DYNAMICS OF ULTRACOLD FERMI GASES: GROSS-PITAEVSKII AND BEYOND

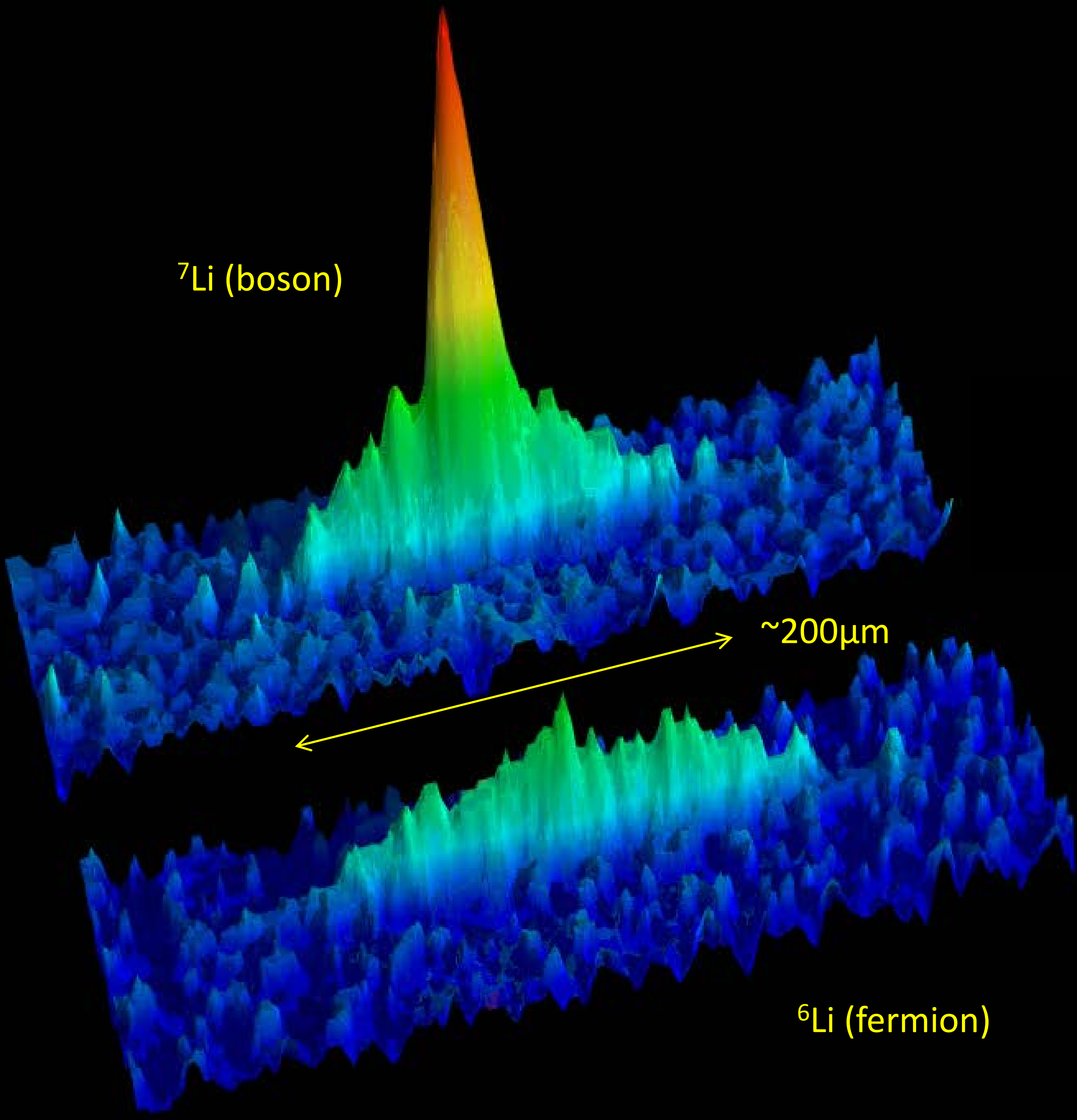
F. Chevy

May 24th, 2019, Les Treilles

Laboratoire Kastler Brossel
Physique quantique et applications

SORBONNE UNIVERSITÉ
CRÉATEURS DE FUTURS DEPUIS 1257
cnrs
PSL UNIVERSITÉ PARIS
ENS
COLLÈGE DE FRANCE 1530
erc


J. Struck
M. Bolhen
C. Salomon
K. Dai
L. Absil
T. Reimann
C. Enessa
M. Pierce
FC
I. Kieffer
S. Jin
T. Yefsah
G. Dash


^{7}Li (boson)
~200μm
^{6}Li (fermion)

BOSE-EINSTEIN CONDENSATES

Dilute Bose-Einstein condensates

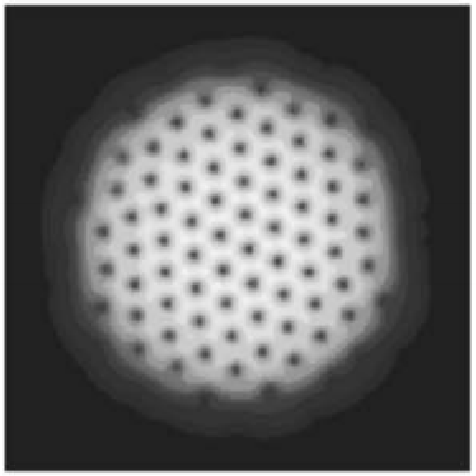
Mean-field approximation (dilute limit ρa^3<<1) : all bosons occupy the same quantum state.

