Mean-field and other effective models in mathematical physics*

Les Treilles, France

May 20–25, 2019

List of participants

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FONDATION DES TREILLES

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Program

Monday

- 19:30 *Welcome dinner*

Tuesday

- 09:30–10:10 Jan Philip Solovej (Univ. Copenhagen, Denmark)
A leading order lower bound on the energy of dilute Bose gases with a sharp error term

- 10:20–11:00 Søren Fournais (Aarhus Univ., Denmark)
The second order correction to the energy of dilute Bose gases

- 11:10–11:40 *Coffee break*

- 11:40–12:20 Alice Sinatra (École Normale Sup., France)
A microscopic derivation of the condensate phase operator evolution in a weakly excited gas

- 13:00–14:00 *Lunch*

- 14:00–16:30 Free discussions

- 16:30–17:00 *Coffee break*

- 17:00–17:40 Antti Knowles (Univ. Genève, Switzerland)
Gibbs measures of nonlinear Schrödinger equations and many-body quantum mechanics

- 17:50–18:30 Phan Thanh Nam (LMU Munich, Germany)
Nonlinear Gibbs measures as the limit of equilibrium quantum Bose gases

- 18:30–19:00 Discussions

- 19:30 *Dinner*

Wednesday

- 09:30–10:10 Elliott H. Lieb (Princeton Univ., USA)
A study of a simple equation that describes the ground-state energy of a Bose gas at low and high density and in dimensions one, two and three

- 10:20–11:00 Markus Holzmann (CNRS Grenoble, France)
The phase transition of a dilute Bose gas in two and three dimensions

- 11:10–11:40 *Coffee break*

- 11:40–12:20 Fernando Brandão (Caltech, USA)
Quantum de Finetti theorems in quantum information theory

- 13:00–14:00 *Lunch*

- 14:00–16:30 Free discussions

- 16:30–17:00 *Coffee break*

- 17:00–17:40 François Golse (École Polytechnique, France)
Empirical Measures and Quantum Dynamics

- 17:50–18:30 Jonas Lampart (CNRS / Univ. Bourgogne, France)
Some properties of the time-dependent potential-to-density mapping

- 18:30–19:00 Discussions

- 19:30 *Dinner*

Thursday

- 09:15–09:55 Robert Seiringer (IST Austria)
The polaron at strong coupling

- 10:05–10:45 Benjamin Schlein (Univ. Zurich, Switzerland)
Dynamics of a polaron: accuracy of the Landau-Pekar equations

- 10:50–11:20 *Coffee break*

- 11:20–12:00 Simona Rota Nodari (Univ. Bourgogne, France)
The relativistic semi-classical equation for a nucleon and its non-relativistic limit

- 12:10–12:50 Thomas Leblé (Courant Institute New York, USA)
Classical systems with Coulomb/Riesz interactions

- 13:00–14:00 *Lunch*

- 14:00–19:00 *Free afternoon / excursion*

- 19:30 *Dinner*

Friday

- 09:30–10:10 Frédéric Chevy (École Normale Sup., France)
Dynamics of ultracold Fermi gases: Gross-Pitaevskii and beyond

- 10:20–11:00 Niels Benedikter (IST Austria)
Bosonization in the High-Density Fermi Gas

- 11:10–11:40 *Coffee break*

- 11:40–12:20 Christian Hainzl (LMU Munich, Germany)
Aspects of the BCS theory of superconductivity

- 13:00–14:00 *Lunch*

- 14:00–16:30 Free discussions

- 16:30–17:00 *Coffee break*

- 17:00–17:40 David Gontier (Univ. Paris-Dauphine, France)
Symmetry Breaking in the Hartree-Fock Homogeneous Electron Gas

- 17:50–18:30 Jakob Yngvason (Univ. Vienna, Austria)
Asymptotic Exactness of Magnetic Thomas-Fermi Theory at Nonzero Temperature

- 18:30–19:00 Discussion

- 19:30 *Conference dinner*

Abstracts

Benedikter, Niels (IST Austria)

Bosonization in the High-Density Fermi Gas

I am going to discuss how excitations of the high-density Fermi gas can be described by a bosonic effective theory, particularly emphasizing the importance of delocalizing particle-hole pairs in momentum space to relax the effect of the Pauli exclusion principle. Then I am going to discuss the properties of the obtained effective theory, focusing on the correlation energy and some open questions.

