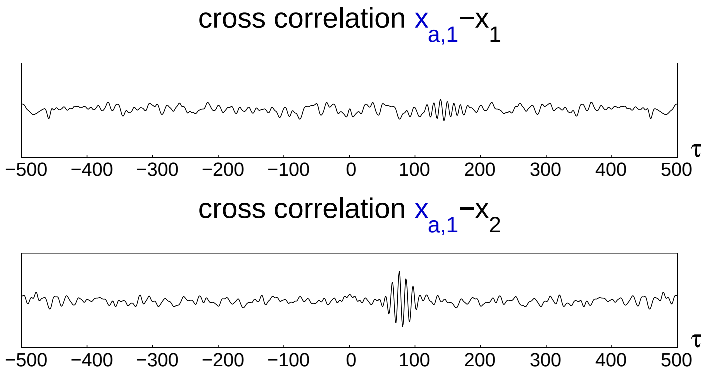
• Here, the cross correlation method does not allow for travel time estimation, because there is not enough "directional diversity".

• Idea (first suggested by M. Campillo [1]): exploit the scattering properties of the medium and use the scatterers as "secondary noise sources".

[1] M. Campillo and L. Stehly, *Eos Trans. AGU* **88**(52) (2007), Fall Meet. Suppl., Abstract S51D-07.

Fourth-order cross correlations for travel time estimation



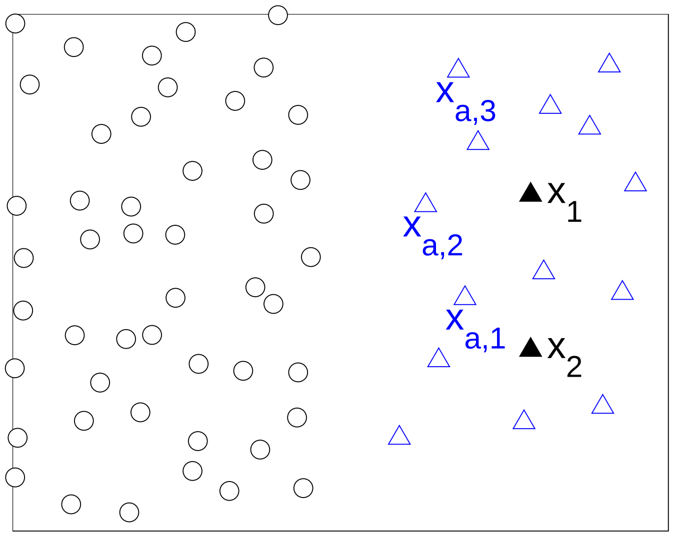


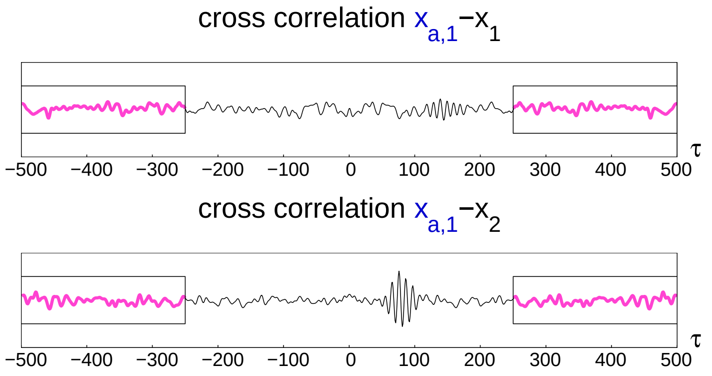
Use of auxiliary sensors $\boldsymbol{x}_{a,j}$, $j = 1, \ldots, N$. Algorithm:

1) for each j, compute the cross correlations $C_T(\tau, \boldsymbol{x}_{a,j}, \boldsymbol{x}_1)$ and $C_T(\tau, \boldsymbol{x}_{a,j}, \boldsymbol{x}_2)$:

$$C_T(\tau, \boldsymbol{x}_{a,j}, \boldsymbol{x}_l) = \frac{1}{T} \int_0^T u(t, \boldsymbol{x}_{a,j}) u(t+\tau, \boldsymbol{x}_l) dt \,, \quad l = 1, 2$$

Fourth-order cross correlations for travel time estimation





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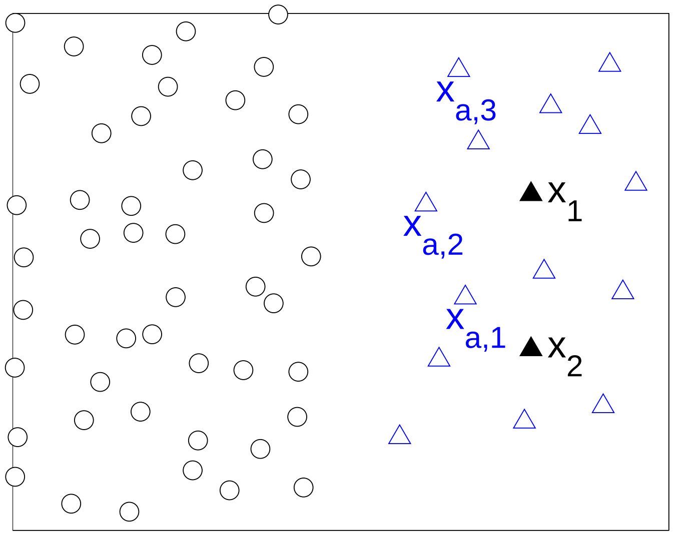
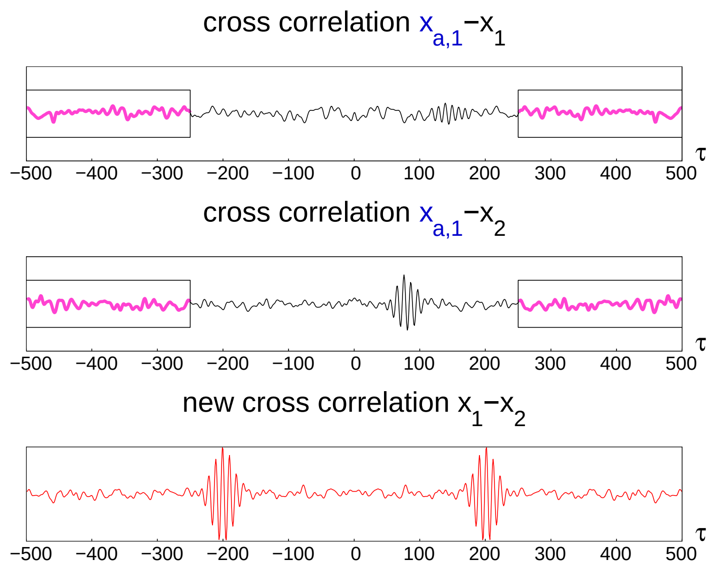
2) consider the tails of $C_T(\tau, \boldsymbol{x}_{a,j}, \boldsymbol{x}_1)$ and $C_T(\tau, \boldsymbol{x}_{a,j}, \boldsymbol{x}_2)$:

