Slow or no heating in many-body quantum systems

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Joint work with

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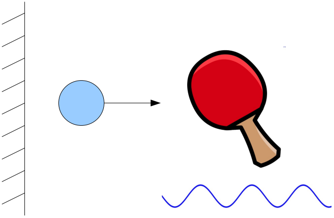
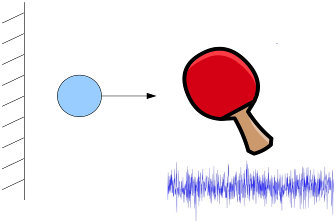
Ergodicity breaking: Ping-pong (... Fermi acceleration)

Periodic player



Hectic player



Ergodicity breaking:

$$E(t) < E_{max}, \quad S(t) < S_{max}$$

(See e.g. D. Dolgopyat and J. De Simoi)

Ergodicity:

$$E(t) \sim t, \quad S(t) \to \infty$$

Ergodicity breaking: many-body quantum systems

Overall question:

Similar dichotomy in many-body quantum systems?

Outline of the talk

- Define ‘ergodicity breaking’
- Linear response
- Beyond linear response: pre-thermalization in ergodic systems
- Beyond linear response: true ergodicity breaking in MBL systems

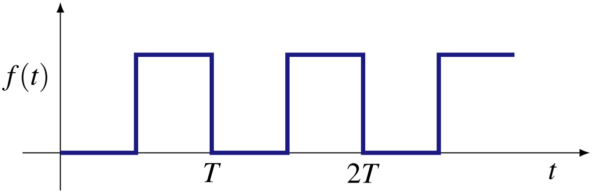
High frequency driving in many-body physics

Time dependent Hamiltonian:

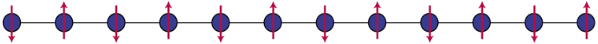
$$H(t) = H^{(0)} + gV(t), \qquad V(t+T) = V(t),$$

where $H^{(0)}$ and V are sum of local terms

E.g.: Spin chain with $V(t) = f(t)\overline{V}$



$$H^{(0)} = \sum_{i\in\Lambda} h_i\sigma_i^{(z)} + J_i\sigma_i^{(z)}\sigma_{i+1}^{(z)}, \qquad \overline{V} = \sum_{i\in\Lambda} \sigma_i^{(x)}\sigma_{i+1}^{(x)}$$



What means “heating up to infinite temperature?”

Define an effective (= Floquet) Hamiltonian H_{eff}

$$U(t_0, t_0 + T) = \mathrm{e}^{-iH_{\mathit{eff}} T}$$

Two cases in the thermodynamic limit $\Lambda \to \infty$:

- H_{eff} is itself a sum of (quasi)-local terms:

  $$H_{\mathit{eff}} = \sum_{i \in \Lambda} H_{\mathit{eff},i}$$

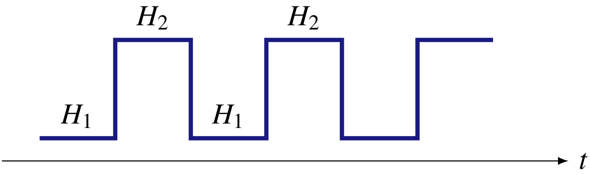
  H_{eff} makes sense in the limit $\lambda \to \infty$,
  Ergodicity breaking: existence of an effective conserved quantity

- H_{eff} has no structure:
  H_{eff} makes no sense in the limit $\Lambda \to \infty$,
  Heating up to infinite temperature (= maximal entropy)

L. D’Alessio and A. Polkovnikov, Ann. of Phys. 333, 2013

Try to settle the issue via BHC expansion

E.g.: Switch protocol



$$U(0,T) = U(0,T/2)U(T/2,T) = \mathrm{e}^{-\mathrm{i}H_1T/2}\mathrm{e}^{-\mathrm{i}H_2T/2}$$
$$= \exp\Big\{ -\mathrm{i}\Big(\frac{H_1+H_2}{2}T - \mathrm{i}[H_1,H_2]\frac{T^2}{2} + \mathcal{O}(T^3)\Big)\Big\}$$

