Optimal Transportation: Economic Applications

Conference in honor of Ivar Ekeland

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U. Paris Dauphine, June 2014

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- This presentation: marriage market only (although some hedonic)

A few relevant questions

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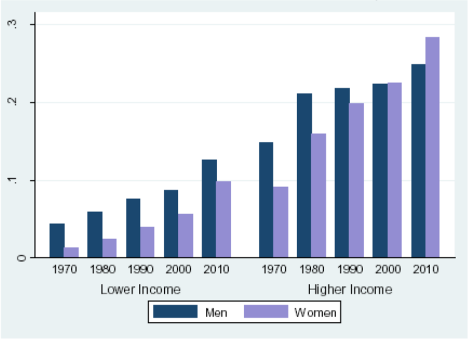
     - Why did correlation change? Did 'preferences for assortativeness' change?
     - How do we compare single-adult households and couples? What about intrahousehold inequality?

A few relevant questions (cont.)

2. College premium and the demand for college education
   Motivation: remarkable increase in female education, labor supply, incomes worldwide during the last decades.

FIGURE 3: FRACTION OF 30- TO 34-YEAR-OLDS WITH COLLEGE EDUCATION, COUNTRIES ABOVE MEDIAN PER CAPITA GDP AND BELOW PER CAPITA GDP, BY SEX

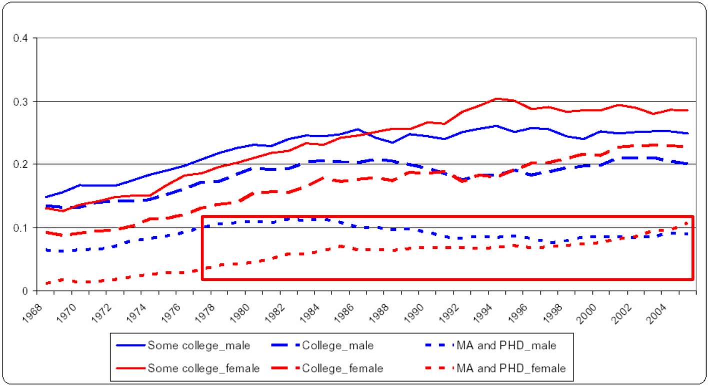


Source: See Figure 1.

Source: Becker-Hubbard-Murphy 2009

In the US:

Figure 13: Completed Education by Sex, Age 30-40, US 1968-2005



Source: Current Population Surveys.

College premium and the demand for college education

Questions:

Why such different responses by gender?

Answer (CIW 2009)

'Marital college premium'

→ how can we compute that?
→ how can we identify that?

→ A structural model is needed!

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   - In particular, what about women:
     - who do want children
     - who would not use abortion (e.g. for religious reasons), etc.
   - ... and what the heck is the relationship between all this and optimal transportation?

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  - two functions $u : X \to \mathbb{R}$ and $v : Y \to \mathbb{R}$ such that:

$$u(x) + v(y) = s(x, y) \quad \forall (x, y) \in \text{Supp}(h)$$

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    ('*how is the surplus allocated?'*)
  - The matching is *pure* if the support of the measure is included in the graph of some function ϕ

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  - Translation:

$$u(x) + v(y) \geq s(x, y) \quad \forall (x, y) \in X \times Y \tag{1}$$

Links with Optimal Transportation

$\rightarrow$ Shapley-Shubik, Becker, Gretsky et al., Ekeland, Ekeland and Carlier, CMcCN, etc.

- Consider the *surplus maximization problem*

$$\max_h \int_{X \times Y} s(x, y)\, dh(x, y)$$

under condition on the marginals (or push forward) of h $(\pi_{\#}^X h = F, \pi_{\#}^Y h = G)$.

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- This is an OT problem, and its dual is:

$$\min \int_X u(x)\, dF(x) + \int_Y v(y)\, dG(y) \quad \text{under}$$

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- Therefore:
    - there exists a stable match if and only if the surplus max problem has a solution (and the value is the same)
    - intracouple allocation determined as the solution to a linear maximization problem!