$$C_{T,\mathrm{coda}}(\tau, \boldsymbol{x}_{a,j}, \boldsymbol{x}_l) = C_T(\tau, \boldsymbol{x}_{a,j}, \boldsymbol{x}_l)\big[\mathbf{1}_{(-T_{c2},-T_{c1})}(\tau) + \mathbf{1}_{(T_{c1},T_{c2})}(\tau)\big], \quad l = 1, 2$$

Fourth-order cross correlations for travel time estimation





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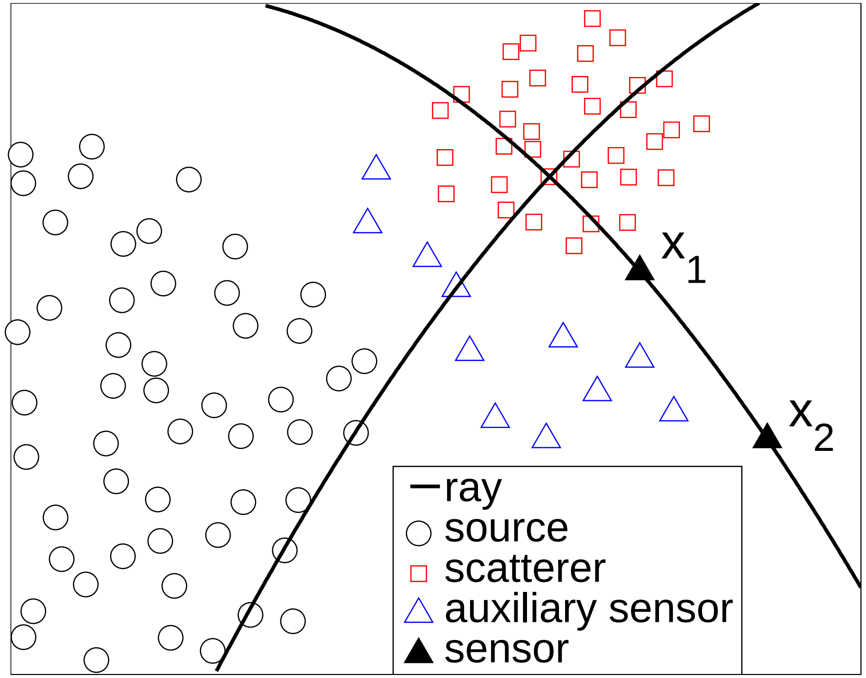
3) compute the cross correlations between the tails and sum over j:

$$C_T^{(3)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_2) = \sum_{j=1}^N \int_{-\infty}^{\infty} C_{T,\mathrm{coda}}(\tau', \boldsymbol{x}_{a,j}, \boldsymbol{x}_1) C_{T,\mathrm{coda}}(\tau'+\tau, \boldsymbol{x}_{a,j}, \boldsymbol{x}_2) d\tau'$$



Using asymptotic analysis [1]: $C^{(3)}$ has singular components if

1) there are scatterers along the ray joining $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$.

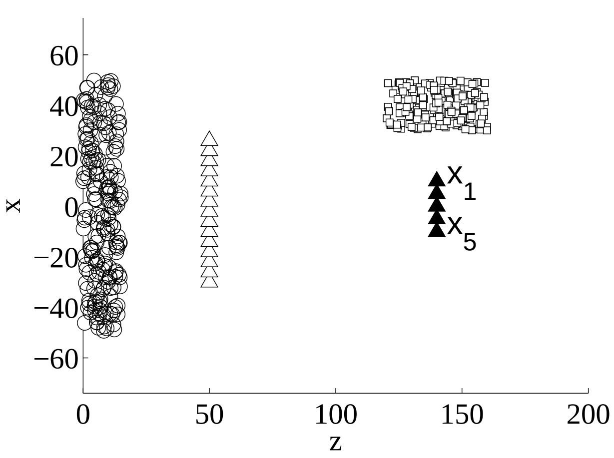
2) there are auxiliary sensors along rays joining sources and scatterers.

These singular components are at $\tau = \pm\mathcal{T}(\boldsymbol{x}_1, \boldsymbol{x}_2)$.

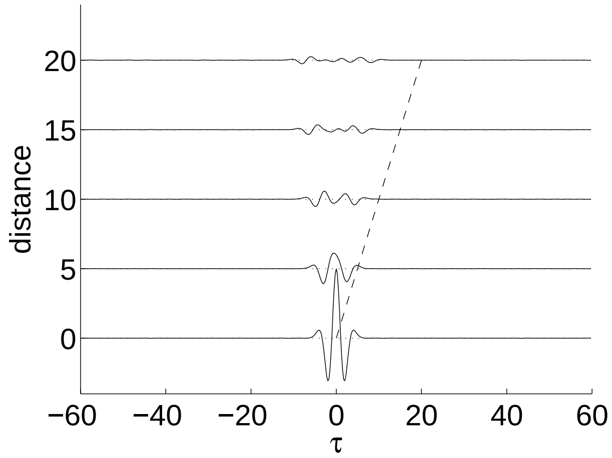
It is not required that the ray joining $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ reaches into the source region !

