Many-body localization: response to thermal spots

François Huveneers

DAUPHINE UNIVERSITÉ PARIS | CEREMADE UMR CNRS 7534

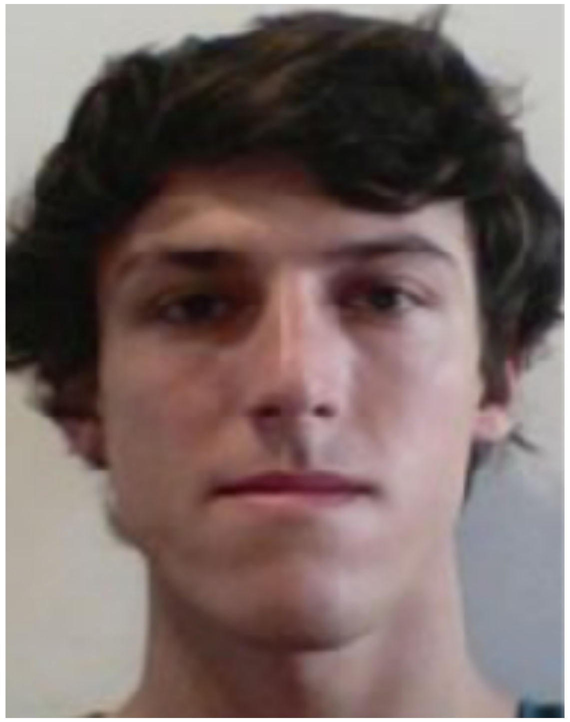
KITP, August 2018

Work in collaboration with

Wojciech
De Roeck
(KUL)

Markus
Müller
(PSI)

David
Luitz
(TUM)

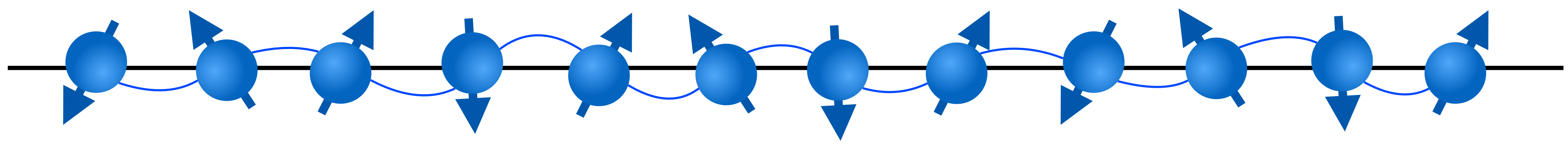


Thimothée
Thiery
(KUL)

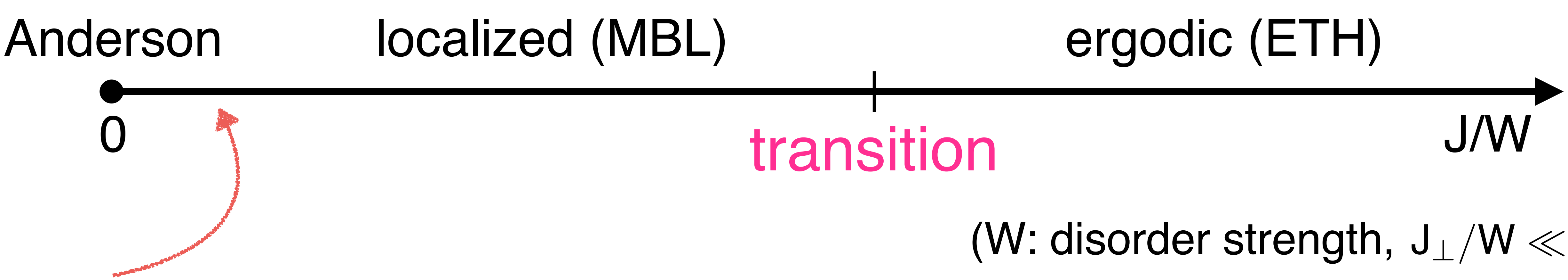
Many-body localization (MBL)

MBL materials:
- non-integrable, interacting, ‘generic’…
- no transport on any time scale
- ergodicity breaking

Example:



$$H = \sum_i h_i \sigma_i^z + J_\perp(\sigma_i^+ \sigma_{i+1}^- + \sigma_i^- \sigma_{i+1}^+) + J \sigma_i^z \sigma_{i+1}^z$$



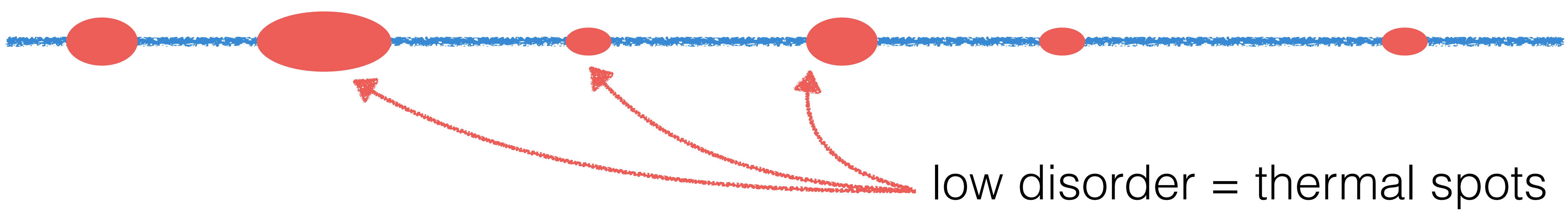
(W: disorder strength, $J_\perp/W \ll 1$)