This *suggests* (ε local energy scale):

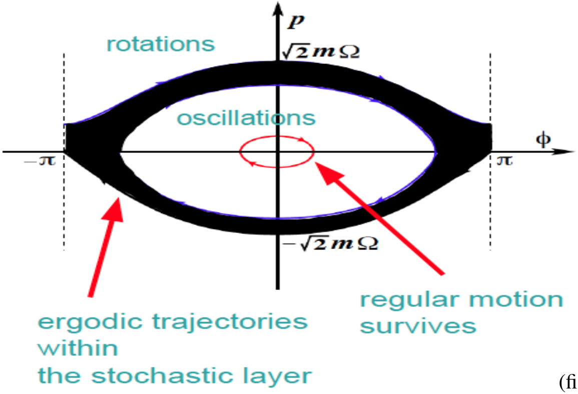
- $T\varepsilon \ll 1$: one may expand in powers of T: ergodicity breaking.
- $T\varepsilon \gtrsim 1$: no effective Hamiltonian as $\Lambda \to 0$: ergodicity

Is this correct?

Asside remark: forced pendulum

Time-dependent one-body classical system

$$H(t) = \frac{p^2}{2m} - m\Omega^2 \cos\phi - g\cos(\phi - \omega t)$$



(figure from D. Basko)

$$|\text{Stochastic layer}| \sim \frac{g}{\Omega}\mathrm{e}^{-\omega/\Omega}, \qquad \omega = \frac{2\pi}{T} \to \infty$$

Free systems: BHC is OK!

Energy absorption remains bounded if

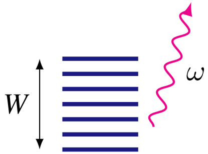
$$\omega = \frac{2\pi}{T} > W$$

frequency

single particle bandwidth

Why? Linear response $\sim$ Golden rule (V_i on site i)

$$\Gamma^{ii}_{\beta}(\omega) = g^2 \sum_{\eta,\eta'} \mathrm{e}^{-\beta E_\eta} |\langle \eta' | V_i | \eta \rangle|^2 \delta(E_\eta - E_{\eta'} - \omega)$$



Interacting systems: BHC is an asymptotic expansion

Why? Linear response again:

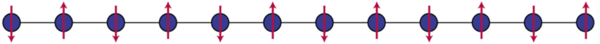
$$\Gamma^{ii}_{\beta}(\omega) \; = \; \sum_{\eta,\eta'} \mathrm{e}^{-\beta E_\eta} |\langle \eta' | V_i | \eta \rangle|^2 \delta(E_\eta - E_{\eta'} - \omega)$$

Imagine N spins satisfying the Eigenstate Thermalization Hypothesis (ETH):

$$|\langle \eta' | V_i | \eta \rangle|^2 \sim \frac{\mathrm{e}^{-|E_{\eta'} - E_\eta|/\varepsilon}}{2^N}$$

(in the middle of the band). Hence

$$\Gamma^{ii}_{\beta}(\omega) \sim \; \mathrm{e}^{-\omega/\varepsilon}$$



N spins with local interactions

Remark: exponential decay comes from locality

Why? You need to modify the configuration in $\mathcal{O}(\varepsilon/\omega)$ sites around i to modify the energy by an amount of order ω:

$$\begin{aligned}\langle\eta'|V_i|\eta\rangle &= \frac{\langle\eta'|[H,V_i]|\eta\rangle}{E_{\eta'}-E_\eta} = \dots = \frac{\langle\eta'|[H,[H,\dots,[H,V_i]]|\eta\rangle}{(E_{\eta'}-E_\eta)^n} \\ &\sim \frac{n!\varepsilon^n}{\omega^n} \sim \mathrm{e}^{-\omega/\varepsilon} \quad \text{(optimize over } n\text{)}.\end{aligned}$$

We derive analytical bounds in linear response for $V(t)=\sum_i V_i(t)$:

$$\Gamma(\omega) \lesssim |\Lambda|\mathrm{e}^{-\omega/\varepsilon}$$

See D. Abanin, W. De Roeck, F. H., W. W. Ho, PRL 115.