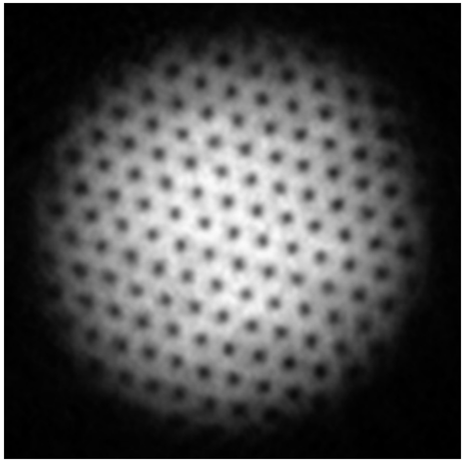
$$\psi(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N,t)=\phi(\mathbf{r}_1,t)\phi(\mathbf{r}_2,t)\ldots\phi(\mathbf{r}_N,t)$$

Gross-Pitaevskii Equation (Non-Linear Shrödinger equation)

$$i\hbar\partial_t\phi=-\frac{\hbar^2}{2m}\nabla^2\phi+V(r)\phi+gN\,|\phi|^2\,\phi$$

I. Danaila, PRA 2005





MIT, Science 2001

Hydrodynamic Approximation

$\phi(\mathbf{r},t) = \sqrt{\rho(\mathbf{r},t)/N}\, e^{i\chi(\mathbf{r},t)}$ $\quad \mathbf{v}(\mathbf{r},t) = \frac{\hbar}{m}\nabla\chi$ (Mabdelung Transform)

$$\partial_t \rho + \nabla \rho \mathbf{v} = 0$$

$$m\left(\partial_t \mathbf{v} + \nabla \mathbf{v}^2/2\right) = -\nabla\left[\mu(\rho) + V(\mathbf{r}) + \cancel{\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}}\right]$$

Long wavelength excitation

$\lambda >> \xi = \hbar/\sqrt{m\mu}$ (healing length)

$\mu(\rho) = g\rho =$ chemical potential

Analytic solutions (polynomial in harmonic trap)

- Ground state (Thomas-Fermi profile): $\rho=(\mu-V(r))/g$
- Low-lying excitations (phonons, center-of-mass oscillations, breathing mode, quadrupole mode…)

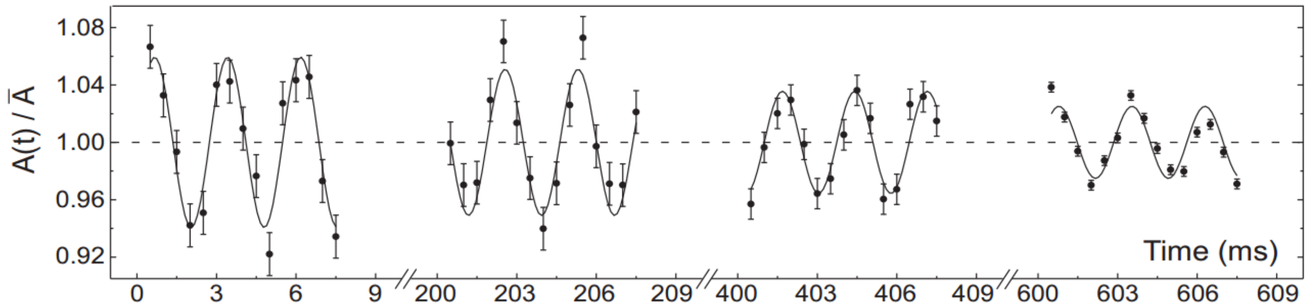
Does **NOT** apply to vortex or solitons (core size~healing length)

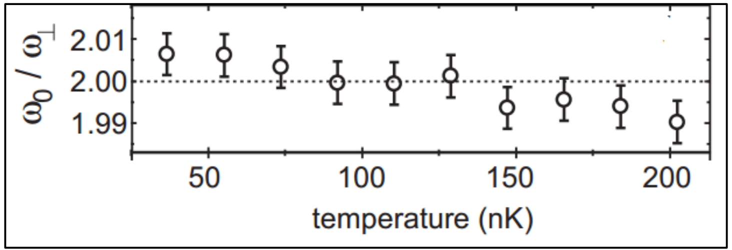
Scaling solution

Assumption: harmonic potential with time varying trapping frequencies $\omega_{x,y,z}(t)$
Scaling Ansatz (Castin-Dum)

$$\rho(x,y,z,t) = \frac{\rho(\lambda_x(t)x, \lambda_y(t)y, \lambda_z(t)z, 0)}{\lambda_x(t)\lambda_y(t)\lambda_z(t)} \quad \Rightarrow \quad \ddot{\lambda}_i = -\omega_i(t)^2 + \frac{\omega_i(0)^2}{\lambda_i \prod_j \lambda_j}$$

Example transverse breathing mode in an elongated trap (ENS 2001): $\omega = 2\omega_\perp$