Links with hedonic models

- Hedonic models: defined by set of buyers X, sellers Y, products Z
- Buyers: utility $u(x,z) - P(z)$ which is maximized over z
- Sellers: profit $P(z) - c(y,z)$ which is maximized over z
- Equilibrium: $P(z)$ such that markets clear ($\rightarrow$ measure over $X \times Y \times Z$)
- Canonical correspondence between QL hedonic models and matching models under TU. Specifically, consider a hedonic model and define surplus:

$$s(x,y) = \max_{z \in Z}(U(x,z) - c(y,z))$$

Let η be the marginal of α over $X \times Y$, $u(x)$ and $v(y)$ by

$$u(x) = \max_{z \in K} U(x,z) - P(z) \text{ and } v(y) = \max_{z \in K} P(z) - c(y,z)$$

Then (η, u, v) defines a stable matching. Conversely, to each stable matching corresponds an equilibrium hedonic price schedule.

Economic applications of OT theory

Three examples

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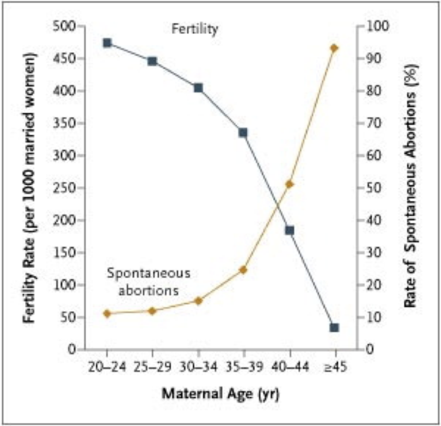
3. The rise of higher education for women (Low 2014)

Reproductive capital and women's demand for higher education

Source: Corinne Low's dissertation (2014)

- Basic remark: sharp decline in female fertility between 35 and 45

Rates of Infertility and Miscarriage Increasing Sharply with Age



Source: Heffner 2004, "Advanced Maternal Age: How old is too old?"

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Spousal Income vs Age at Marriage (1955-1966 birth cohort, 2010 ACS)

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- Impact on marital prospects?

Model

- Two commodities, private consumption and child expenditures; utility:

$$u_i = c_i\,(Q+1)\,,\;\; i = h, w$$

and budget constraint (y_i denotes i's income)

$$c_h + c_w + Q = y_h + y_w$$

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$$u_i = c_i (Q + 1), \; i = h, w$$

and budget constraint (y_i denotes i's income)

$$c_h + c_w + Q = y_h + y_w$$

- Transferable utility: any efficient allocation maximizes $u_h + u_w$; therefore surplus with a child

$$s(y_h, y_w) = \frac{(y_h + y_w + 1)^2}{4}$$

and without a child ($Q = 0$)

$$s(y_h, y_w) = y_h + y_w$$

therefore, if π probability of a child:

$$s(y_h, y_w) = \pi \frac{(y_h + y_w + 1)^2}{4} + (1 - \pi)(y_h + y_w)$$

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  - but investment implies fertility loss
    - $\pi = p$ if invest
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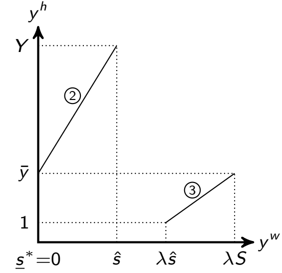
1. Negative assortative