OK: Gornyi et al. ’05, Basko et al. ’06, Oganesyan et al. ’07, Serbyn et al. ’13, Huse et aL ’14, Imbrie ’16, etc…

Plan of the talk

Mechanism for thermalization:

instability of the MBL phase to the inclusion of thermal spots



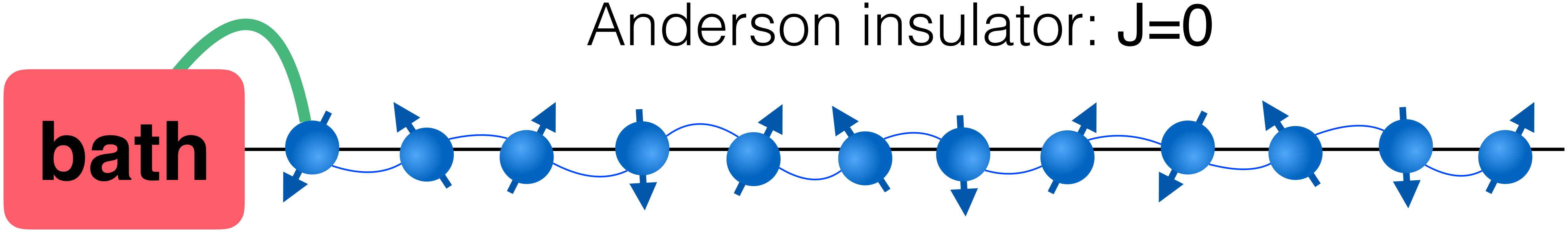
1) Response to a single spot (microscopic)
2) General considerations on the transition
3) Picture of the transition through a multi-scale analysis (RG)

Part I:

Single spot

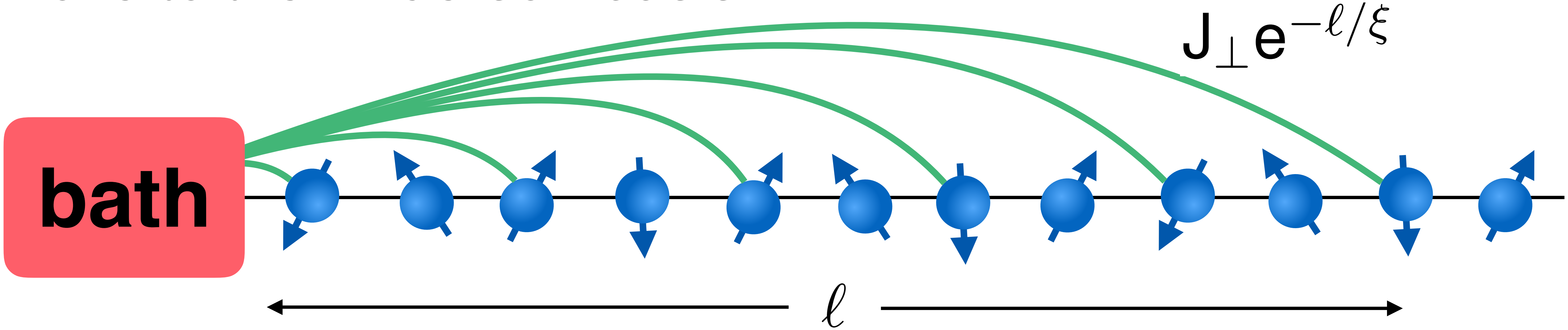
Anderson insulator coupled to an imperfect bath

Anderson insulator: **J=0**



ξ: localization length

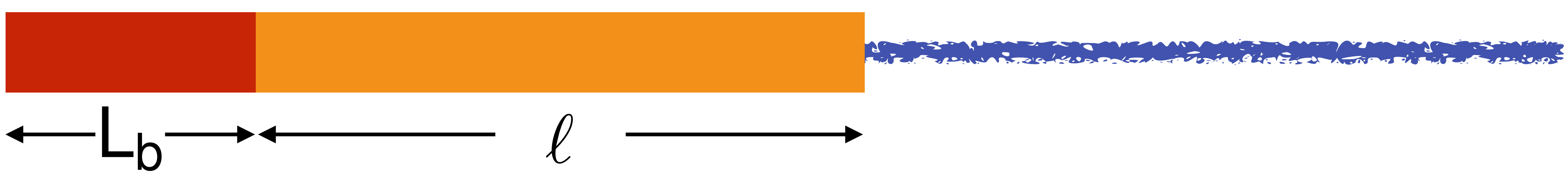
Move to the Anderson basis:



Bath: $\mathbf{L_b}$ spins with *random matrix* interaction ($\mathbf{L_b}$ fixed)

Risk of avalanche

The bath thermalizes near spins, becomes larger and closer to ideal… eventually thermalizes the whole chain!



$$\mathcal{G} = \frac{\text{matrix element}}{\text{level spacing}} \sim \frac{e^{-\ell/\xi}\, e^{-s(T)(L_b+\ell)/2}}{e^{-s(T)(L_b+\ell)}}$$

s(T): entropy density, $s(T=\infty) = \log 2$

RM assumption

Upper bound on the localization length

Avalanche stops when $\mathcal{G}(\ell) < 1$, i.e. for

$$\ell \sim \frac{1}{\xi^{-1} - \frac{\log 2}{2}} L_b$$

Write $\xi_c = 2/\log 2$

$\xi < \xi_c$: The avalanche will eventually stop

$\xi > \xi_c$: MBL is unstable

The value of ξ_c depends on the lattice:

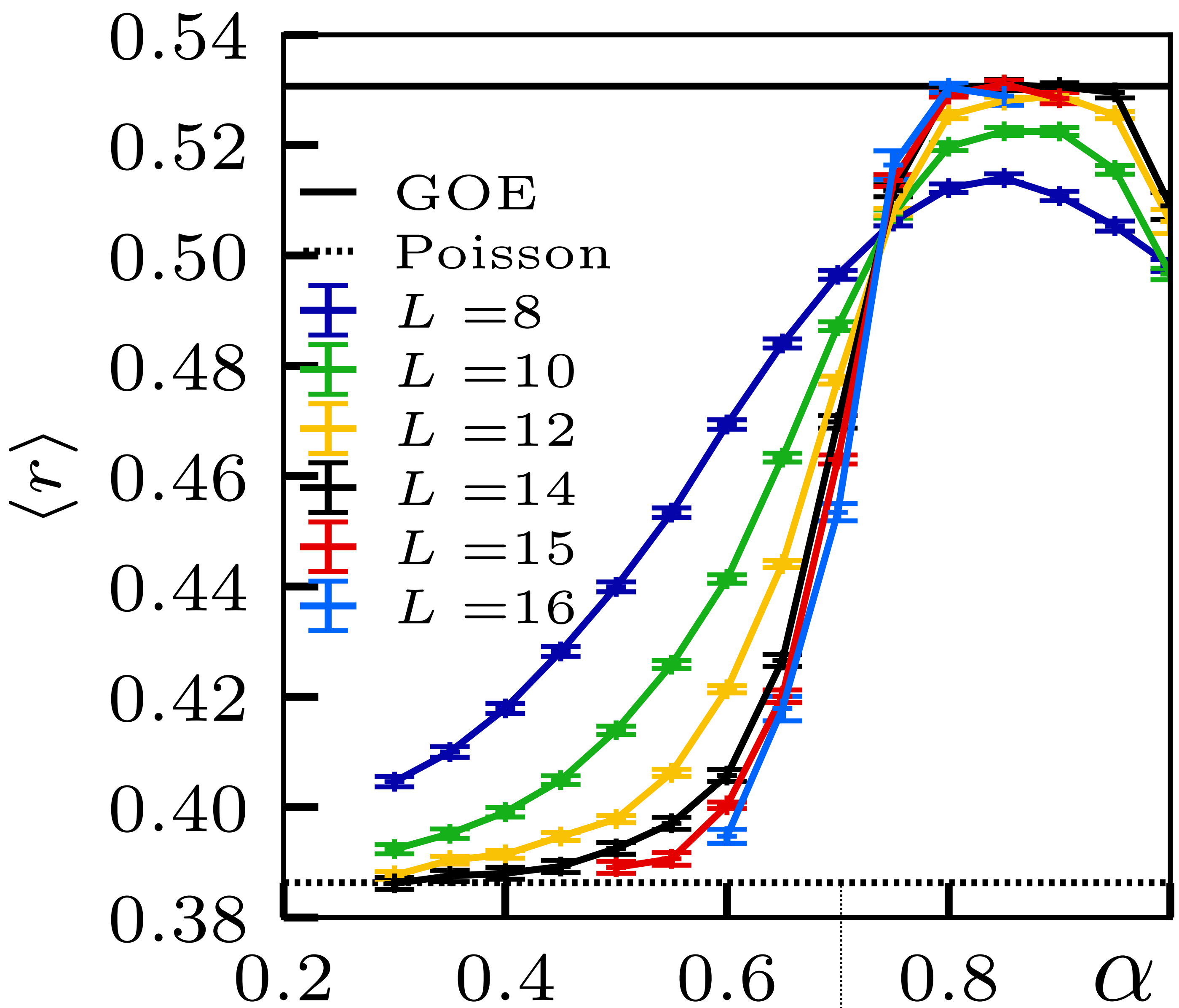
- spins on both sides of the spot: $\xi_c = 1/\log 2$
- $d > 1$: $\xi_c = 0$

cfr. W. De Roeck and F. H., PRB ‘17

see also M. Znidaric and M. Ljubotina, PNAS ‘18

Numerical check

$L_b = 3 \ : \ H_{bath} = GOE(8 \times 8)$



$\alpha = e^{-1/\xi}$ $\alpha_c = 0.71$

$$r_i = \frac{\min\{\Delta E_i, \Delta E_{i+1}\}}{\max\{\Delta E_i, \Delta E_{i+1}\}}$$

$$\Delta E_i = E_i - E_{i-1}$$

$$\langle r_{GOE} \rangle = 0,53$$

$$\langle r_{Poisson} \rangle = 0,38$$

(Oganesyan et Huse '06)

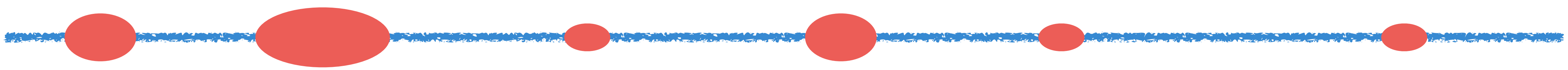
cfr. D. Luitz, F. H. et W. De Roeck, PRL 117, 2017