If the scattering region covers the region of interest or surrounds it, then $C^{(3)}$ has singular components at $\tau = \pm\mathcal{T}(\boldsymbol{x}_1, \boldsymbol{x}_2)$!

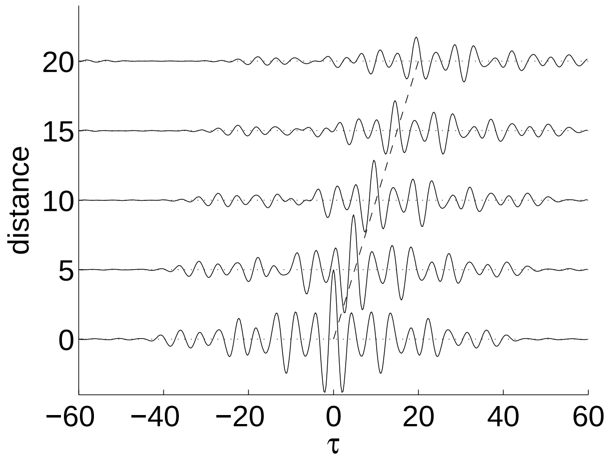
[1] J. Garnier and G. Papanicolaou, *SIAM J. Imaging Sciences* **2**, 396 (2009).



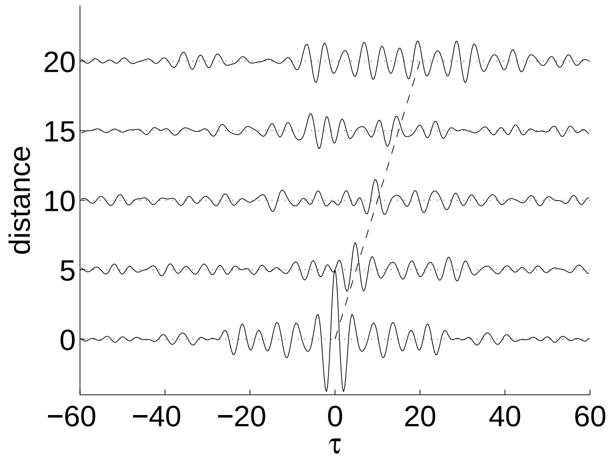
a) Configuration



b) $C^{(1)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_j)$



c) $C^{(3)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_j)$
with auxiliary sensors

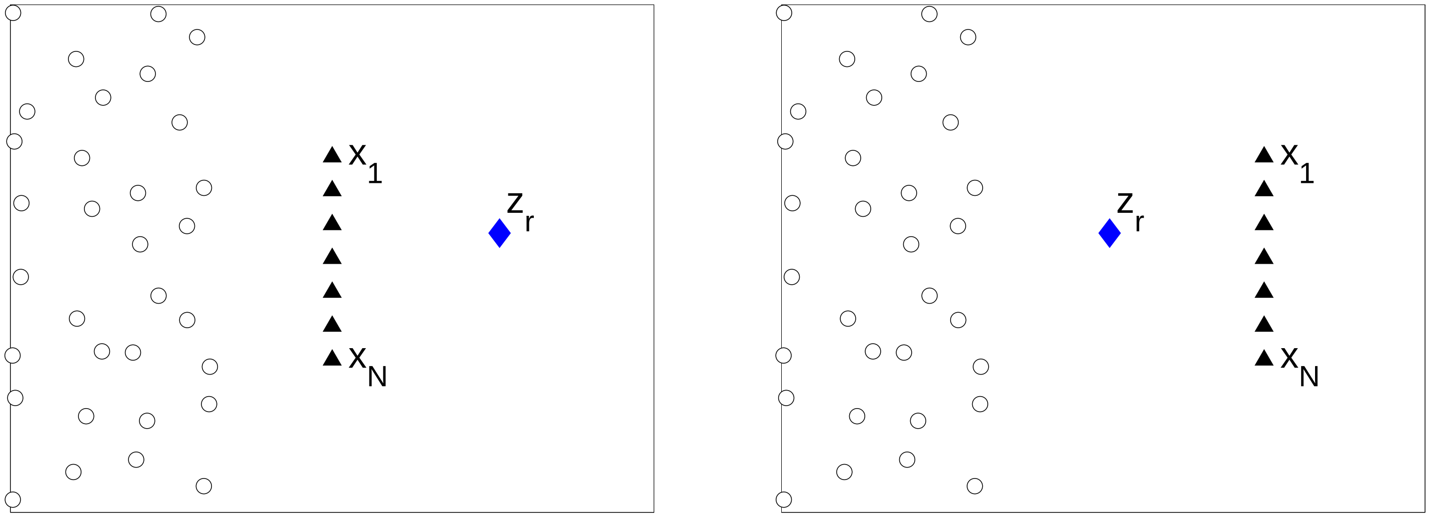


d) $C^{(3)}(\tau, \boldsymbol{x}_1, \boldsymbol{x}_j)$
without auxiliary sensors

Part II: Passive sensor imaging of reflectors

- Array of passive sensors $\boldsymbol{x}_j$, $j = 1, \dots, N$
- Ambient noise sources ($\circ$) emitting stationary random signals
- Target at $\boldsymbol{z}_{\rm r}$ (small reflector to be imaged)

- Two different illumination configurations:



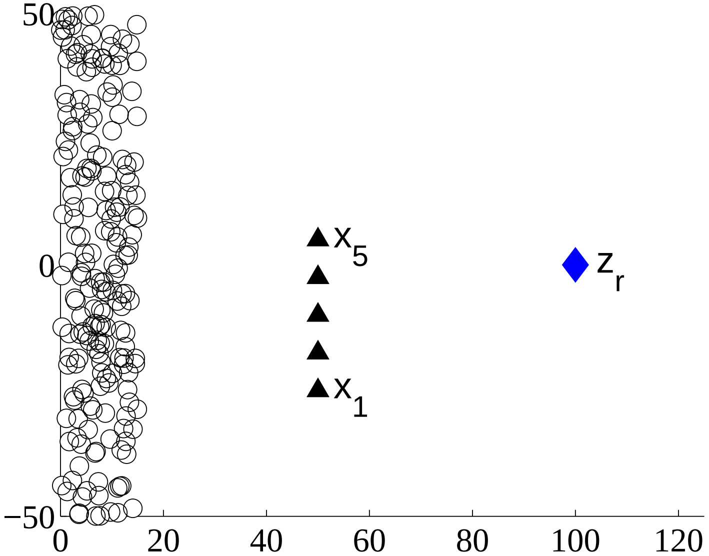
Daylight illumination   Backlight illumination

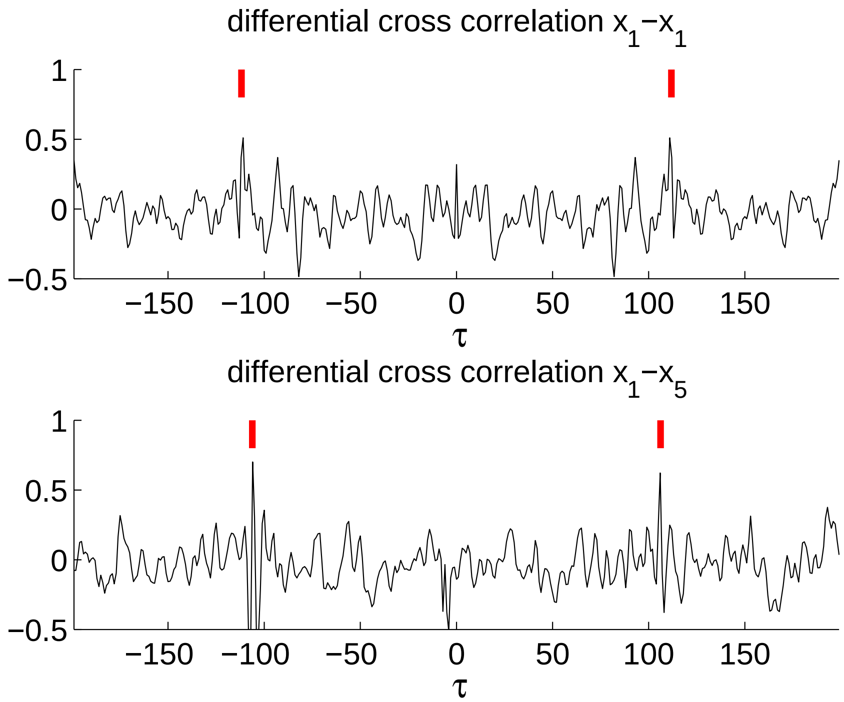
- Here:

The background medium is homogeneous or smooth, and the travel times between the sensors and points in the search region are known.
Data in the absence (C_0) and in the presence (C) of the reflector are available.

Daylight illumination

• Data in the absence ($C_0(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l)$, $j, l = 1, \ldots, N$) and in the presence ($C(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l)$, $j, l = 1, \ldots, N$) of the reflector





Differential cross correlation:

$$\Delta C(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l) = (C - C_0)(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l)\,, \quad j, l = 1, \ldots, N$$

Theory: $\Delta C(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l)$ *has singular components at* $\tau = \pm[\mathcal{T}(\boldsymbol{x}_j, \boldsymbol{z}_{\rm r}) + \mathcal{T}(\boldsymbol{x}_l, \boldsymbol{z}_{\rm r})]$.

Daylight illumination - migration

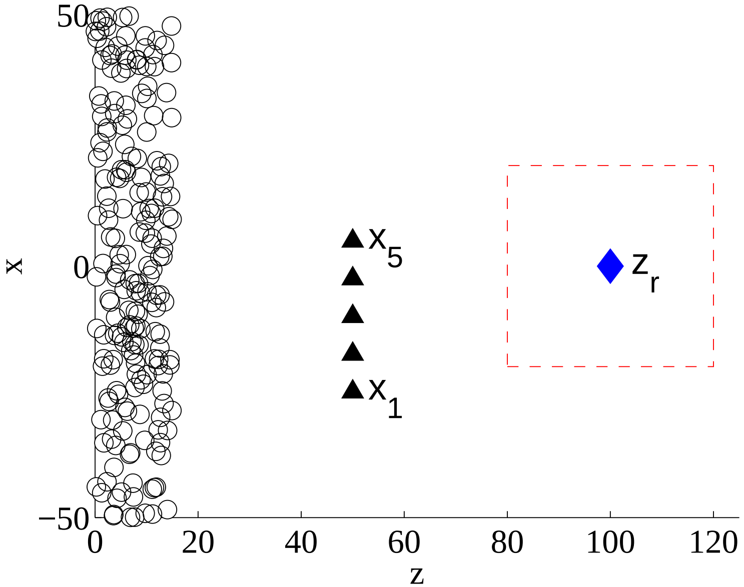
$$\Delta C(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l) = (C - C_0)(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l)$$