Brandão, Fernando (Caltech, USA)

Quantum de Finetti theorems in quantum information theory

I will discuss the connection of quantum de Finetti theorems, relating extendability to tensor product structure, with topics in quantum information theory and show how tools from quantum information can be used to prove such theorems.

Chevy, Frédéric (École Normale Sup., France)

Dynamics of ultracold Fermi gases: Gross-Pitaevskii and beyond

With Bose-Einstein condensation, fermionic superfludity is one of the main paradigms in quantum many-body physics. In the early 80's, Leggett, Nozières and Schmitt-Rink suggested that they could be unified within a single model, the so-called BEC-BCS crossover describing the ground state of an ensemble of attractive fermions. In this scenario, weakly attractive fermions are described by Bardeen-Cooper and Schrieffer's (BCS) theory while in the strongly attractive regime they form a Bose-Einstein condensate of deeply bound bosonic dimers. The BEC-BCS crossover hypothesis was confirmed experimentally in 2003 using ultracold vapours. However, an accurate and comprehensive theoretical description of the properties is still missing. In my talk, I will present an overview of recent advances in the experimental and theoretical study of strongly correlated fermions. I will show how Gross-Pitaevskii equation can be extended to address the dynamics of these systems within the hydrodynamic approximation and I will discuss numerical strategies pursued to capture the dynamics of solitons or vortices in these systems.

Fournais Søren (Aarhus Univ., Denmark)

The second order correction to the energy of dilute Bose gases

For a dilute system of non-relativistic bosons interacting through a positive L^1 potential v with scattering length a we prove that the ground state energy density satisfies the bound

$$e(\rho) \geq 4\pi a \rho^2 \left(1 + \frac{128}{15\sqrt{\pi}}\sqrt{\rho a^3} + o(\sqrt{\rho a^3}\,)\right),$$

thereby proving the Lee-Huang-Yang formula for the energy density. This is joint work with Jan Philip Solovej (Copenhagen).

Golse, François (École Polytechnique, France)

Empirical Measures and Quantum Dynamics

The notion of phase-space empirical measure associated to a classical N particle system, and its evolution under the dynamics defined by Newton's equations of motion, are a key ingredient in the derivation of the mean-field limit in classical mechanics. We define a quantum analogue of this notion of empirical measures and explain how it can be used to obtain a rate of convergence for the mean-field limit in quantum mechanics that is uniform in the Planck constant. Joint work with T. Paul

Gontier, David (Univ. Paris-Dauphine, France)

Symmetry Breaking in the Hartree-Fock Homogeneous Electron Gas

In this talk, we discuss the phase diagram of the Hartree-Fock (HF) Homogeneous Electron Gas (HEG), both theoretically and numerically. We first recall the Overhauser spatial symmetry breaking which occurs for $T = 0$ at all densities and we present a new lower bound on the energy gain due to this symmetry breaking. At high density we can prove that this gain can only be exponentially small. The argument reduces to estimating the first eigenvalue of a one-particle Coulomb operator with kinetic energy degenerating on a sphere. By the same method we prove that the critical temperature above which the gas is the paramagnetic fluid, is exponentially small as well.

Then we focus on the fluid phase of the HF HEG and discuss spin symmetry breaking. We find two regions where the gas is either ferromagnetic or paramagnetic. We can partly justify the paramagnetic phase theoretically.

This is joint work with Mathieu Lewin and Christian Hainzl.

Hainzl, Christian (LMU Munich, Germany)

Aspects of the BCS theory of superconductivity

We review various results about the BCS theory of superconductivity.