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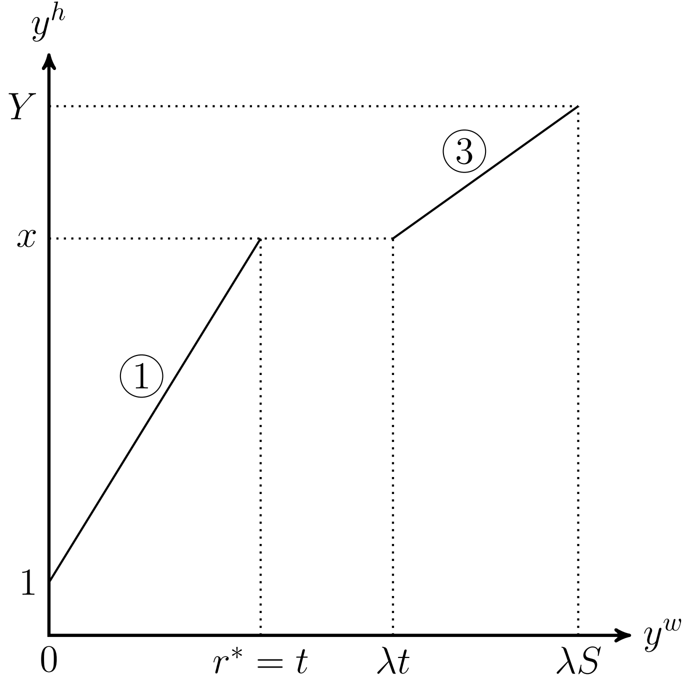

y^h
Y
x
1
0
③
①
$r^* = t$
λt
λS
y^w

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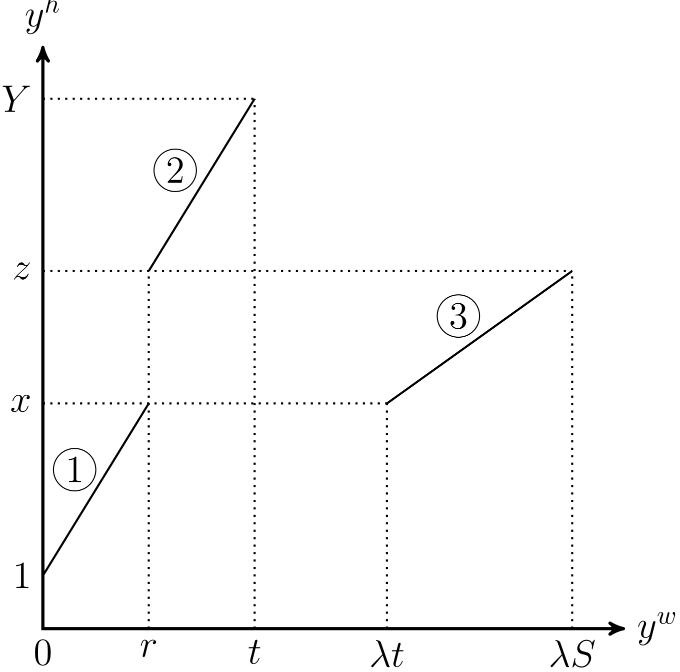

y^h
Y
z
x
1
0
r
t
λt
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y^w
1
2
3

