MARKET FRICTIONS AND CORPORATE FINANCE

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THE CONTEXT

- Market finance has been revolutionized by the use of Arbitrage Pricing Markets
- Financial mathematicians have designed and developed these methods in many directions (incomplete markets, transaction costs, incomplete information)
- Following Merton (1973), corporate finance has started using similar methods to study financing and liquidity management policies of corporations.
- Frictions have to be introduced to get out of the Modigliani and Miller paradox: without frictions, financing policy is irrelevant!

FRICTIONS IN CORPORATE FINANCE

- Bankruptcy costs and tax-deductibility of debt (Trade-Off theory)
- Internal frictions (corporate governance problems, moral hazard)
- Market frictions (issuance costs, market breakdowns,...)

I will focus here on the third type of frictions.

MODEL

Filtered probability space (Ω, F_t, ρ)

- $t = 0$: entrepreneur creates company
  Investment I $\rightarrow$ ... cash flow
  $\mu dt + \sigma dW_t \quad t \geq 0$
  W_t = standard Brownian motion (F_t-adapted)
  I is financed by issuing securities (typically debt + equity) on primary market.
- $0 < t < \tau$: company pays out $dL_t \geq 0$ specified by securities' contracts.
- $t = \tau$: company defaults on its contractual obligations. Liquidation (zero value).

RESEARCH QUESTION

- Optimal financing method L_t?
- IMPORTANT RESTRICTION $dL_t \geq 0$ because securities have limited liabilities ($\neq$ credit lines, venture capital,...)

Examples:

- $dL_t = cdt \quad t \leq \tau$ (straight debt)
- $L_t =$ Non predictable dividend process (equity)

MARKET FRICTIONS

- Issuing new securities is costly (underwriting fees, legal costs, taxes).
- In practice (Ross et al. 2003, US data) $\sim 7\%$ of gross proceeds.
- Fixed cost component $K \Rightarrow$ new issues are lumpy and infrequent (for simplicity: no variable costs)
- Cumulated (net) issuance strategy (F_t adapted)

$$T_t = \sum_{i=1}^{\infty} x_i \mathbb{1}_{\{t \geq t_i\}}$$

i^{th} issuance: date $t_i >$ net amount x_i (gross amount: $x_i + K$)

FIRST BENCHMARK: $K = \infty$

- No new issuance $\Rightarrow$ Firm defaults when runs out of cash to cover operating losses.
- Cash reserves: $X_t = x$

$$dX_t = \mu dt + \sigma dW_t - dL_t$$

$$\tau = Inf\{t, x_t < 0\}$$

- Shareholder value only depends on X_t:

$$V^L(x) = \mathbb{E}[\int_0^{\tau} e^{-rt} dL_t \quad | \quad X_t = x]$$

NB: Shareholders are risk neutral.

OPTIMAL FINANCING POLICY

Jeanblanc - Shiryaev (1995) Radner - Shepp (1996)

$$V_\infty(x) = \max_{dL_t \geq 0} V^L(x)$$

- Proposition 1: V_∞ is the unique $\mathscr{C}^2$ solution of

$$max\{\frac{\sigma^2}{2} V'' + \mu V' - rV, 1 - V'\} = 0$$

  such that $V(0) = 0$.
- Intuition:

$$rV_\infty(x) = \max_{l \geq 0} l[1 - V'_\infty(x)] + \mu V'_\infty(x) + \frac{\sigma^2}{2} V''_\infty(x)$$

OPTIMAL FINANCING POLICY (2)

- Proposition 2: $\exists ! x^*_\infty / rV_\infty(x) = \mu V'_\infty(x) + \frac{\sigma^2}{2} V''_\infty(x) \quad x < x^*_\infty$

$$V'_\infty(x) = 1 \qquad x > x^*_\infty$$

$$V'_\infty(x^*_\infty) = 1, V''_\infty(x^*_\infty) = 0).$$

- Intuition: pay out everything above x^*_∞, nothing below.
- Variant of the Skorokhod problem: X_t is stopped at x^*_∞, but killed at 0.

FINANCIAL IMPLICATIONS

- Debt ($dL_t = cdt$) is sub-optimal
- More generally, any security that draws upon cash reserves when $V'_\infty(x) > 1$ is sub-optimal
- Optimal financing = 100% equity
- Note that dividends dL_t are non-predictable
  $\Rightarrow$ even contingent debt $c(X_t)$ is sub-optimal

FINITE ISSUING COST K

- Shareholder value:

$$V_K^{L,J}(x) = \mathbb{E}[\int_0^T e^{-rt}(dL_t - dJ_t^K)]$$

$$dX_t = \mu dt + \sigma dW_t - dL_t + dJ_t$$

$$= Inf\{f, X_{t^+} < 0\}$$

$$J_t = \sum_{t_i<t} x_i \quad J_t^K = \sum_{t_i<t}(x_i + K)$$

$$V_K(x) \equiv \max_{\substack{dL_t \geq 0 \\ dJ_t \geq 0}} V_K^{L,J}(x)$$

SECOND BENCHMARK: $K = 0$

- When there are no frictions the solution in trivial: immediately distribute all cash, choose arbitrarily L_t and J_t s.t. $dL_t - dJ_t = \mu dt + \sigma dW_t$ (Modigliani Miller)

$$V_0(x) = x + \mathbb{E}[\int_0^\infty e^{-rt}(\mu dt + \sigma dW_t)] = x + \frac{\mu}{r}$$