Part II:

General ‘facts’ about the transition

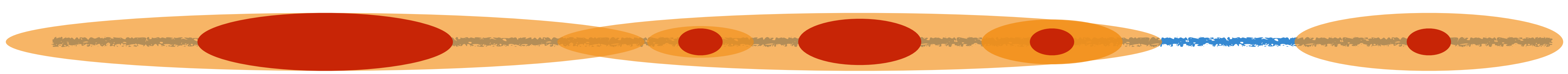
spots all over the chain

Griffiths regions:



Simple, microscopically motivated, rules to deal with them?

main issue: spot-spot interactions



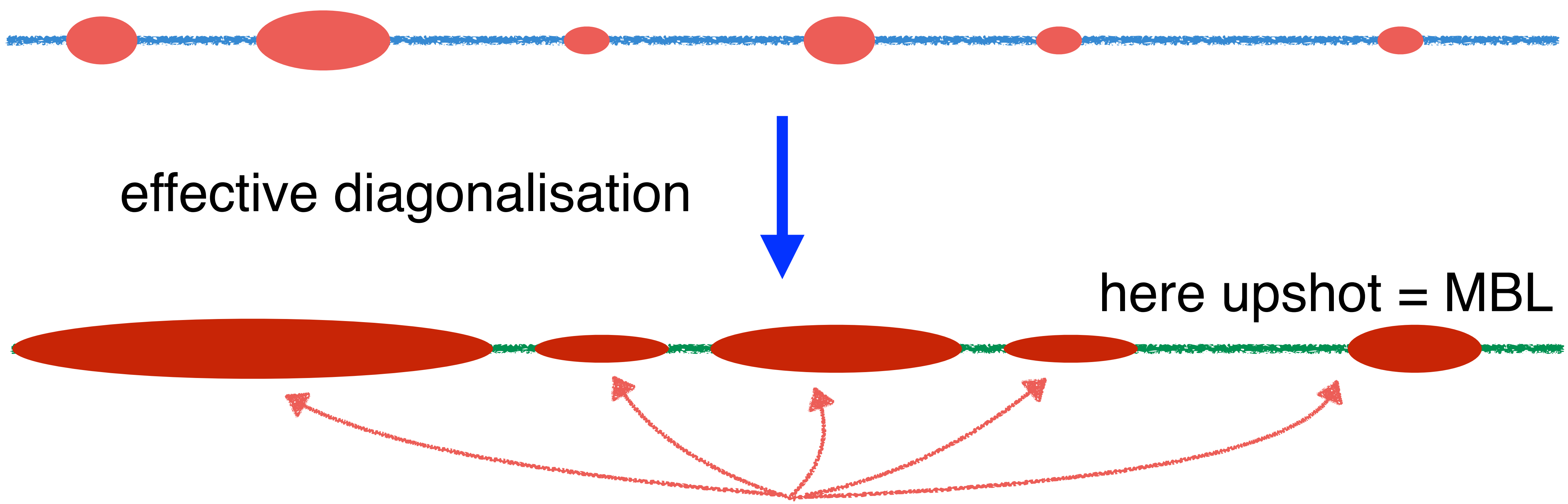
related issue: What spots to deal with first?

Hard task: Vosk and Altman ’14, V., Huse and A. ’15, Potter, Vasseur and Parameswaran ’15, Imbrie ’16, Zhang, Zhao, Devakul, H. ’16, Dumitrescu, V. and P. ’17, Goremykina, V. and Serbyn ‘18

Imbrie’s approach: main source of inspiration for our RG

Two basic assumptions

ε: density of spots *initially* (inverse disorder strength)



T(L): thermal region, *after diagonalization*

A1: $\varepsilon \;\to\; \langle |\mathrm{T(L)}| \rangle_{\varepsilon}$ is continuous and non-decreasing

A2: $\mathrm{T(L)} \subset \mathrm{T(L')}$ if we enlarge the system from **L** to **L'**

Note: **A2** might not truly hold microscopically (proximity effects)

consequences

Thermal density: $\rho(\mathsf{L}) = |\mathsf{T}(\mathsf{L})|/\mathsf{L}$

C1: For any ε, $\langle\rho(\mathsf{L})\rangle_\varepsilon \to \rho^\star(\varepsilon)$ as $\mathsf{L} \to \infty$

Follows from **A2** by Fekete's superadditivity lemma

C2: concentration around the mean:

$$\mathsf{P}(|\rho(\mathsf{L}) - \rho^\star(\varepsilon)| > \delta) \to 0 \qquad \forall\delta > 0 \quad \text{as} \quad \mathsf{L} \to \infty$$