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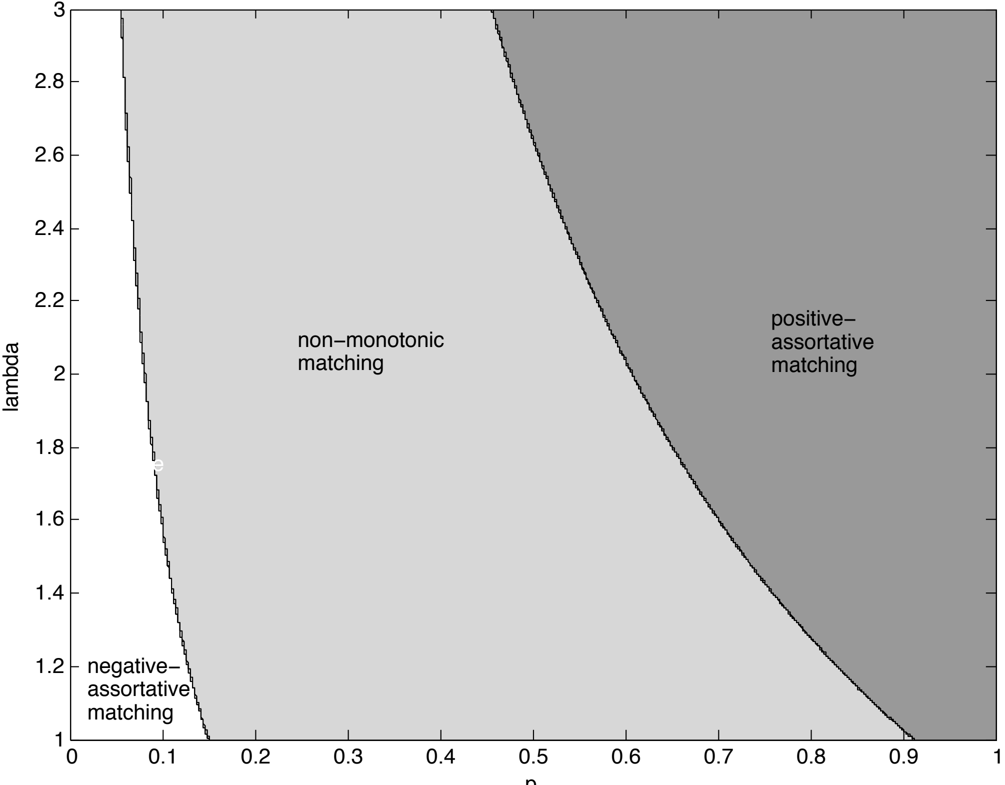
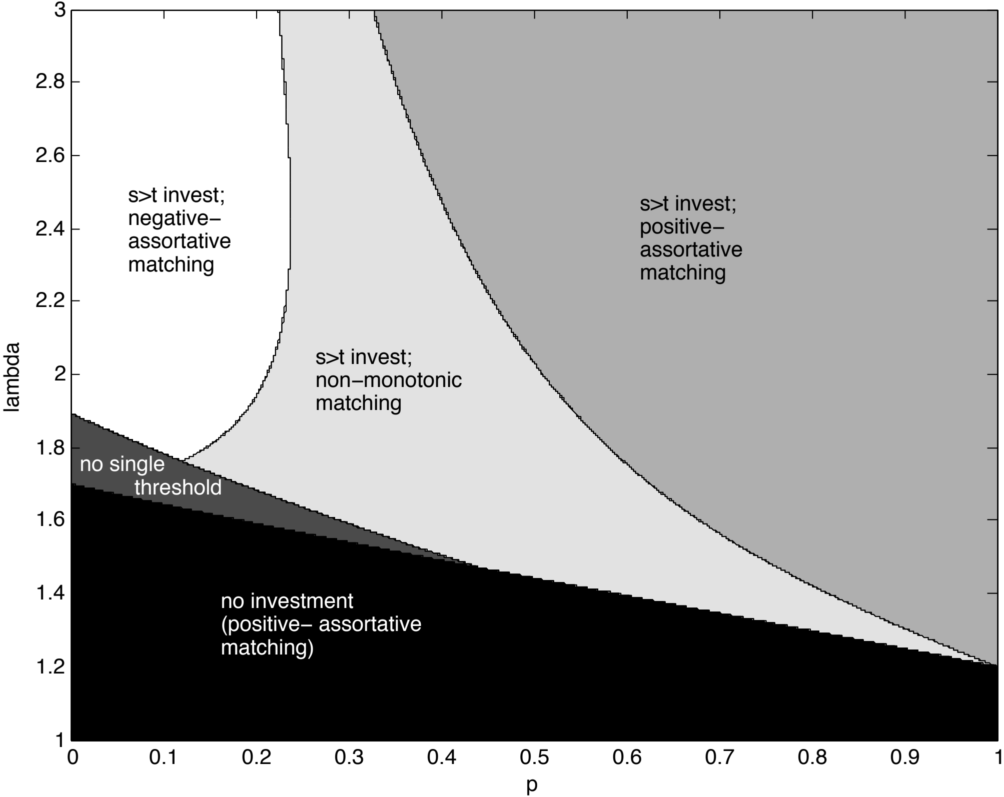


negative–
assortative
matching
non–monotonic
matching
positive–
assortative
matching
lambda
p
e

Stage 1: investment choice

$\longrightarrow$ Graph



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negative−
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lambda
p