- Cash is useless; Optimal financing is under intermediate
- Value of the firm = cash + ENPV (future cash flows)

SOLUTION WHEN $K > 0$

Decamps-Mariotti-Rochet-Villeneuve (JoF 2011)

- Proposition 3: $\exists! x^*(K)$ such that

$$rV_K(x) = \mu V_K'(x) + \frac{\sigma^2}{2} V_K''(x) \quad 0 < x < x^*(K)$$

$$V_K(0) = [V_K(x^*(K)) - x^*(K) - K]_+$$

$$V_K'(x) = 1 \quad x > x^*(K); \quad V_K \in \mathscr{C}^2(\mathbb{R})$$

- NB: $x^*(.)$ increasing, $x^*(K) = x^*$ for K large enough, then V_K coincides with V_∞.

OPTIMAL FINANCING

- Still 100% equity.
- Target cash level increases in K.
- Dividends distributed more often when $K \downarrow$.
- New issuance only takes place when $X_t = 0$ (not true in practice).
- No variable issuance costs $\Rightarrow$ optimal to collect (in net) the target level of cash $= x^*(K)$

STOCK PRICE IMPLICATIONS

- Shareholder value (we drop index K)

$$V(X_t) = N_t S_t$$

where $N_t = \#$ outstanding shares, $S_t =$ stock price

- N_t is locally constant. Only increases when $X_t = 0$

$$dN_t = (\Delta N) \mathbb{1}_{X_t = 0}$$

- When new issue, S_t cannot jump (no arbitrage)

$$\begin{cases} V(0) = N_t S_t \\ V(x^*) = (N_t + \Delta N) S_t = V(0) + x^* + K \end{cases}$$

$$\Rightarrow \frac{\Delta N}{N_t} = \frac{V(x^*) - V(0)}{V(0)} = \frac{x^* + K}{V(0)} = \delta \text{ constant dilution factor}$$

STOCK PRICE IMPLICATIONS (2)

- In between two issuances, S_t=deterministic function of x_t:
  $S_t = S(N, X_t) = \frac{V(X_t)}{N}$
- S_t varies between $S_0 = \frac{V(0)}{N}$ and $S^* \equiv \frac{V(x_t)}{N}$
  $\frac{dS_t}{S_t} = \pi dt + \sigma(t, S_t) dW_t$
- $\frac{dS_t}{S_t} = \frac{V'(x_t)}{V(x_t)} dx_t + \frac{\sigma^2}{2} \frac{V''(x_t)}{V(x_t)} dt$
  where $\sigma(t, S) = \sigma \frac{V'}{V}(N_t V^{-1}(s))$ $\quad \downarrow$ in t and S
  local volatility model.

THE CASE WHERE $K = 0$

- Shareholder value is constant: $N_t S_t \equiv \frac{\mu}{r}$
- However the number of shares evolves to cover financing needs: $S_t dN_t + \mu dt + \sigma dW_t = 0$

$$\Rightarrow \frac{dS_t}{S_t} = \frac{N_t dS_t}{N_t S_t} = -\frac{r}{\mu} StdN_t = rdt + \frac{r\sigma}{\mu} dW_t$$

CONSTANT VOLATILITY

FINANCING SiFis

Rochet-Zargari(2014): Work in progress

SiFis are financial institutions whose liquidation would entail large social costs (large, complex banks or financial utilities like Clearing Houses or Exchanges)

$\Rightarrow$ Whenever shareholders want to close them, government is compelled to intervene: capital injection + reconstruction $\Rightarrow$ generates large costs γ
We use the previous model with K very large

CONFLICTING OBJECTIVES

Shareholders $V(x) = \mathbb{E}[\int_0^\tau e^{-rt} dL_t | X_t = x]$

Regulator $R(x) = V(x) - [\gamma + x_R - V(x_R)]\mathbb{E}[e^{-rt} | X_t = x]$

$\gamma =$ cost of public intervention.

SiFi is temporalily nationalized, restructured and privatized again.

PRIVATELY OPTIMAL FINANCING

Same as above:

- 100% equity
- Dividends are only distributed when $X_t > x^*_\infty$.
- Firm is liquidated when X_t falls below 0.

Cost γ of public intervention: not internalized by shareholders $\Rightarrow$ Frequent failures.

SOCIALLY OPTIMAL FINANCING

- $rR(x) = \mu R'(x) + \frac{\sigma^2}{2} R''(x) \quad 0 < x < x^R$
- $R'(x) = 1 \quad x > x^R$
- $R \in \mathscr{C}^2(\mathbb{R}_+)$
- $R(0) = V(x^R) - x^R - \gamma < 0$

- Still 100% equity.
- Target cash level is higher: $x^R > x^*_\infty$.
- Implemented by capital/liquidity regulation:
  firm not allowed to distribute dividends if $x_t \leq x^R$.

INTRODUCING DEBT

In practice, corporations issue a lot of debt (especially banks). Something is missing in our model:

- Tax deductibility of coupons
- debt reduces the Free Cash Flow problem (Jensen)
- debt may provide incentives to managers (?!)

But debt financing has many drawbacks:

- Increases default probabilities
- Limits future investment (debt overhang)
- May generate systematic risk

FUTURE RESEARCH

In practice companies default either for liquidity reasons (like in our paper) or for solvency reasons.
A natural extension would be a two dimensional model with two state variables: cash reserves X_t and profitability μ_t.