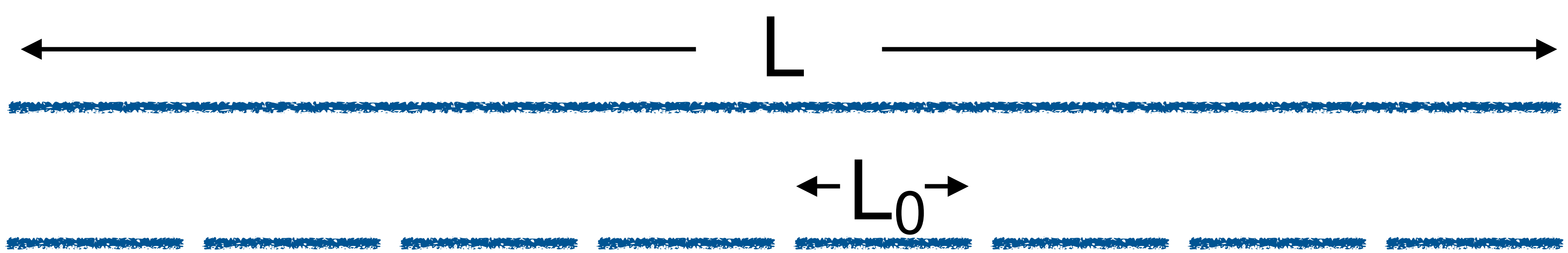
In particular, 2 possibilities at criticality:

1) MBL *with probability 1* if $\rho^\star(\varepsilon_c) < 1$

2) Thermal *with probability 1* if $\rho^\star(\varepsilon_c) = 1$

Why **C2**?

Compare with a systems cut into blocks of size $\mathbf{L_0}$



$\mathsf{L_0}$ large enough so that $\langle \rho(\mathsf{L_0}) \rangle_\varepsilon \sim \rho^\star(\varepsilon)$ (by **C1**)

C2 holds true for the ‘block’ system, hence by **A2**,

$$\mathsf{P}(\rho(\mathsf{L}) - \rho^\star(\varepsilon) > -\delta) \;\rightarrow\; 0 \quad \text{as} \quad \mathsf{L} \rightarrow \infty$$

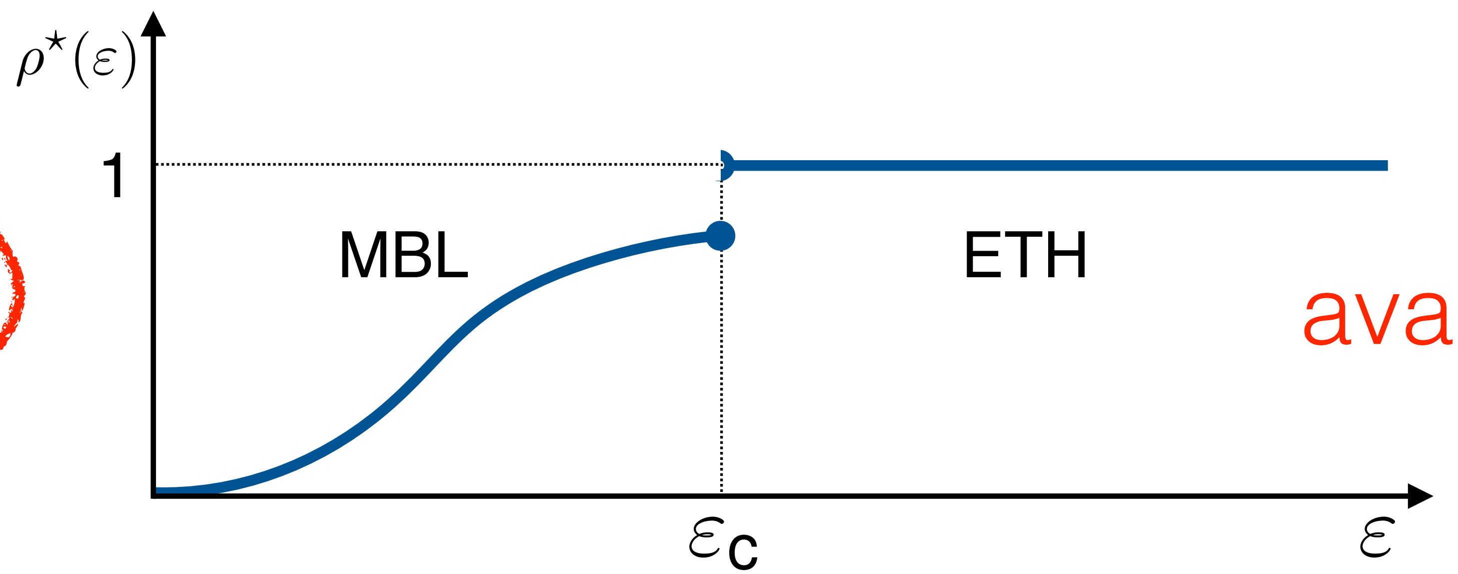
concentration in the other direction: $\rho^\star(\varepsilon)$ is the average

MBL fixed point

C3: $\varepsilon \;\to\; \rho^\star(\varepsilon)$ is left-continuous and non-decreasing