Remark: The bound is not always optimal; e.g. Araki analyticity in $d=1$ guarantees faster than exponential decay.

Moving to the rotating frame

We want to go beyond linear response! Floquet representation:

$$U(t_0, t) = P(t)\mathrm{e}^{-\mathrm{i}H_{\mathit{eff}}t}, \quad \text{with} \quad P(t+T) = P(t), \quad P(t) \text{ unitary.}$$

Analogy with diagonalization:

- Diagonalize H(static) : find a base change U such that

$$U^\dagger H U = H_{diag}.$$

- Find H_{eff} : find a rotating frame $P(t)$ such that

$$P^\dagger(t)\Big(H(t) - \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\Big)P(t) = H_{\mathit{eff}}.$$

Very useful: $P(t)$ can be constructed perturbatively/iteratively (inspiration: Imbrie's work on MBL)

The rotating frame in three examples

1. Quasi-conserved quantities in ergodic driven systems:
   H_{eff} is nor perturbative nor interesting (no locality),
   Some ‘asymptotic’ $\widetilde{H}_{\mathit{eff}}(t)$ is much more useful!

2. Quasi-conserved quantities in non-driven systems:
   Singlons and doublons in the Fermi-Hubbard model.

3. True effective conserved quantity in MBL systems:
   ETH is violated (previous reasoning does not apply).
   “Locally, MBL system remains finite-dimensional as $\Lambda \to \infty$”.
   True H_{eff} (sum of local terms) if ω/ε is high enough.

1. Quasi-conserved quantities in driven systems (I)

We remind the set-up: we take the Hamiltonian

$$H(t) = H^{(0)} + gV(\omega t), \quad H^{(0)} = \sum_{i\in\Lambda} H_i^{(0)}, \quad V(\omega t) = \sum_{i\in\Lambda} V_i(\omega t)$$

We assume that the frequency is high

$$g/\omega \ll 1, \qquad \varepsilon/\omega \ll 1.$$

We try to find a periodic unitary $P(\omega t)$ that preserves locality so that

$$P^\dagger(\omega t)\Big(H(\omega t) - \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\Big)P(\omega t) = \widetilde{H}_{\mathit{eff}}(\omega t).$$

1. Quasi-conserved quantities in driven systems (II)

We look for periodic $P(\omega t)$ of the form

$$P(\omega t) = \mathrm{e}^{-\mathrm{i}\frac{g}{\omega}A(\omega t)}, \qquad A(\omega t) = \sum_{i\in\Lambda} A_i(\omega t).$$

- Locality:

$$P^\dagger(\omega t) O_i P(\omega t) = \sum_{A\supset\{i\}} O_A(\omega t), \quad \|O_A(\omega t)\| \leq \varepsilon \mathrm{e}^{-|A|/\ell_0}$$

- Choose $A(t)$ properly:

$$P^\dagger(t)\Big(H(t) - \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\Big)P(t) = H^{(0)} + \underline{gV(\omega t) + g\frac{\mathrm{d}A}{\mathrm{d}t}(\omega t)} + \mathcal{O}(g/\omega)(\varepsilon + g)$$

Choose A to cancel this part

OK: $g/\omega \ll 1$

We take

$$A_i(\omega t) = \int_0^{\omega t} \mathrm{d}s\, V_i(s)$$

1. Quasi-conserved quantities in driven systems (III)

Iterate this procedure n times... but not at infinitum! Trade-off:

- Taking n large, the time-dependent part becomes $(g/\omega)^n \ll 1$,
- Taking n large, $A_i^{(n)}$ are on n sites, and combinatorial factors $n!$ pop out.