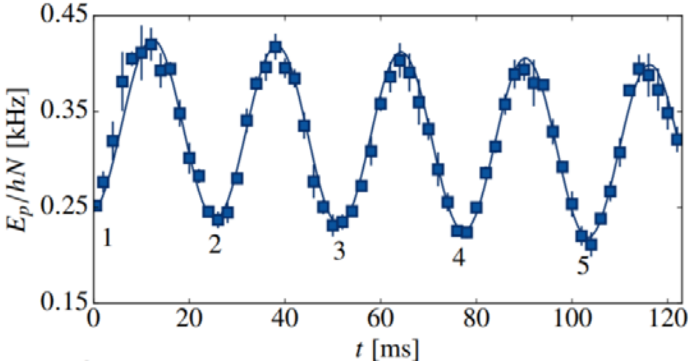


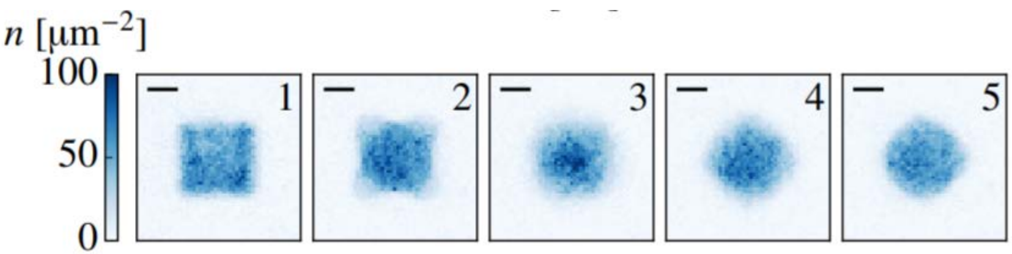


Scaling solutions (2D)

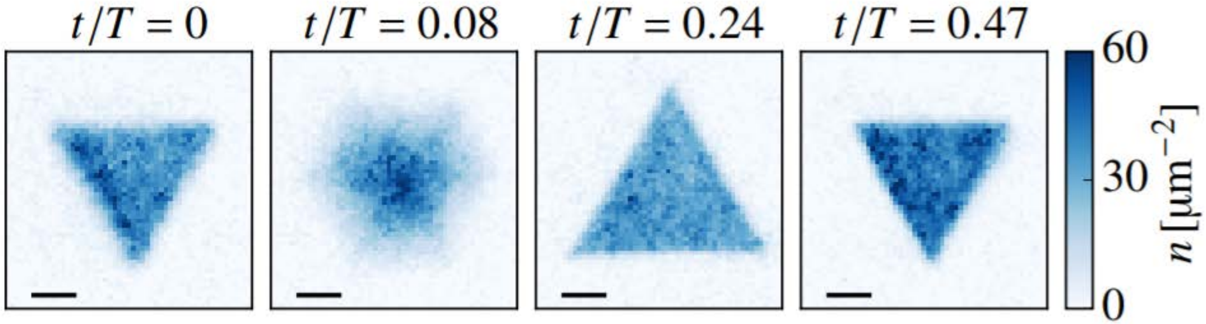
Saint-Jalm *et al.* **arXiv:1903.04528**

SO(2,1) dynamical symmetry (Pitaevski 1997)





2D breathers

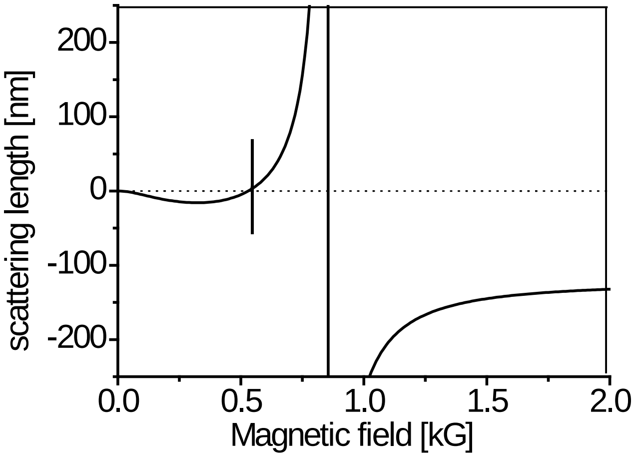


FERMIONIC SUPERFLUIDS

The zero range model

Physical assumptions:

Spin ½ fermions
No spin polarization
No interaction between same-spin fermions



For «resonant» fermions, scattering length >> range of the potential

Universality hypothesis: the effect of interactions on macroscospic properties is fully encapsulated in a

$$H = \sum_i -\frac{\hbar^2}{2m}\Delta_i + V(\mathbf{r}_i)$$

$$\psi(\mathbf{r}_1 \ldots \mathbf{r}_{2N}, t) \underset{r_{i\le N} - r_{j>N} \to 0}{=} A(r_{k\neq i,j})\left(\frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} - \frac{1}{a} + \ldots\right)$$ (Bethe-Peierls)