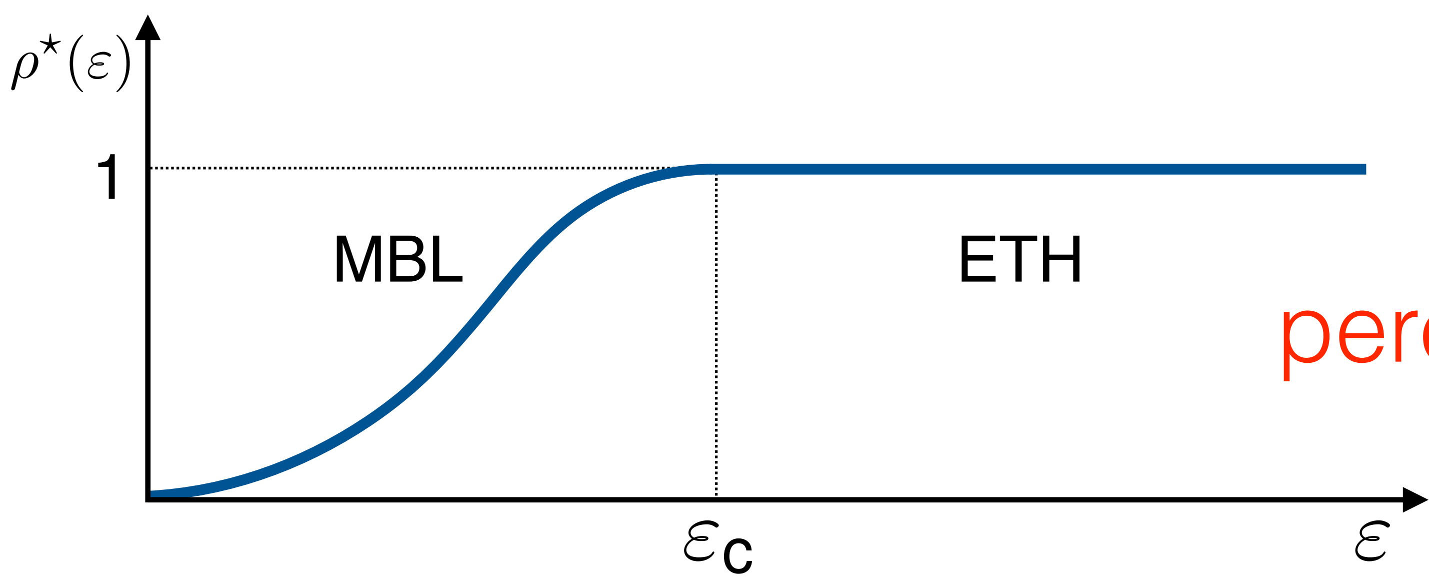
Follows by standard arguments from **A1** and **C1**

MBL fixed point



avalanche

Thermal fixed point



percolation

We know also $\xi < \xi_c$ and obviously $\xi \to \infty$ as $\rho \to 1$

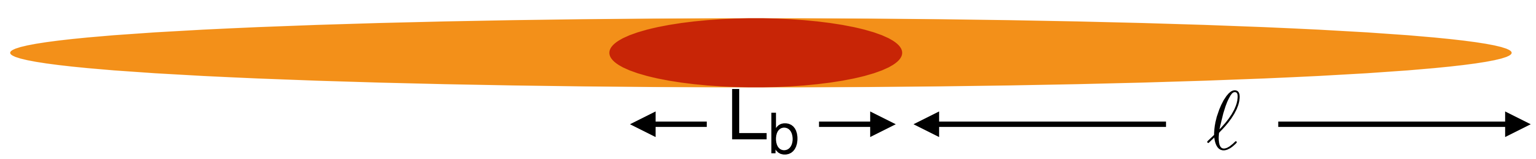
Part III:

Multi-scale analysis (RG)

The need for RG

- Develop a picture for how the transition happens
- Is $\xi_c = 1/\log 2$ still the critical localization length?

Avoid paradoxes: resonances percolate at some $\xi_\star < \xi_c$



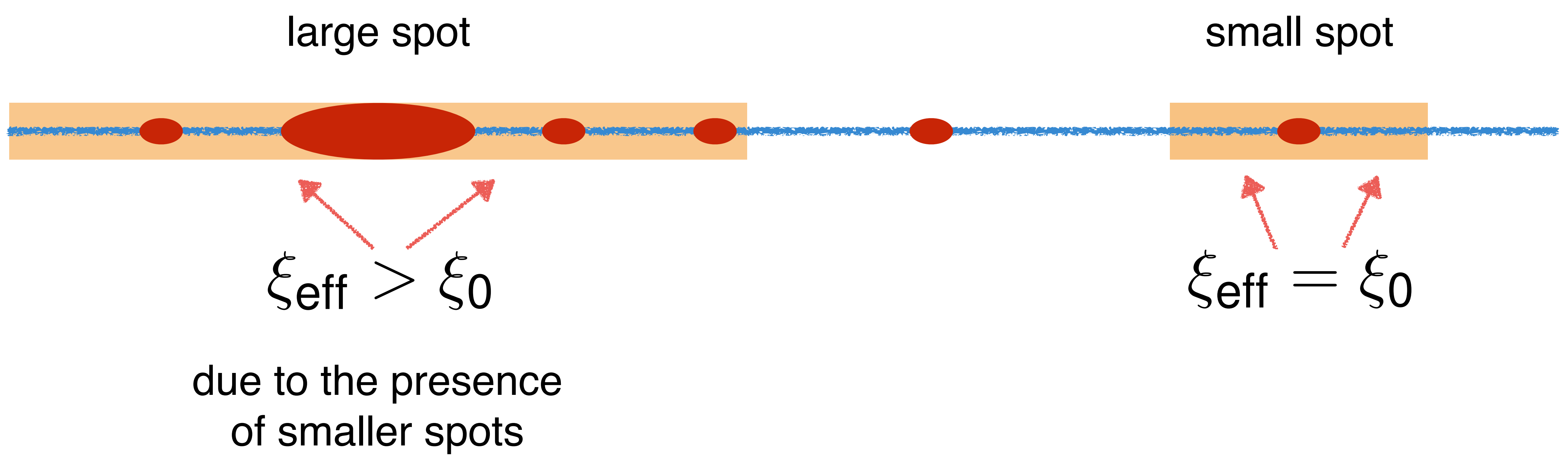
$$\ell \sim \frac{L_b}{\xi_c - \xi_\star}$$

Impossible: $\rho_c < 1$!