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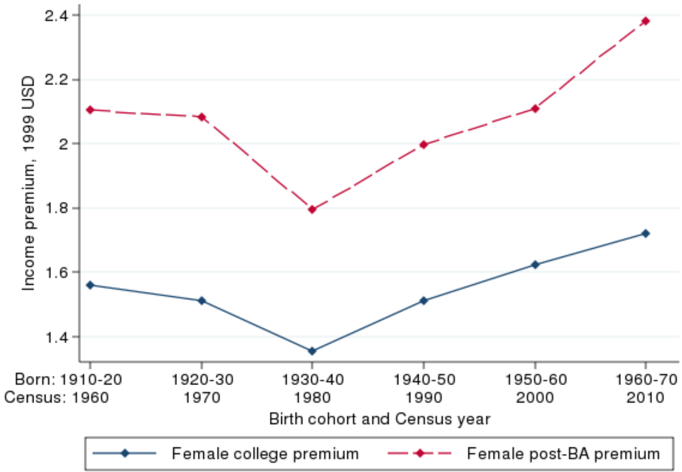
- Increase in λ: dramatic increase in 'college + premium'

Wage income premium over women with some college



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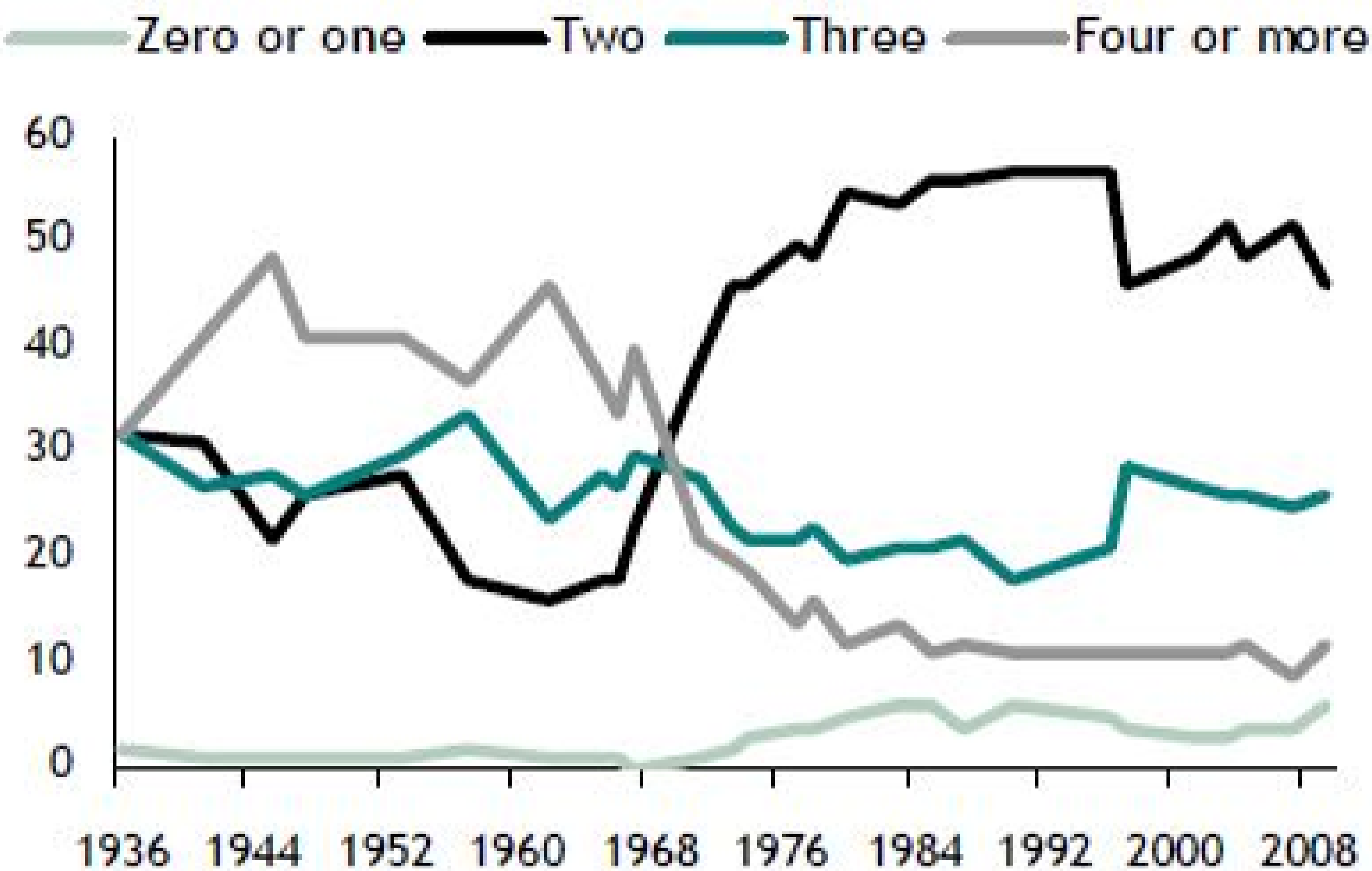
(%)