Holzmann, Markus (CNRS Grenoble, France)

The phase transition of a dilute Bose gas in two and three dimensions

I will try to review some of the results concerning the phase transitions of a homogeneous dilute Bose gas: the Bose-Einstein transition in three dimensions and the Berenzinski-Kosterlitz-Thouless transition in (quasi) two dimensions. I will focus on the scaling behavior of thermodynamic functions around the phase transition and the shift of the critical temperature as a function of the interaction.

Knowles, Antti (Univ. Genève, Switzerland)

Gibbs measures of nonlinear Schrödinger equations and many-body quantum mechanics

Many time-dependent nonlinear Schrödinger equations admit an invariant Gibbs measure, which is a probability measure on the space of distributions that is left invariant by the time evolution. Such measures have been extensively studied as tool to construct global solutions of time-dependent nonlinear Schrödinger equations with rough initial data. I review some recent progress in collaboration with Jürg Fröhlich, Benjamin Schlein, and Vedran Sohinger on deriving these measures in dimensions 1,2,3 as high-temperature limits of many-body quantum mechanics. In one dimension, I also explain how time-dependent correlation functions of the nonlinear Schrödinger equation arise as limits of corresponding quantum many-body correlation functions.

Lampart, Jonas (CNRS / Univ. Bourgogne, France)

Some properties of the time-dependent potential-to-density mapping

In time-dependent density functional theory one considers the mapping that associates to a time-dependent potential the one-particle density of the solution to Schrödinger's equation, for a given initial condition. The Runge-Gross theorem states that this map is one-to-one. I will discuss the hypothesis for this to hold and some properties of the range of the map.

Leblé, Thomas (Courant Institute New York, USA)

Classical systems with Coulomb/Riesz interactions

I will review recent results concerning gases of classical particles with Coulomb, or more general Riesz interactions. We use a "statistical physics" mindset to derive a free energy functional that governs their microscopic behavior in the thermodynamic limit. It also allows us to study the fluctuations in the finite systems, which exhibit remarkable rigidity properties.

I will mention some open questions about the phase diagrams, charge fluctuations, and universality results.

Lieb, Elliott H. (Princeton Univ., USA)

A study of a simple equation that describes the ground-state energy of a Bose gas at low and high density and in dimensions one, two and three

I will start with a quick review of the simple equation derived in 1964 to calculate the ground state energy E of a dilute Bose gas with 2-body repulsive interactions. It yielded the famous LHY term. Beyond that it suprisingly works reasonably well for all densities. It was then shown to work very well for the 1D delta function gas where the exact E is known and shown to be in error by at most 19%. It works well for quantum jellium also.

In current work with Eric Carlen and Ian Jauslin further results are obtained, partially with the help of modern computers:

(a) Rigorous proof of existence of a solution in all D, especially 2D where Schick had derived the answer with great labor (later proved correct with Jakob Yngvason);

(b) estimates of the condensate fraction which have an interesting, unforeseen twist at high density;

(c) Proof, by computation only, so far, of thermodynamic stability.

This is work in progress. Many open problems remain.

Phan Thanh Nam (LMU Munich, Germany)

Nonlinear Gibbs measures as the limit of equilibrium quantum Bose gases

We consider the Gibbs state of an interacting Bose gas just above the critical temperature of Bose-Einstein phase transition. We show that, in a mean-field limit, a superposition state of condensates emerges and it can be effectively described by a nonlinear Gibbs measure. Except in the one-dimensional case, the Gibbs measure is supported on singular distributions. On the other hand, the quantum problem is completely well defined but it is involving non-commutative objects. Our derivation is based on a novel method to control the particle correlation of the quantum Gibbs state, which allows to take into account the Wick renormalization in the limit. This is joint work with Mathieu Lewin and Nicolas Rougerie.

Rota Nodari, Simona (Univ. Bourgogne, France)

The relativistic semi-classical equation for a nucleon and its non-relativistic limit

In this talk, we consider one relativistic nucleon interacting with classical σ and ω mesons fields in an atomic nucleus. I will explain that, in the non-relativistic limit of nuclear physics, the model converges to a specific nonlinear Schrödinger-type equation with a mass depending on the solution itself. I will also discuss open problems concerning the case of several nucleons. Joint work with Mathieu Lewin.