The two-body problem

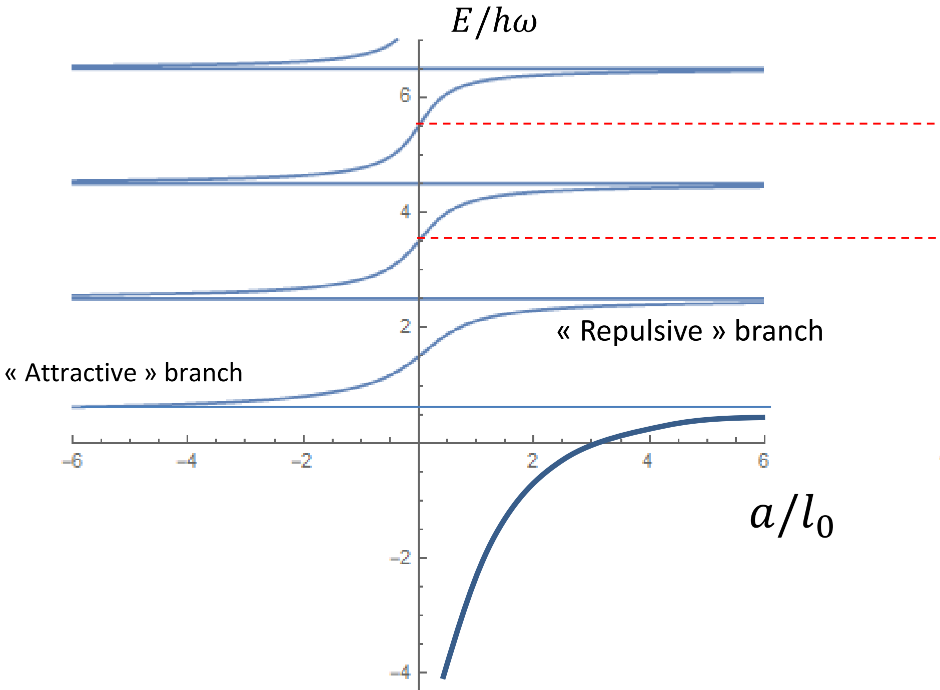
Bound state (E<0) when a is positive

$$E = -\frac{\hbar^2}{ma^2} \qquad \psi(r) = \frac{e^{-r/a}}{r} \qquad \text{(a=size of the dimer)}$$

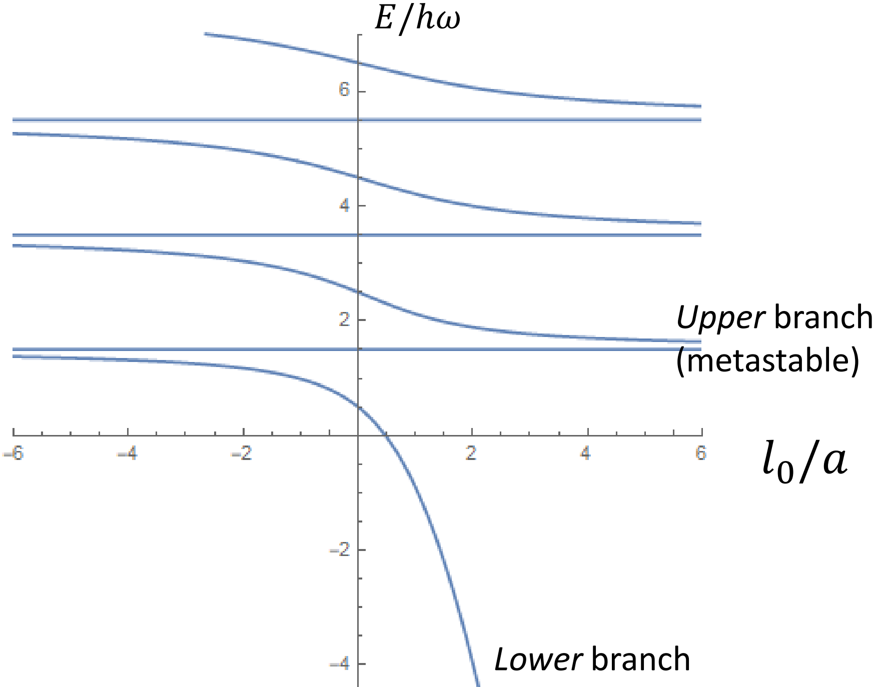
In a harmonic trap (T. Busch *et al.* Found. Phys. 28, 549–559 (1998))

$$\sqrt{2}\,\frac{\Gamma(-E/2\hbar\omega + 3/4)}{G(-E/2\hbar\omega + 1/4)} = \frac{\ell_0}{a}$$

'Mean-field' point of view



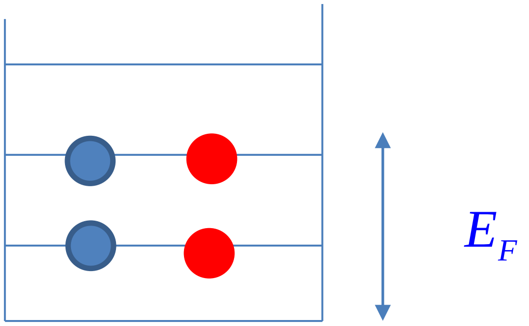
'Crossover' point of view



The many-body ground state: scaling approach

What is the zero-temperature equation of State $\mu(\rho)$?