Upshot: We have constructed quasi-conserved quantity $\widetilde{H}_{\mathit{eff}}$ such that, for all stroboscopic times kT,

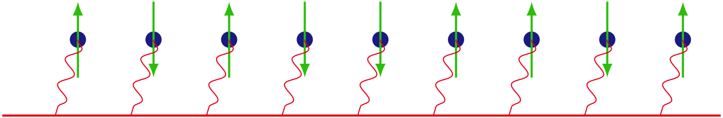
$$\left\| U^\dagger(kT)\widetilde{H}_{\mathit{eff}}U(kT) - \widetilde{H}_{\mathit{eff}} \right\| \leq C \cdot |\Lambda| \cdot kT \cdot g e^{-\omega/\varepsilon},$$

and $\widetilde{H}_{\mathit{eff}}$ is extensive and of the form

$$\widetilde{H}_{\mathit{eff}} = \sum_{i\in\Lambda} \widetilde{H}_{\mathit{eff},i}.$$

Example of use: Driven system coupled to a bath

$$H(t) = H_{\mathrm{S}}(t) + H_{\mathrm{B}} + g_{\mathrm{BS}} H_{\mathrm{BS}}$$



Q: Does the system reach a thermal ensemble? Which one?

A: assume that

$$g_{\mathrm{BS}} \gg g e^{-\omega/\varepsilon},$$

then the system equilibrates to a state close to the Gibbs state

$$\frac{e^{-\beta_{\mathrm{B}} \widetilde{H}_{\mathit{eff}}}}{Z}, \qquad \beta_{\mathrm{B}} = (\text{bath temperature})^{-1}$$

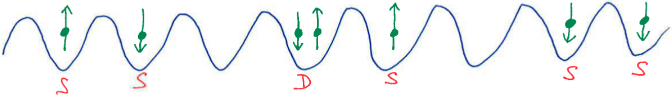
Why? Timescale for thermalization is g_{BS}^{-1}

2. Fermi-Hubbard model (I)

Similar phenomenon in non-driven systems, e.g.

$$H = U \sum_i n_{i\uparrow} n_{i\downarrow} + \tau \sum_i \left(a_i^\dagger a_{i+1} + a_i a_{i+1}^\dagger \right)$$

Particles come in singlons or doublons:



Very hard to create/destroy a doublon in the strongly interacting regime

$$U/\tau \gg 1$$

because of the big energy mismatch $\Delta E = U \gg \tau$.

2. Fermi-Hubbard model (II)

Three exactly conserved quantities:

$$H, \qquad N_\uparrow = \sum_i a^\dagger_{i\uparrow} a_{i\uparrow}, \qquad N_\downarrow = \sum_i a^\dagger_{i\downarrow} a_{i\downarrow}.$$

We exhibit an extra approximately conserved, extensive, quantity

$$\mathcal{N}_D = \sum_i \mathcal{N}_{D,i}$$

("dressed" number of doublons) such that

$$\frac{1}{\Lambda}\|\mathrm{e}^{-\mathrm{i}Ht}\mathcal{N}_D\mathrm{e}^{\mathrm{i}Ht}-\mathcal{N}_D\| \leq C\cdot t\cdot \tau\mathrm{e}^{-U/\tau}$$

Same method: successive transformations to eliminate the terms in the Hamiltonian responsible for non-conservation of doublons.

See also N. Strohmaier et al., PRL 104, R. Sensarma et al., PRB 82, . . .

3. MBL systems (I)

1-*d* disorder spin chain as a typical example:

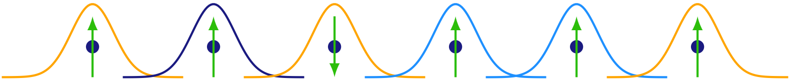
$$H^{(0)} = \sum_i \left(h_i \sigma_i^{(z)} + J \sigma_i^z \sigma_{i+1}^{(z)} + \tau_i \sigma_i^{(x)} \right)$$

Described by a full set of local integrals of motion (LIOMs):

$$H = \sum_i J_i \tau_i^{(z)} + \sum_{i<j} J_{i,j} \tau_i^{(z)} \tau_j^{(z)} + \sum_{i<j<k} J_{i,j,k} \tau_i^{(z)} \tau_j^{(z)} \tau_k^{(z)} + \dots$$

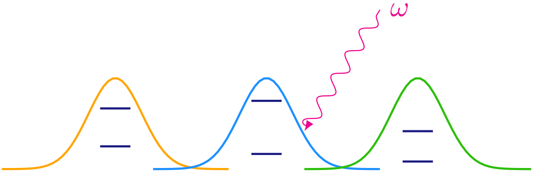
with

$$[\tau_i^{(z)}, \tau_j^{(z)}] = 0, \qquad |J_{i_1,\dots,i_n}| \lesssim \mathrm{e}^{-(i_n - i_1)/\xi}.$$



3. MBL systems (II)

Previous argument for heating up invalidated!