Schlein, Benjamin (Univ. Zurich, Switzerland)

Dynamics of a polaron: accuracy of the Landau-Pekar equations

In this talk, we will discuss recent results concerning the time-evolution of a polaron, ie. an electron interacting with the quantized polarisation field of an underlying lattice (phonons), in the regime of strong coupling $\alpha \gg 1$. For initial data where the electron minimizes the energy associated with the phonon field, we show that the nonlinear Landau-Pekar equations provide a good approximation of the dynamics, up to times of order α^2. The proof makes use of an adiabatic theorem for the solution of the Landau-Pekar equations. This talk is based on joint work with Nikolai Leopold, Simone Rademacher and Robert Seiringer.

Seiringer, Robert (IST Austria)

The polaron at strong coupling

We review old and new results on the Fröhlich polaron model. The discussion includes the validity of the (classical) Pekar approximation in the strong coupling limit, quantum corrections to this limit, as well as the divergence of the effective polaron mass.

Sinatra, Alice (École Normale Sup., France)

A microscopic derivation of the condensate phase operator evolution in a weakly excited gas

A fundamental property of macroscopic quantum systems such as Bose-Einstein condensates is time coherence or phase coherence. To study the intrinsic limits of phase coherence in an isolated condensed gas at thermal equilibrium, it is necessary to treat the condensate phase as an operator and to describe its dynamics including the thermal excitations of the system.

In this talk, I will explain how we can derive an evolution equation for the phase operator $\hat{\theta}_0$ of a condensate, and relate $\mathrm{d}\widehat{\theta}_0/\mathrm{d}t$ to a "chemical potential operator" at non-zero temperature, thus generalizing to the multi-mode quantum case the second Josephson relation established for the phase of the order parameter of a superconductor. We will limit ourselves to the "weakly excited" case, where the thermal depletion of the condensate is weak and the quasi-particles are in the so called collisionless regime.

While the bosonic case is relatively smooth and can be studied within the Bogoliubov theory with a fixed total number of particles, the case of a condensate of pairs of fermions in two spin states, where we lack a systematic approach such as Bogoliubov's theory, is more complicated. The contribution of the fermionic excitation branch to $\mathrm{d}\widehat{\theta}_0/\mathrm{d}t$ can be obtained by the linearized equations of motion for the fluctuations of the pair operators $\widehat{\psi}_\downarrow\widehat{\psi}_\uparrow$, $\widehat{\psi}_\uparrow^\dagger\widehat{\psi}_\downarrow^\dagger$ and $\widehat{\psi}_\sigma^\dagger\widehat{\psi}_\sigma$ around their mean field value (Anderson's RPA). On the other hand, for the contribution of the phonon branch, we had to use a semiclassical variational approach.

H. Kurkjian, Y. Castin, A. Sinatra, Brouillage thermique d'un gaz cohérent de fermions *(Thermal blurring of a coherent Fermi gas)*, Comptes Rendus Physique **17**, 789 (2016). English version: *arXiv:1502.05644*

Solovej, Jan Philip (Univ. Copenhagen, Denmark)

A leading order lower bound on the energy of dilute Bose gases with a sharp error term

I will discuss how to obtain a lower bound on the ground state energy of Bose gases, which in the dilute limit has the correct leading order term with an error of the order predicted by the Lee-Huang-Yang formula. The assumptions on the interaction potential are that it is positive and has finite scattering length. In particular, the bound holds for the hard core potential. The work is based on work with Fournais and work with Brietzke and Fournais.

Yngvason, Jakob (Univ. Vienna, Austria)

Asymptotic Exactness of Magnetic Thomas-Fermi Theory at Nonzero Temperature

Thomas-Fermi theory for matter in strong magnetic fields at nonzero temperature has since long been used by astrophysicists for approximate computations of the equation of state of neutron star crusts. I will review some rigorous results about this approximation and point out possible extensions.