Notes: “Don’t know/refused” responses not shown. Respondents were asked: "What is the ideal number of children for a family to have?"

Source: Gallup, 1936-2007; Pew Research Center, 2009

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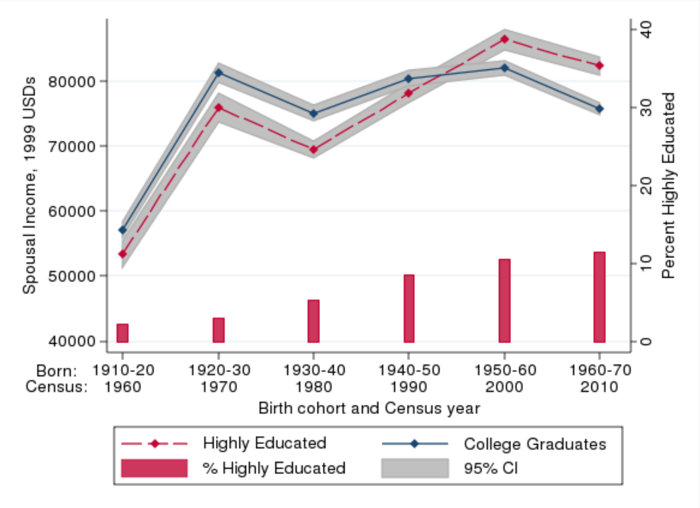
- What about data?

Spousal income by wife's education level, white women 41-50



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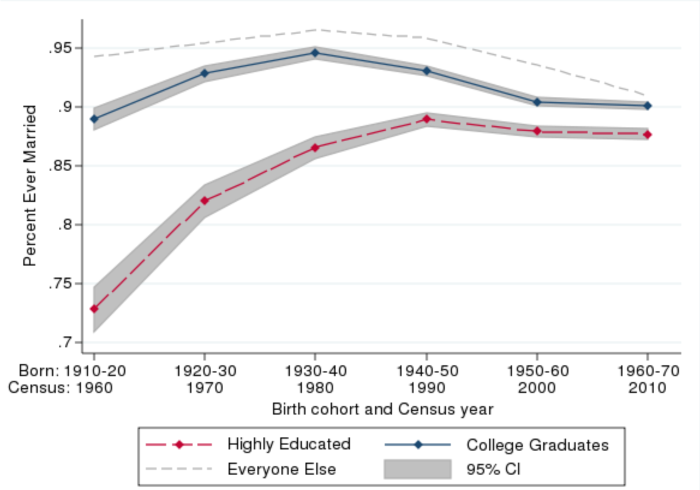

