Spectra of random regular hypergraphs

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Joint work with Yizhe Zhu

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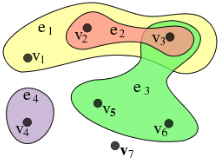
Hypergraphs

- Hypergraph: V =vertex set, E =edge set



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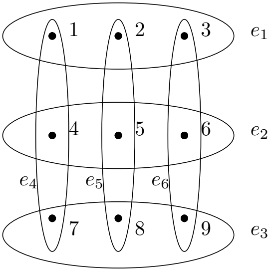
- $k = 2$: d-regular graphs.



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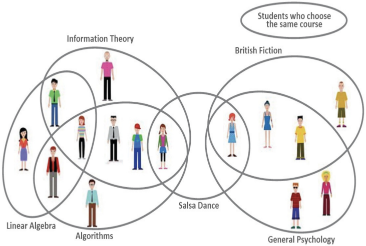

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- As with graphs, one main object of study is *expansion* (edge, vertex, spectral)

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- Local laws, eigenvectors, etc.

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- Applications in optimization, etc.

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- Incidentally, general hypergraphs' eigenvalues have been connected to diameters, random walks, Ricci curvature (Banerjee '17)

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Theorem (Feng-Li 1996)

Let G_n be any sequence of connected (d,k)-regular hypergraphs with n vertices. Then

$$\lambda_2(A_n) \geq k - 2 + 2\sqrt{(d-1)(k-1)} - \epsilon_n.$$

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- Algebraic construction: Martínez-Stark-Terras (2001), Li (2004), Sarveniazi (2007).

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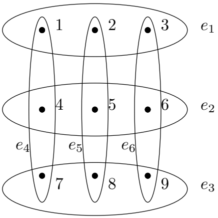

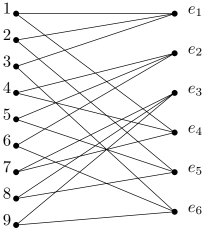

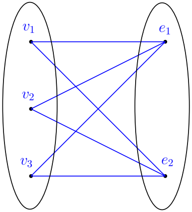

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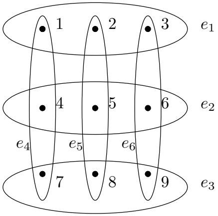

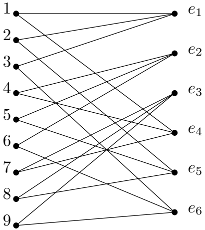

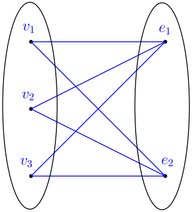

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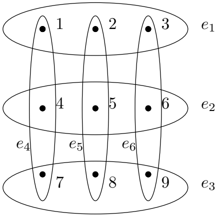

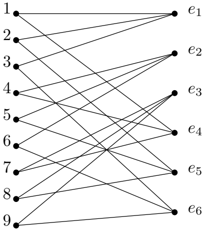

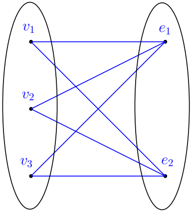

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- Any event F holds whp for random bipartite biregular graphs
  $\Leftrightarrow$ F holds whp for the uniform measure over S_1
  $\Leftrightarrow$ corresponding F' holds whp for random regular hypergraphs.

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Let G_n be a random (d,k)-regular hypergraphs with n vertices. Then with high probability for any eigenvalue $\lambda \neq d(k-1)$,

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- Uses Brito, D., Harris ('20)
- What does λ_2 tell us about H?

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Let H be a (d,k)-regular hypergraph and $\lambda = \max\{\lambda_2, |\lambda_n|\}$. Then the following holds: for any subsets $V_1, V_2 \subset V$,

$$\left| e(V_1, V_2) - \frac{d(k-1)}{n}|V_1|\cdot|V_2| \right| \leq \lambda \sqrt{|V_1|\cdot|V_2|\left(1-\frac{|V_1|}{n}\right)\left(1-\frac{|V_2|}{n}\right)}.$$

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$e(V_1, V_2)$: number of hyperedges between V_1, V_2 with multiplicity $|e \cap V_1| \cdot |e \cap V_2|$ for any hyperedge e.

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- a non-backtracking walk of length ℓ in a hypergraph is a sequence

$$w = (v_0, e_1, v_1, e_2, \ldots, v_{\ell-1}, e_\ell, v_\ell)$$

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- a NBRW of length ℓ from v_0: a uniformly chosen member of all non-backtracking walks of length ℓ starting at v_0.
- How fast does the NBRW converge to a stationary distribution? Mixing rate:

$$\rho(H) := \limsup_{\ell\to\infty} \max_{i,j\in V} \left|(P^{(\ell)})_{ij} - \frac{1}{n}\right|^{1/\ell}.$$

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Theorem (D.-Zhu 2019)

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- NBRWs mix faster than simple random walks.

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d, k constant	$f(x) = \frac{1+\frac{k-1}{q}}{(1+\frac{1}{q}-\frac{x}{\sqrt{q}})(1+\frac{(k-1)^2}{q}+\frac{(k-1)x}{\sqrt{q}})} \frac{1}{\pi}\sqrt{1-\frac{x^2}{4}}$ with $q=(k-1)(d-1)$. $k=2$: Kesten-McKay law
$d \to \infty, \frac{d}{k} \to \alpha > 0$ $d = o(n^{\epsilon})$ for any $\epsilon > 0$	$f(x) = \frac{\alpha}{1+\alpha+\sqrt{\alpha}x} \frac{1}{\pi}\sqrt{1-\frac{x^2}{4}}$ Marčenko-Pastur law
$\frac{d}{k} \to \infty, d = o(n^{\epsilon})$	$f(x) = \frac{1}{\pi}\sqrt{1-\frac{x^2}{4}}$ semicircle law

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- A cycle in a hypergraph is a cycle in the RBBG. A non-backtracking cycle in the hypergraph is a non-backtracking cycle in the RBBG.

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  - more refined, $(\lambda_2^2 - d_1) = O(\sqrt{d_1(d_2 - 1)})$ when $d_2 = O(1)$.

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- Enough to calculate fluctuations for A when d_1/d_2 bounded in both directions, but if $d_2/d_1 \to 0$, only good enough when d_2 constant. More work needed.

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