Ideal Fermi gas



$$\mu = E_F,$$

$$k_F = (6\pi^2\rho)^{1/3} \qquad E_F = \hbar^2 k_F^2 / 2m$$

Equation of state with zero-range interactions

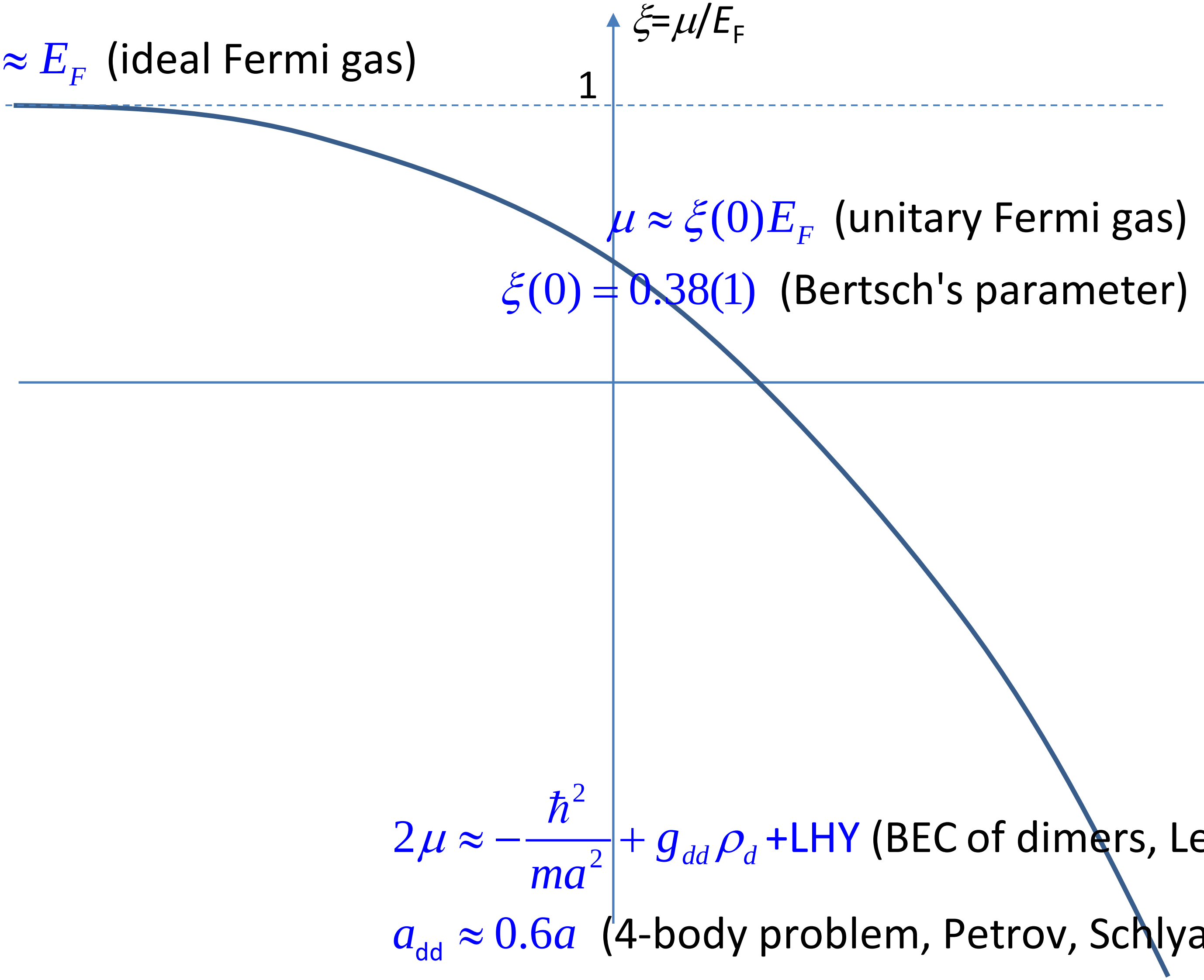
$$\mu = f(\hbar, m, a, \rho)$$

Dimensional analysis

$$\mu = E_F \xi(1/k_F a)$$

The BEC-BCS crossover

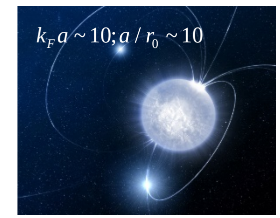
(Nozière-Legget-Schmitt-Rink)



$\mu \approx E_F$ (ideal Fermi gas)

$\mu \approx \xi(0) E_F$ (unitary Fermi gas)