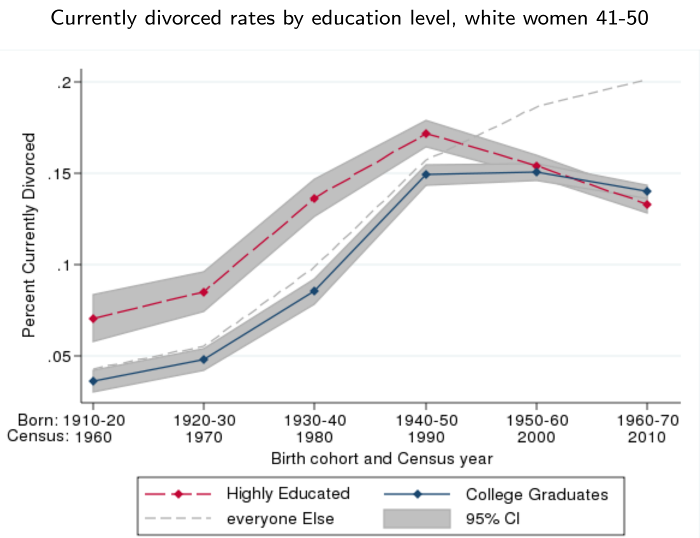

Currently divorced rates by education level, white women 41-50
Percent Currently Divorced
.2
.15
.1
.05
Born: 1910-20
Census: 1960
1920-30
1970
1930-40
1980
1940-50
1990
1950-60
2000
1960-70
2010
Birth cohort and Census year
Highly Educated
College Graduates
everyone Else
95% CI

Generalization: the 'true' bidimensional model

Source: Chiappori, McCann, Pass (in progress)

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$$u(x_1, x_2) = \max_y s(x_1, x_2, y) - v(y)$$

Assume purity, then $y = f(x_1, x_2)$ and envelope theorem:

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- CDR give the pdf in f

$$\frac{\partial^2 s}{\partial x_1 \partial y} \frac{\partial f}{\partial x_2} = \frac{\partial^2 s}{\partial x_2 \partial y} \frac{\partial f}{\partial x_1}$$

Generalization: the 'true' bidimensional model

Actually, if ϕ defined by

$$f(x_1, x_2) = y \to x_2 = \phi(x_1, y)$$

then DE in ϕ:

$$\frac{\partial \phi}{\partial x_1} = \frac{\frac{\partial^2 s(x_1, \phi(x_1, y), y)}{\partial x_1 \partial y}}{\frac{\partial^2 s(x_1, \phi(x_1, y), y)}{\partial x_2 \partial y}}$$

In our case:

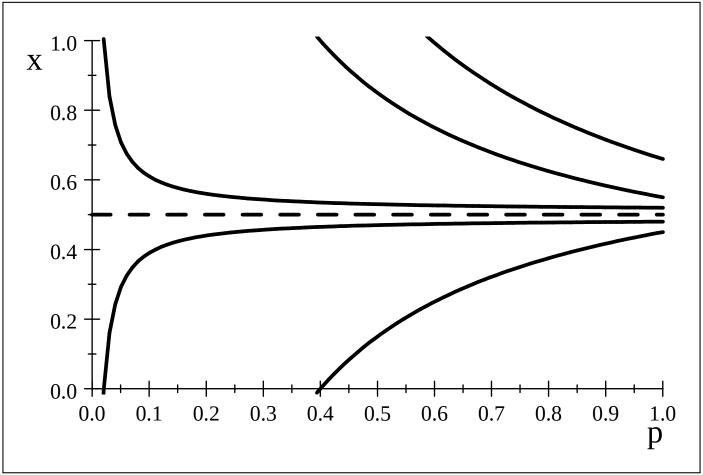
$$\frac{\partial \phi}{\partial p} = -\frac{1}{p}(\phi(p, y) + y - 1)$$