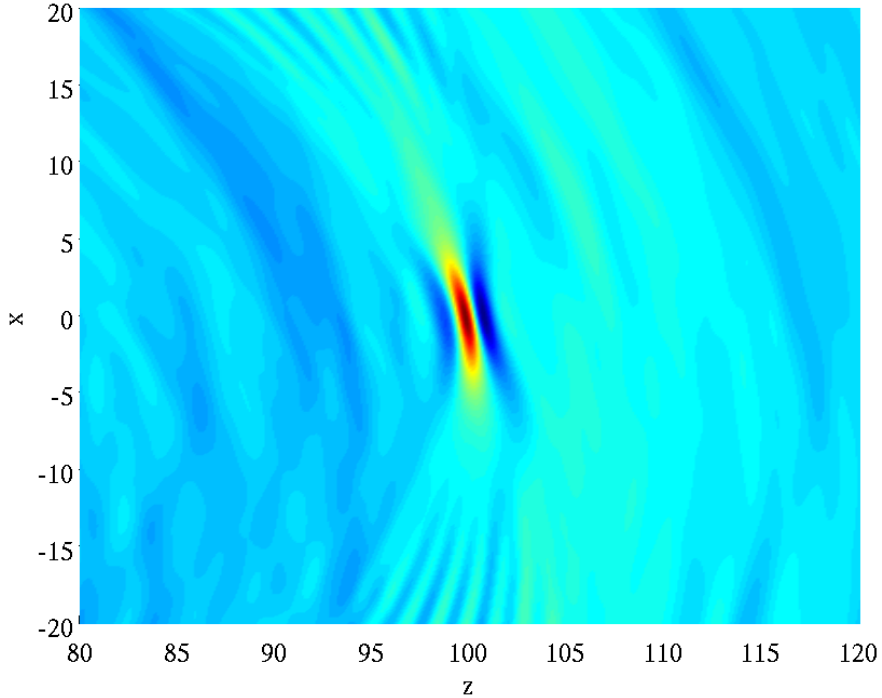
Theory: $\Delta C(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l)$ *has singular components at* $\tau = \pm[\mathcal{T}(\boldsymbol{x}_j, \boldsymbol{z}_{\mathrm{r}}) + \mathcal{T}(\boldsymbol{x}_l, \boldsymbol{z}_{\mathrm{r}})]$.

- Migration of the differential cross correlations ΔC.

Migration functional for the search point $\boldsymbol{z}^S$:

$$\mathcal{I}^{\mathrm{D}}(\boldsymbol{z}^S) = \sum_{j,l=1}^{N} \Delta C\big(\mathcal{T}(\boldsymbol{x}_j, \boldsymbol{z}^S) + \mathcal{T}(\boldsymbol{x}_l, \boldsymbol{z}^S), \boldsymbol{x}_j, \boldsymbol{x}_l\big)$$





Backlight illumination - migration

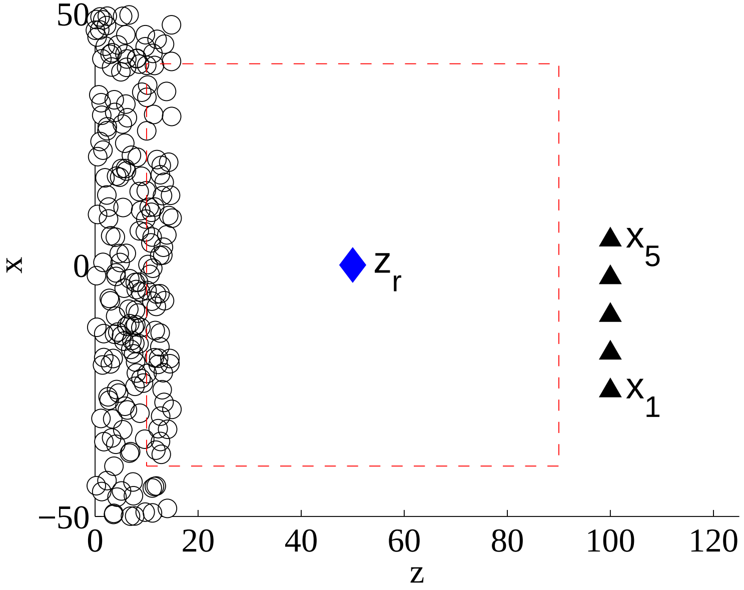
$$\Delta C(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l) = (C - C_0)(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l)$$

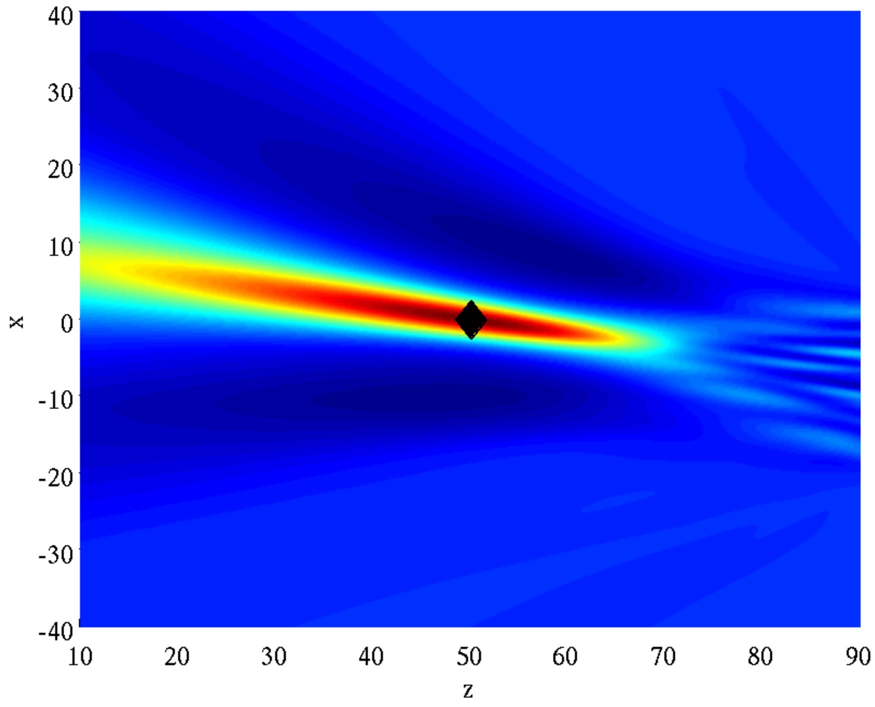
Theory: $\Delta C(\tau, \boldsymbol{x}_j, \boldsymbol{x}_l)$ *has singular components at* $\tau = \mathcal{T}(\boldsymbol{x}_l, \boldsymbol{z}_{\rm r}) - \mathcal{T}(\boldsymbol{x}_j, \boldsymbol{z}_{\rm r})$.