- Finite size scalings (mostly numerical)

Effective localization length

localization length depends on the scale:



- Deal with the smallest spots first, to avoid non-sense.
- Rule of *halted decay*: no decay through thermal regions

$$e^{-\ell/\xi_{\text{eff}}} = e^{-(\ell-\ell_{\text{th}})/\xi_0} \quad \Rightarrow \quad \xi_{\text{eff}} = \xi_0 \frac{1}{1-\ell_{\text{th}}/\ell}$$

ℓ_{th} : number of spins thermalized at previous scales : bare spots + collar

Diverging response to thermal inclusions

Define: scale k: bare spot of size k.

ξ_k: typical effective localization length at scale k.

collar length:

$$\ell_k \sim \frac{k}{\xi_c - \xi_k}, \qquad \xi_c = 1/\log 2$$

$$\text{MBL}: \ \ell_k/k \to \ell^\star \quad \text{as} \quad k \to \infty$$

$$\text{critical}: \ \ell_k/k \to \infty \quad \text{as} \quad k \to \infty$$

$$\text{thermal}: \ \ell_k/k = \infty \quad \text{for some } k < \infty$$

avalanche: at some scale, a large enough spot shows up and the full material becomes thermal.

Simplified scheme

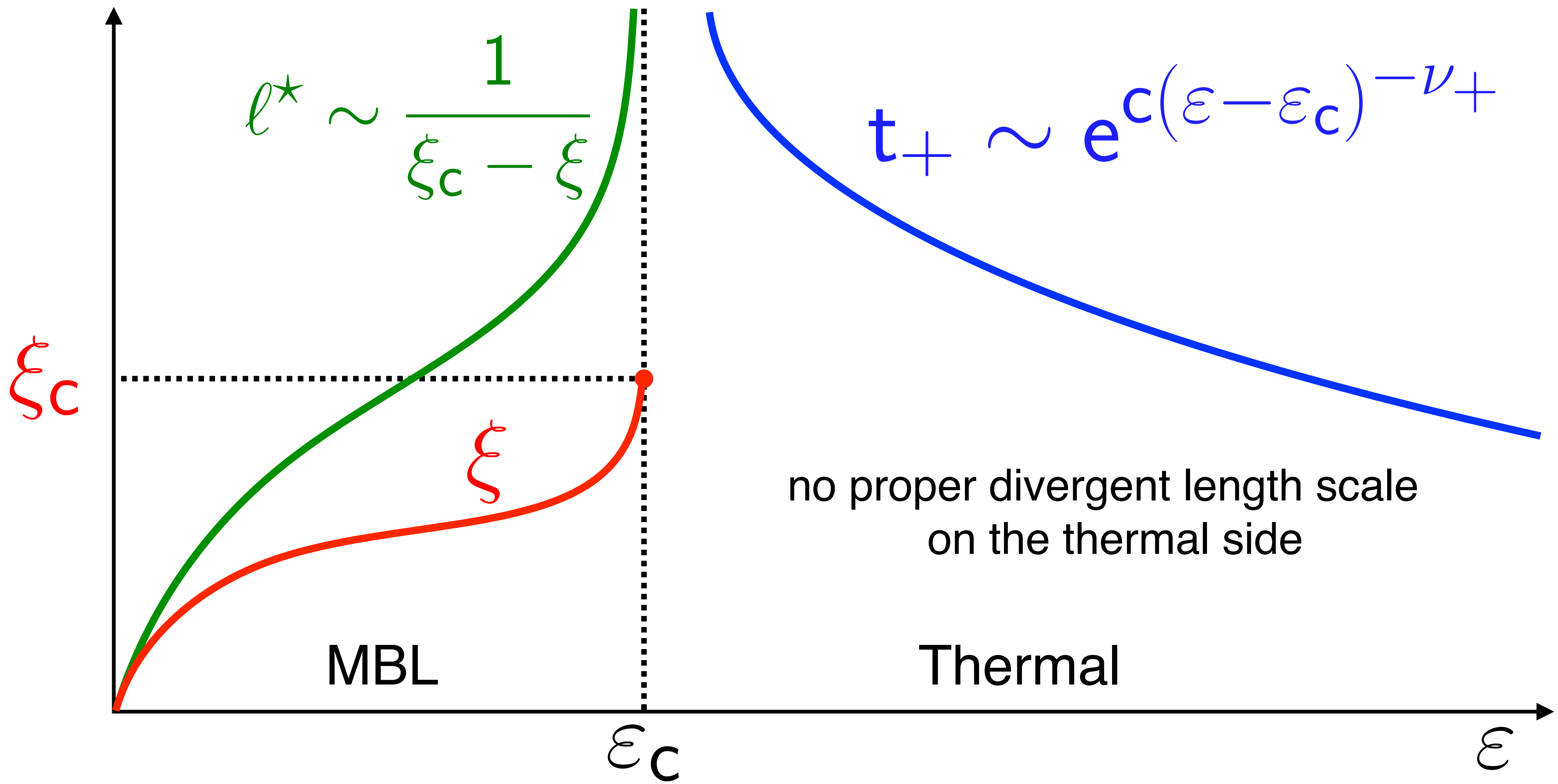
Flow on 3 parameters: $\xi_k,\ \rho_k,\ \ell_k$.