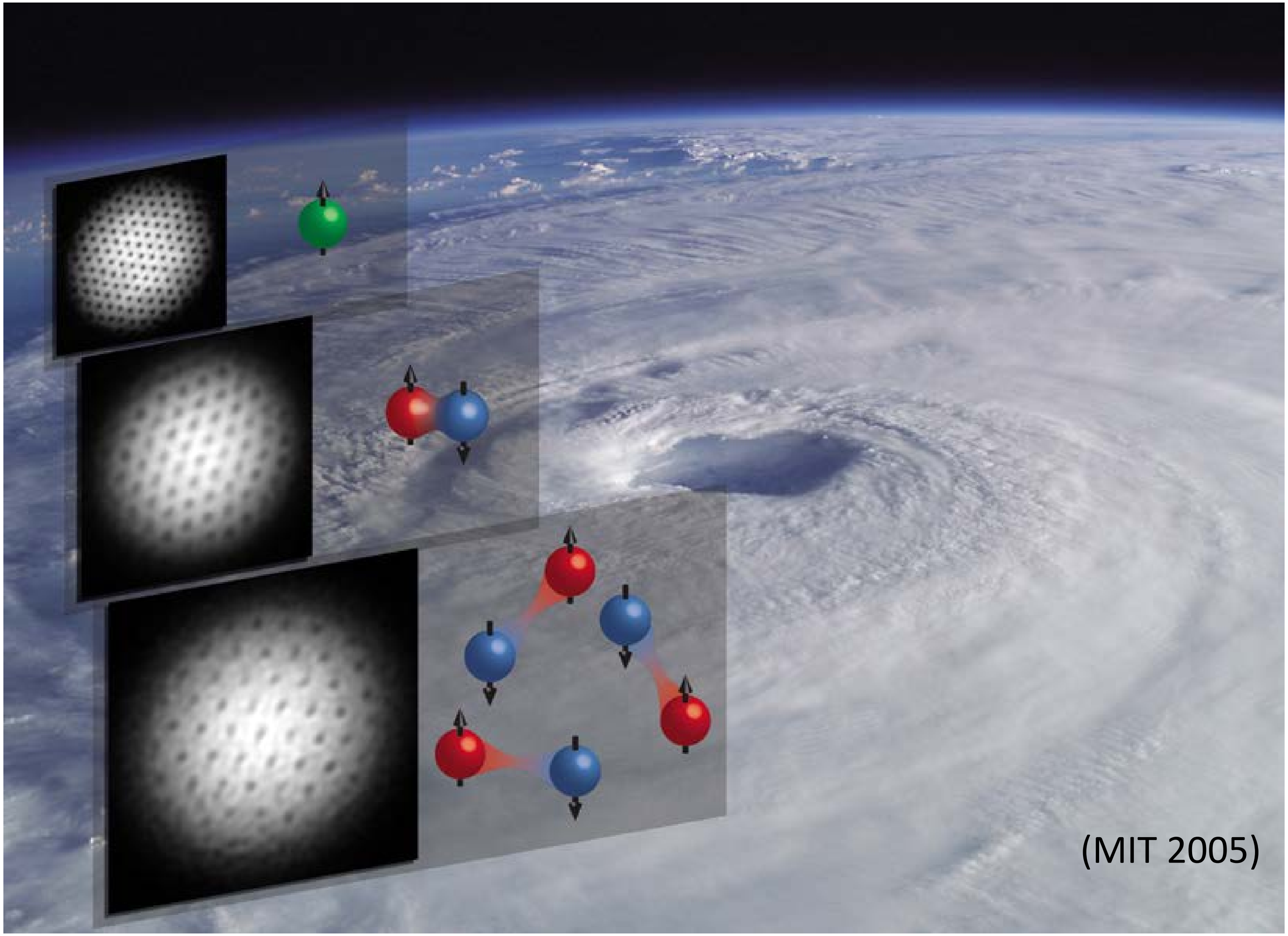
$\xi(0) = 0.38(1)$ (Bertsch's parameter)



$2\mu \approx -\frac{\hbar^2}{ma^2} + g_{dd}\rho_d$ +LHY (BEC of dimers, Leyronas PRL 07)

$a_{dd} \approx 0.6a$ (4-body problem, Petrov, Schlyapnikov)

Dynamics of fermionic superfluids



Dynamics in the mean-field regime

Weakly attractive limit: BCS theory

Bogoliubov-de Gennes-Anderson equations

$$i\hbar\partial_t \begin{pmatrix} u_s \\ v_s \end{pmatrix} = \begin{pmatrix} h_0 & \Delta \\ \Delta^* & -h_0^* \end{pmatrix} \begin{pmatrix} u_s \\ v_s \end{pmatrix}$$