- Migration of the differential cross correlations ΔC.

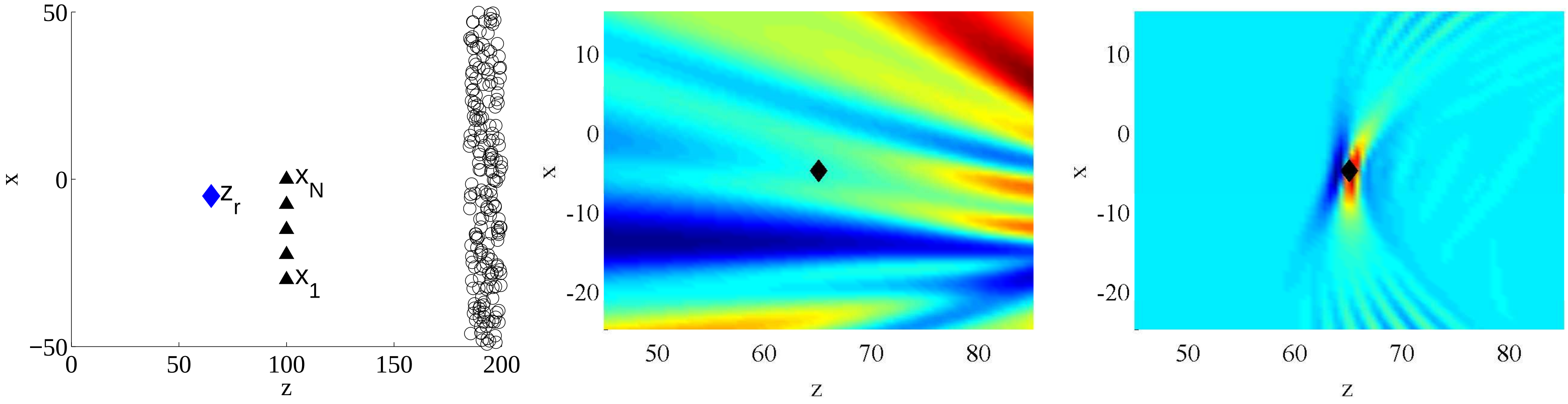
Migration functional for the search point $\boldsymbol{z}^S$:

$$\mathcal{I}^{\rm B}(\boldsymbol{z}^S) = \sum_{j,l=1}^{N} \Delta C\big(\mathcal{T}(\boldsymbol{x}_l, \boldsymbol{z}^S) - \mathcal{T}(\boldsymbol{x}_j, \boldsymbol{z}^S), \boldsymbol{x}_j, \boldsymbol{x}_l\big)$$





Imaging with daylight illumination



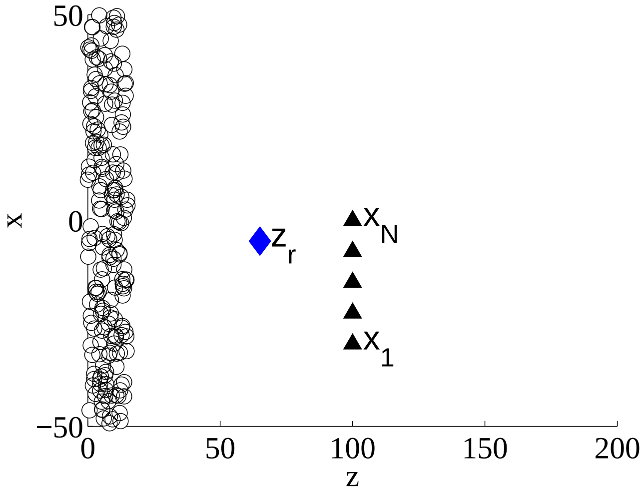
Configuration $\mathcal{I}^{\mathrm{B}}$ $\mathcal{I}^{\mathrm{D}}$

Passive sensor imaging using the differential cross correlation technique in a homogeneous medium. The daylight illumination configuration is plotted in the left figure: the circles are the noise sources and the triangles are the sensors.

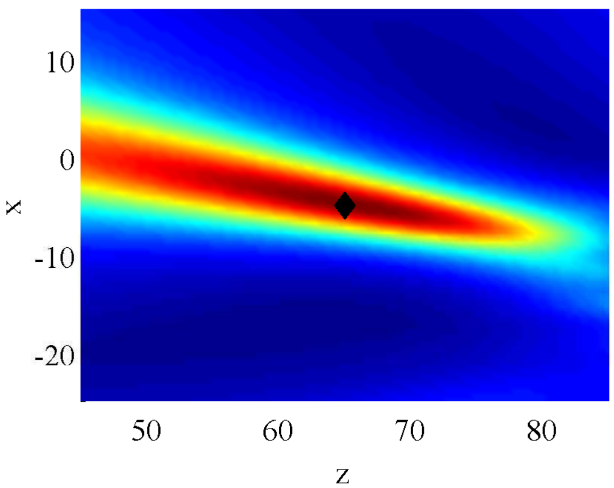
Good range resolution ($\sim c_0/B$), good cross range resolution ($\sim \lambda_0 L/a$) for $\mathcal{I}^{\mathrm{D}}$ (sum of travel times is used in $\mathcal{I}^{\mathrm{D}}$, very sensitive to range).