For the local perturbation $V_i(t)$, only a finite number of levels to match with ω.

Upshot: If

$$g/W \ll 1, \qquad g/\omega \ll 1,$$

then

$$H_{\mathit{eff}} = \sum_i H_{\mathit{eff},i}$$

is well defined as $\Lambda \to \infty$ and is itself MBL.

3. MBL and linear response

For MBL systems, linear response and Floquet regime are different:

Linear response:

- Free systems (Anderson insulator): Mott law for the AC conductivity:

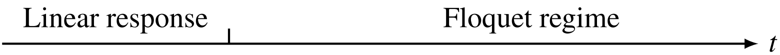
$$\sigma(\omega) \sim \omega^2 > 0 \qquad \text{as} \qquad \omega \to 0.$$

- Generalization to interacting MBL systems (S. Gopalakrishnan et al., PRB 92):

$$\sigma(\omega) \sim \omega^\alpha > 0 \qquad \text{as} \qquad \omega \to 0.$$

What happend?

- $\sigma(\omega)$ is computed in equilibrium
- For ergodic systems, equilibrium is preserved upon heating up
- MBL systems go out of equilibrium after a transient time



Rotating frame for MBL systems (I)

We want more than an asymptotic expansion; we need to improve the scheme

- Ergodic systems: $1/\omega$ expansion in disguise:

$$1 \rightsquigarrow 1/\omega \rightsquigarrow 1/\omega^2 \rightsquigarrow \dots \rightsquigarrow 1/\omega^n.$$

- MBL systems: Renormalization Group approach:

$$g \rightsquigarrow (g/\delta E)g \rightsquigarrow (g/\Delta E)^{2^2-1}g \rightsquigarrow \dots \rightsquigarrow (g/\Delta E)^{2^n-1}g \rightsquigarrow \dots$$

It turns out that

- Allows to overcome combinatorial problems (aka KAM scheme, Imbrie's schme)
- Possible thanks to the MBL structure of the eigenstates of $H^{(0)}$.

Rotating frame for MBL systems (II)

Again try $P(\omega t)$ of the form

$$P(\omega t) = \mathrm{e}^{-gA(\omega t)}, \qquad A(\omega t) = \sum_{i\in\Lambda} A_i(\omega t).$$

and expand

$$\begin{aligned} P^{\dagger}(t)\Big(H(t) - \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\Big)P(t) = & H^{(0)} + gV(\omega t) + g[V(t), H^{(0)}] + g\omega\frac{\mathrm{d}A}{\mathrm{d}t}(\omega t) \\ & + \mathcal{O}(g/\min\{\omega, W\})g. \end{aligned}$$

We can now take A to cancel the whole expression in red:

$$\langle\eta'|\hat{A}(k)|\eta\rangle = \frac{\langle\eta'|\hat{V}(k)|\eta\rangle}{\Delta E^{(0)}_{\eta,\eta'} + 2\pi k\omega}$$

where η, η' are eigenstates of $H^{(0)}$. This expression is fine since

- We assumed $g \ll \omega$ and $g \ll W$.
- $\langle\eta'|V_i|\eta\rangle$ is very small unless η and η' differ only on a few LIOMs near point i.