h_0: single particle Hamiltonian

$$\Delta(\mathbf{r}) = -g_0 \sum_s u_s(\mathbf{r}) v_s^*(\mathbf{r}).$$

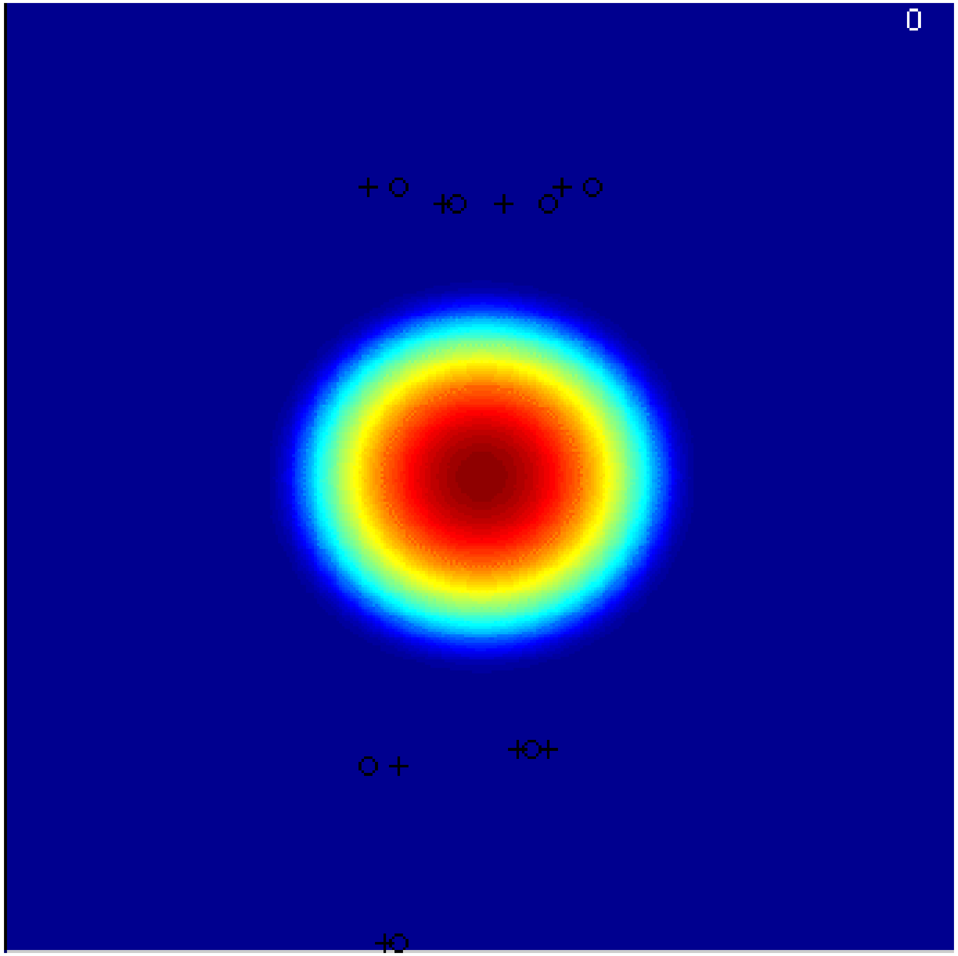
Long wavelength excitations: superfluid hydrodynamics, valid for any interaction regime (see Tonini *et al.*, EPJD '06 for derivation in mean-field regime)

$$\partial_t \rho + \nabla \rho \mathbf{v} = 0$$

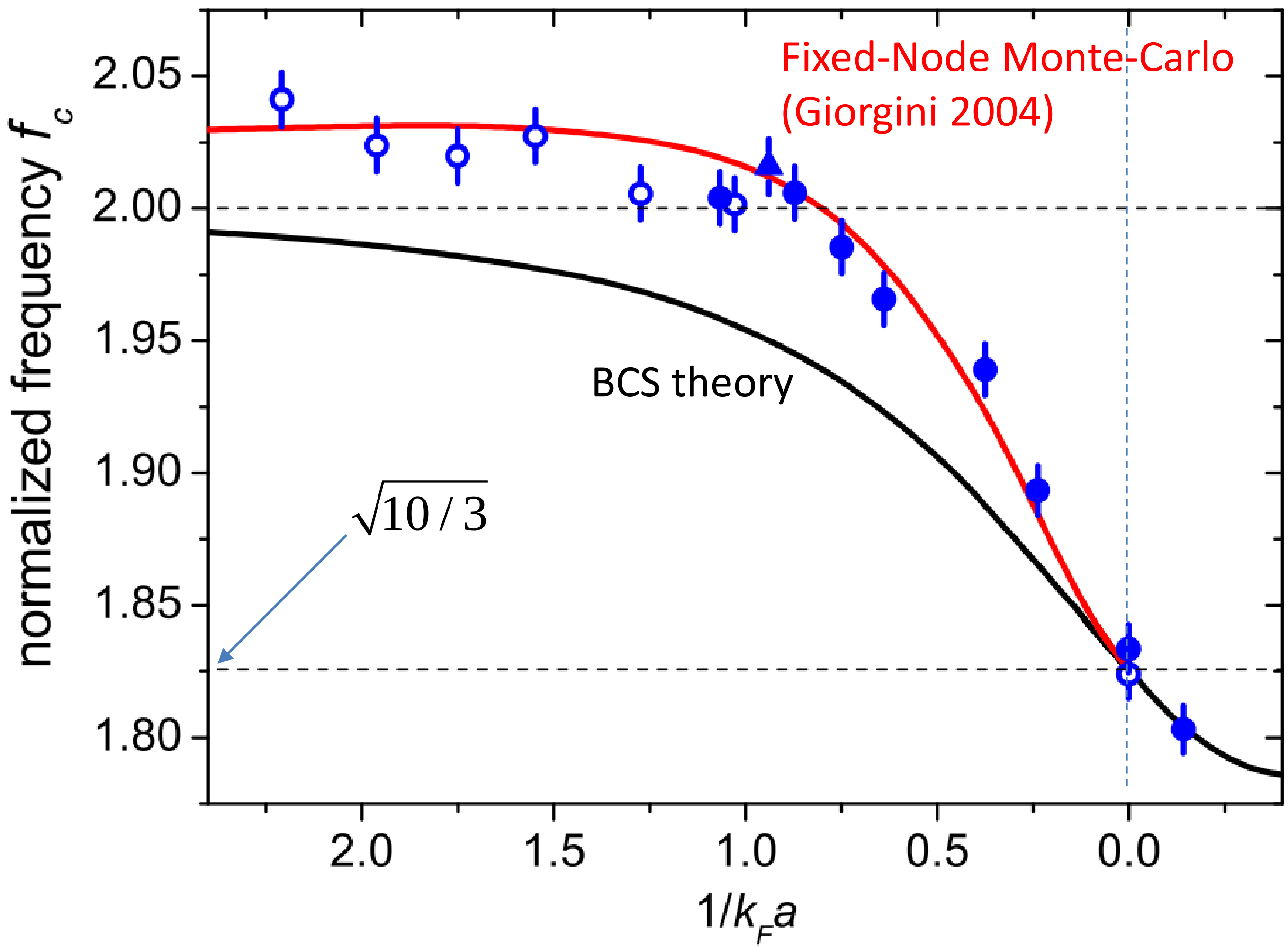
$$m\left(\partial_t v + \nabla v^2 / 2\right) = -\nabla(\mu(\rho) + V)$$