J. Garnier and G. Papanicolaou, *SIAM J. Imaging Sciences* **2**, 396 (2009).

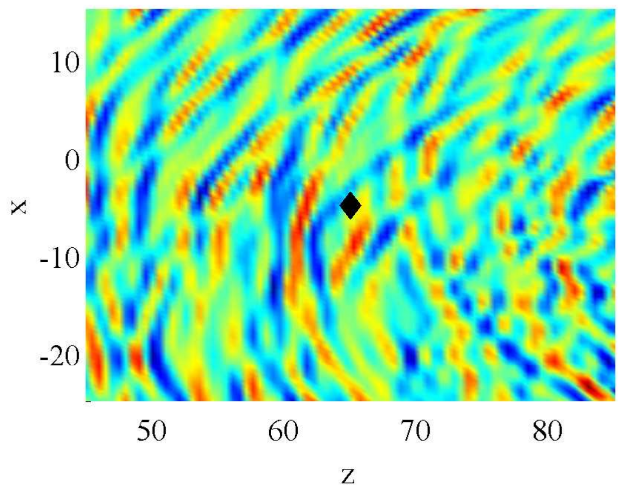
Imaging with backlight illumination



Configuration



$\mathcal{I}^{\mathrm{B}}$



$\mathcal{I}^{\mathrm{D}}$

Passive sensor imaging using the differential cross correlation technique in a homogeneous medium. The backlight illumination configuration is plotted in left figure: the circles are the noise sources and the triangles are the sensors.

Poor range resolution ($\sim \lambda_0 L^2/a^2$), good cross range resolution ($\sim \lambda_0 L/a$) for $\mathcal{I}^{\mathrm{B}}$ (difference of travel times is used in $\mathcal{I}^{\mathrm{B}}$, not sensitive to range).

J. Garnier and G. Papanicolaou, *SIAM J. Imaging Sciences* **2**, 396 (2009).

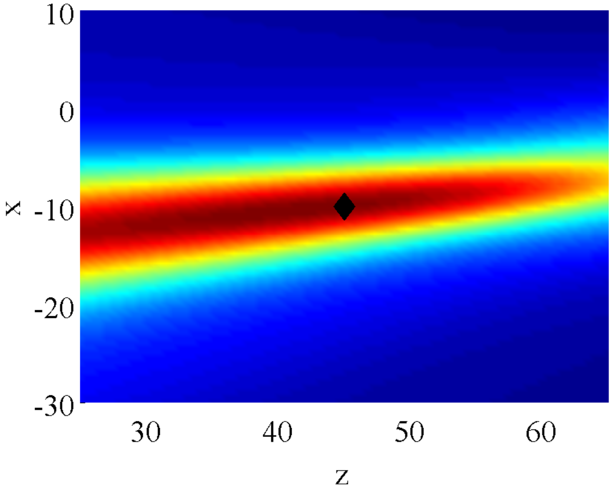
Imaging in a randomly scattering medium

What if the background is not homogeneous (or smoothly varying) but scattering ?

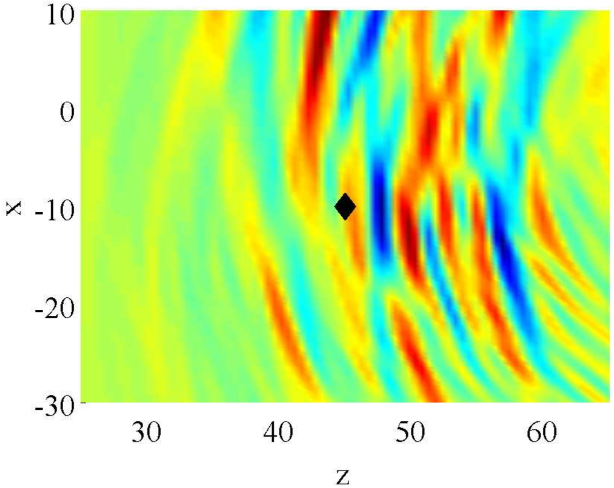
A scattering medium plays a dual role: it enhances the directional diversity of the illumination but it blurs the image.

Migration of $C^{(1)}$ and/or $C^{(3)}$ with backlight and/or daylight imaging functionals.

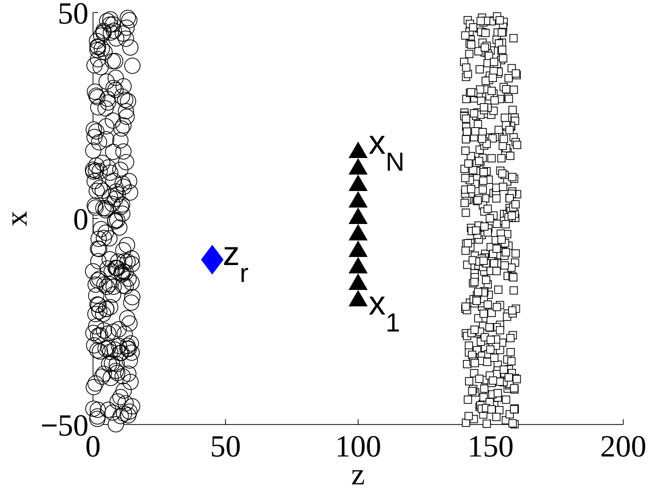
Imaging with backlight illumination and with scattering