$$\begin{cases} \xi_{k+1}^{-1} = (1-\rho_k)\,\xi_k^{-1} & \text{rule of halted decay} \\ \rho_k = \varepsilon^k (k+\ell_k) & \text{thermal density from bare spots of size } \mathbf{k} \\ \ell_k = \dfrac{k}{\xi_c - \xi_k} & \text{collar length for bare spots of size } \mathbf{k} \end{cases}$$

Issues:
- fluctuations of ξ_k are ignored
- thermal density ρ_k is underestimated: $\varepsilon^k \to \varepsilon^{k^{\alpha(\varepsilon)}}$

- still needs to be solved numerically

Qualitative diagram



$\xi_c = 1/\log 2$

Cfr. T. Thiery, F. H., M. Mueller, W. De Roeck, arXiv:1706.09338

Finite size scalings

$\mathsf{p}(\varepsilon, \mathsf{L})$: probability that a system of size L is thermal

MBL side: $\mathsf{p}(\varepsilon, \mathsf{L}) \sim \mathsf{F}(\mathsf{L}/\mathsf{L}_-)$, $\quad \mathsf{L}_- \sim (\varepsilon_\mathsf{c} - \varepsilon)^{-\nu_-} \quad (\mathsf{L} \to \infty)$

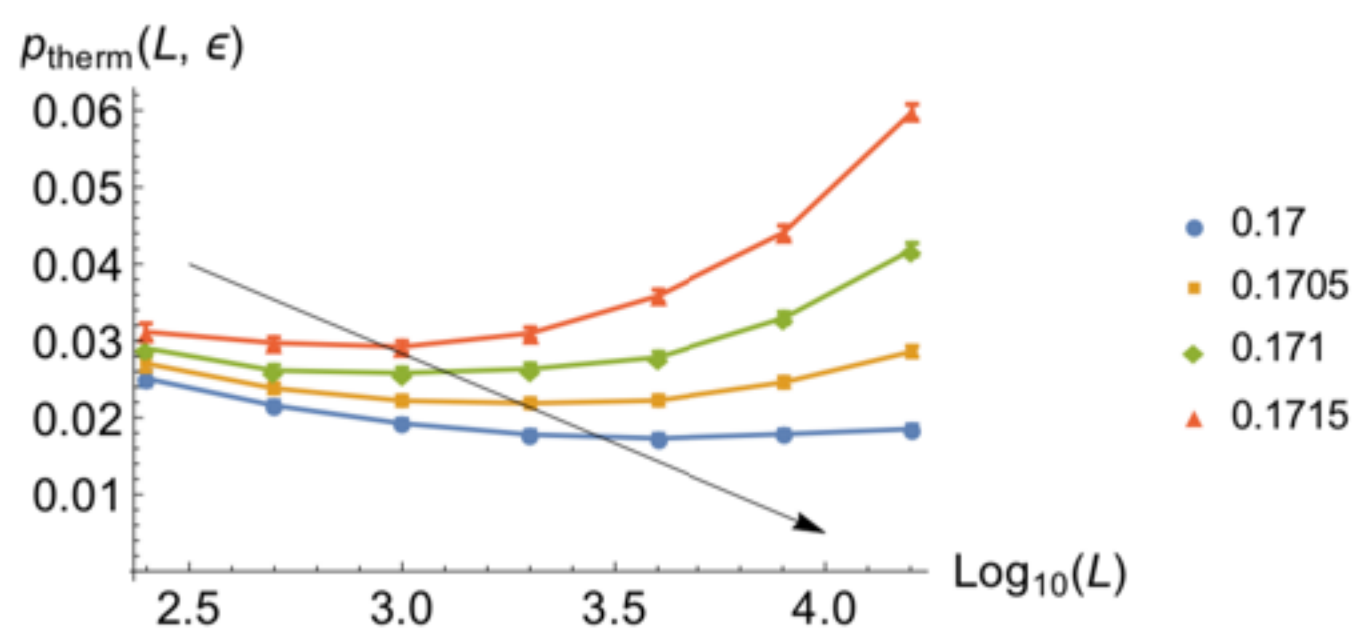
thermal side: $\quad \mathsf{L}_+ \sim (\varepsilon_\mathsf{c} - \varepsilon)^{-\nu_+}$

(different mechanisms: we expect $\nu_- \neq \nu_+$)

at criticality: $\mathsf{p}(\varepsilon_\mathsf{c}, \mathsf{L}) \sim \mathsf{L}^{-\beta}$

Non-monotonic behavior in the thermal phase close to the transition

avalanche



Closer to exact scheme

- Abandon the reduced description with a few parameters
- Fix precise rules to deal with the fusion of resonant spots
- Solve numerically

Upshot:

- confirm the picture from the simplified scheme
- fix some issues (mainly: critical exponents satisfy Harris)

cfr. T. Thiery, M. Müller and W. De Roeck, arXiv:1711.09880

Conclusions

Instability of the MBL phase:

- A single imperfect bath can destabilize MBL
- Localized transition point, with finite loc. length
- Discontinuity of the thermal density at the transition (unlike percolation)
- Physical picture from RG, scale dependent loc. length
- Divergent response to the inclusion of thermal spots