Numerical results on MBL systems

Consider the protocol where we switch between two Hamiltonians

$$U(0,T) = \mathrm{e}^{-\mathrm{i}H_0T_0}\mathrm{e}^{-\mathrm{i}H_1T_1}$$

with

$$H_0 = \sum_i \big(h_i\sigma_i^{(z)} + J_z\sigma_i^{(z)}\sigma_{i+1}^{(z)}\big), \quad H_1 = J_x\sum_i \big(\sigma_i^{(x)}\sigma_{i+1}^{(x)} + \sigma_i^{(y)}\sigma_{i+1}^{(y)}\big)$$

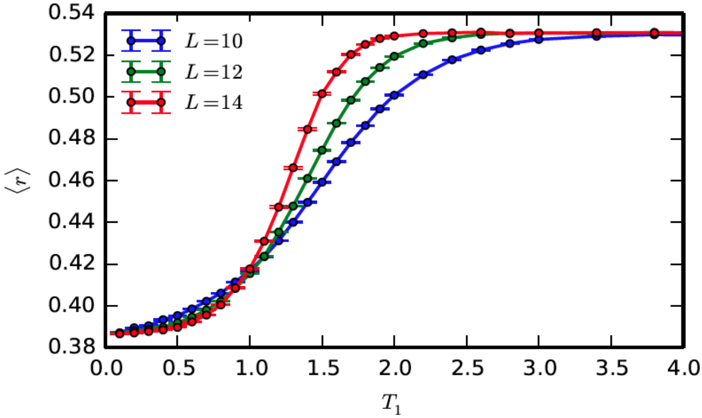
We look at several signatures for MBL

1. Level statistics. Sort the quasi-energy levels $\theta_i \in [0,2\pi)$ by increasing order and consider

$$r = \frac{\min\{\delta_i, \delta_{i+1}\}}{\max \delta_i, \delta_{i+1}}, \qquad \delta_i = \theta_{i+1} - \theta_i.$$

- Poisson statistics (no level repulsion $\sim$ MBL): $r \simeq 0.38$.
- GOE statistics (level repulsion $\sim$ ergodic phase): $r \simeq 0.53$

Numerical results on MBL systems



MBL persists at “high frequency” (small T_1).

(from P. Ponte, Z. Papic, F. H. and D. Abanin, PRL 114)

Numerical results on MBL systems

2. Entanglement entropy:

- Split the system into two halfs: $\Lambda = A \cup B$

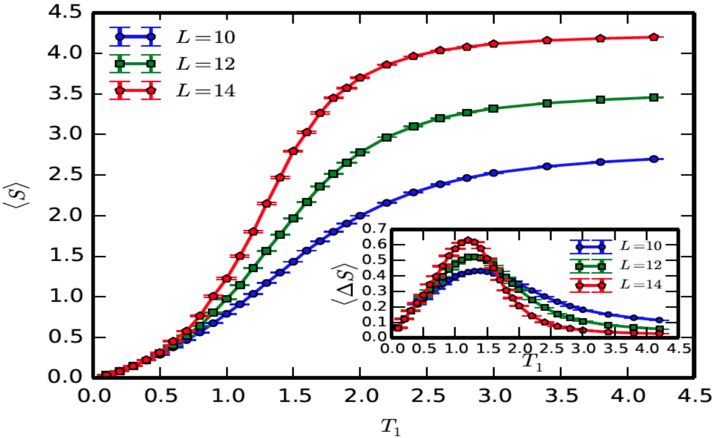
- Take a eigenstate $|\psi\rangle$
- Trace over half of the space: $\rho_B = \mathrm{Tr}_A(|\psi\rangle\langle\psi|)$
- Compute the entropy of ρ_B:

$$S_A = -\mathrm{Tr}(\rho_B \log \rho_B).$$

Look then at S_A in function of the system size L:

- Ergodic phase $\sim$ Volume law: $S_A(L) \sim L$ (in $d = 1$)
- MBL phase $\sim$ Area law: $S_A(L) \sim 1$ (in $d = 1$)

Numerical results on MBL systems



Area law at small T_1, volume law at large T_1

(from P. Ponte, Z. Papic, F. H. and D. Abanin, PRL 114)

Conclusions

- Heating up to an infinite temperature state can generically (i.e. for ergodic systems) not be avoided.
- However, generically there exists a broad interesting transient regime of pre-thermalization.
- Many-Body Localized system are exceptional in that respect; ergodicity breaking at high frequency.