$\mathcal{I}^{\mathrm{B}}$ with $C^{(1)}$

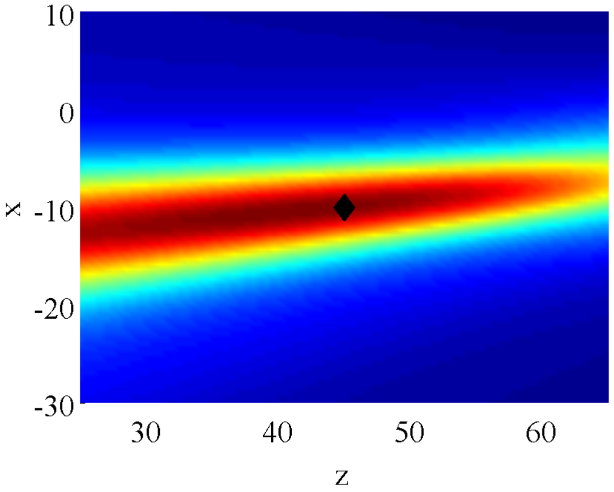


$\mathcal{I}^{\mathrm{D}}$ with $C^{(1)}$

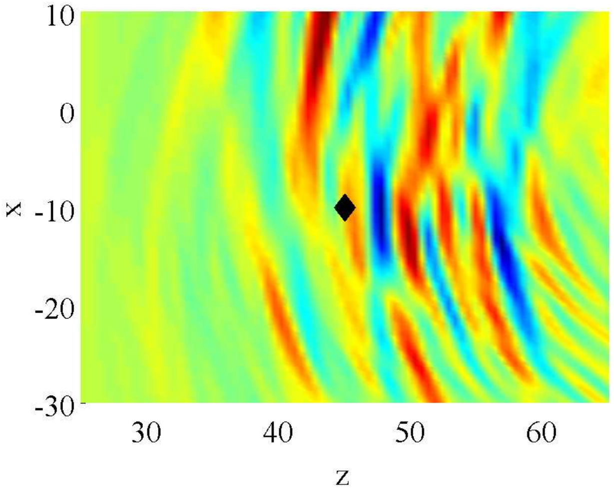


Configuration

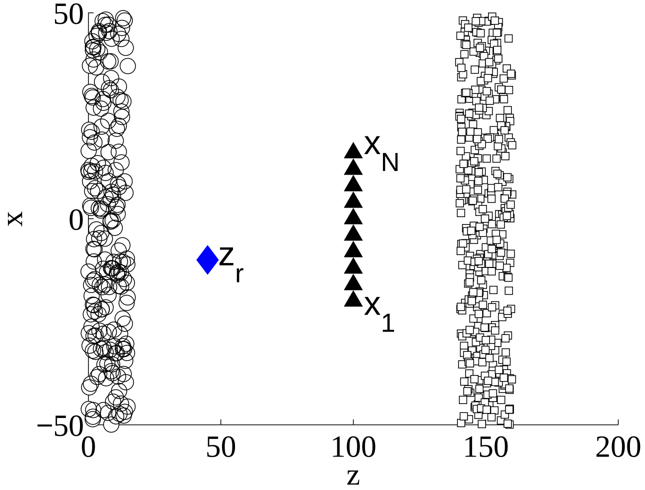
Imaging with backlight illumination and with scattering



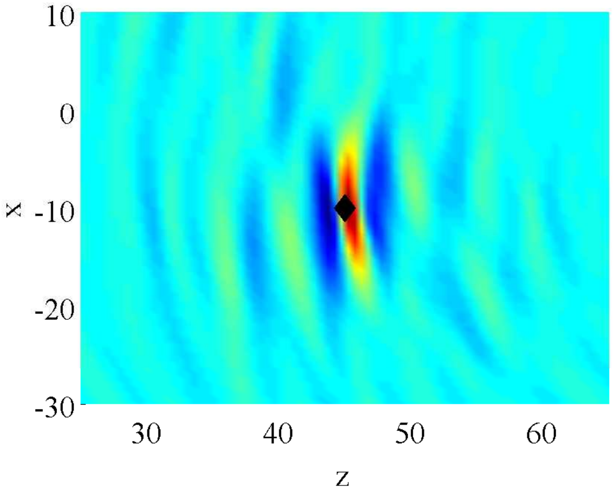
$\mathcal{I}^{\mathrm{B}}$ with $C^{(1)}$



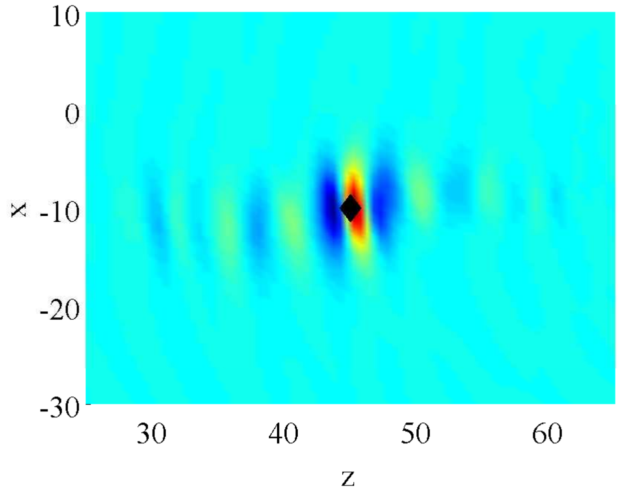
$\mathcal{I}^{\mathrm{D}}$ with $C^{(1)}$



Configuration



$\mathcal{I}^{\mathrm{D}}$ with $C^{(3)}$



$\mathcal{I}^{\mathrm{D}}$ with $C^{(3)}$

times $\mathcal{I}^{\mathrm{B}}$ with $C^{(1)}$

Conclusion

- Travel time estimation, source localization, and imaging of reflectors are possible using cross correlation of ambient noise signals.

- It is possible to exploit the scattering properties of the medium for travel time estimation using special fourth-order cross correlation.

- It is possible to exploit the scattering properties of the medium for imaging by cross correlating and migrating the coda cross correlations ($\mathcal{I}^{\rm D}$ and/or $\mathcal{I}^{\rm B}$ migration with $C^{(1)}$ and/or $C^{(3)}$).

Conclusion

- Travel time estimation, source localization, and imaging of reflectors are possible using cross correlation of ambient noise signals.

- It is possible to exploit the scattering properties of the medium for travel time estimation using special fourth-order cross correlation.

- It is possible to exploit the scattering properties of the medium for imaging by cross correlating and migrating the coda cross correlations ($\mathcal{I}^{\rm D}$ and/or $\mathcal{I}^{\rm B}$ migration with $C^{(1)}$ and/or $C^{(3)}$).

- The use of generalized Radon transform improves migration resolution (better than Kirchhoff migration).

- Other propagation regimes can be analyzed: parabolic approximation, radiative transfer, randomly layered media.

- Main applications in geophysics (global, regional, and local scales: volcano monitoring, oil reservoir monitoring). Also in microwave imaging.