Formation of a vortex lattice

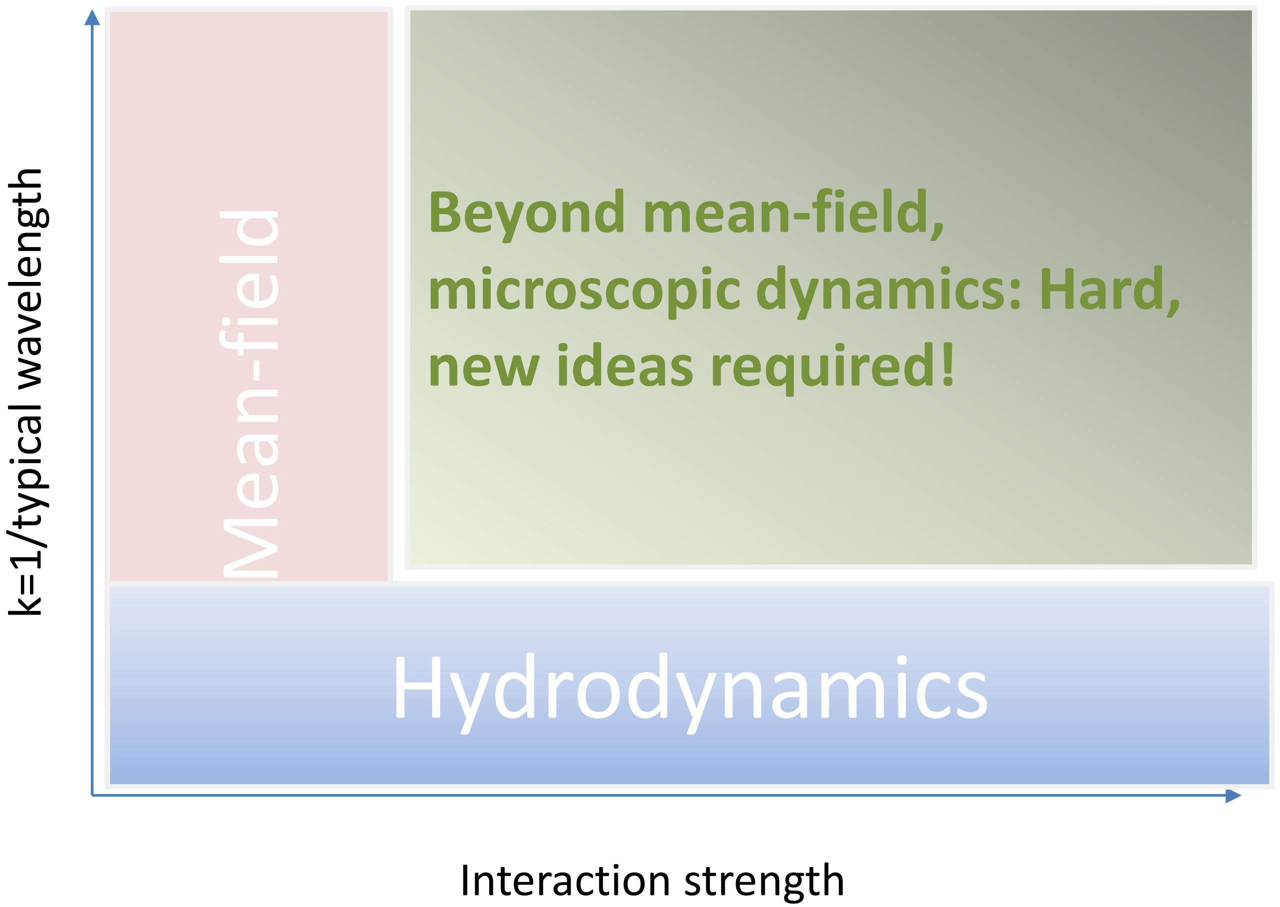
(Tonini et al, EPJD 2006)



Breathing mode of a fermionic superfluid (Innsbruck 2006)



CONCLUSION


Mean-field
Beyond mean-field, microscopic dynamics: Hard, new ideas required!
Hydrodynamics
k=1/typical wavelength
Interaction strength