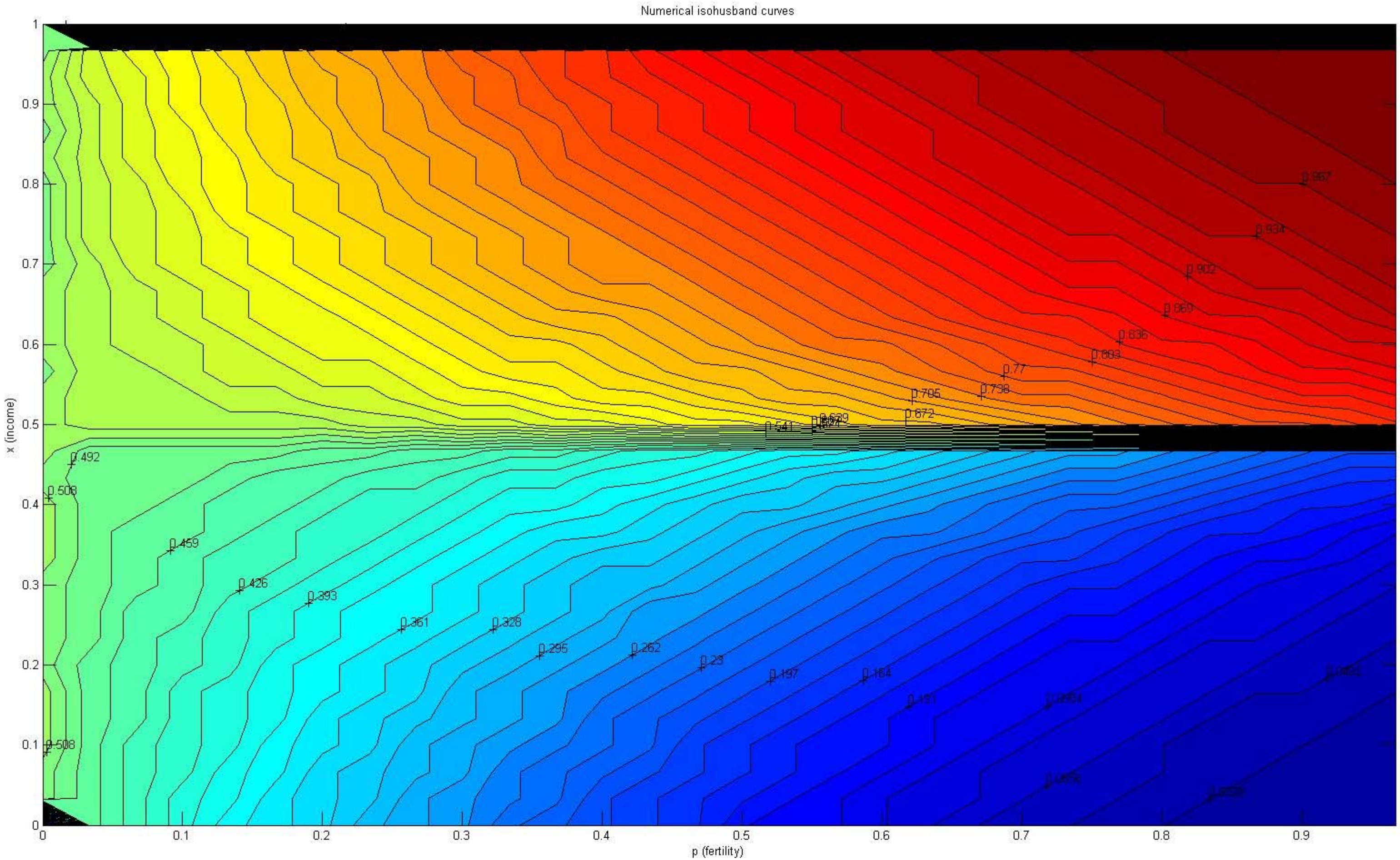
gives

$$\phi(p, y) = 1 - y + \frac{K(y)}{p}$$

and $K(y)$ pinned down by the measure conditions

The uniform case: iso-husband curves





Numerical isohusband curves
x (income)
p (fertility)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.492
0.508
0.459
0.426
0.393
0.361
0.328
0.295
0.262
0.23
0.197
0.164
0.705
0.738
0.77
0.672
0.967
0.934
0.902

A stochastic version

Finally, how can we capture traits that are unobservable (to the econometrician)?
$\longrightarrow$ Usual idea: unobserved heterogeneity represented by a random component (say, in the surplus function)
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  - both:

$$\varepsilon_{i,j} = \alpha_i^J + \beta_j^I + \eta_{ij}$$

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  Theorem: *In the Choo Siow specification, there exists* $U^{I,J}$ *and* $V^{I,J}, I, J = 1, ..., K$, *with* $U^{I,J} + V^{I,J} = Z^{I,J}$, *such that for any matched couple* $(i \in \bar{I}, j \in \bar{J})$

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  $\rightarrow$ can compute

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