The Polynomially Self-Repelling Motion

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Recent Progress on Self-Interacting Processes
and Non-Reversible Monte Carlo

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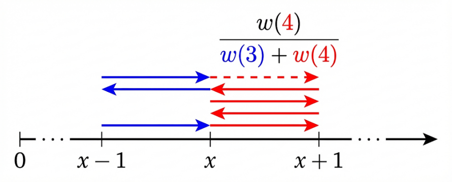
Self-interacting Random Walk on $\mathbb{Z}$



Let $w : \mathbb{N} \cup \{0\} \to (0, \infty)$ be a monotone weight function, $X_0 = 0$, and

$$P(X_{n+1} = X_n + 1 \mid X_0, \ldots, X_n) = 1 - P(X_{n+1} = X_n - 1 \mid X_0, \ldots, X_n)$$
$$= \frac{w\left(\# \text{ of crossings of } \{X_n, X_n + 1\}\right)}{\sum_{i \in \{-1,1\}} w\left(\# \text{ of crossings of } \{X_n, X_n + i\}\right)}$$

If $w \nearrow$ the process X is "self-attractive" (or "reinforced").
If $w \searrow$ the process X is "self–repelling".
Polynomially self-repelling random walks: $\frac{1}{w(n)} = n^{\alpha}\left(1 + \frac{2B}{n} + O\left(\frac{1}{n^2}\right)\right)$.

Our Results

Theorem (Yu, 26+)

A polynomially self–repelling walk satisfies a Joint Ray–Knight theorem with diffusive scaling (space and local time scaled by n) and limiting curves $\{\mathcal{S}_{r,\cdot}(a)\}_{a\geq 0, r\in\mathbb{R}}$ having joint distribution given by

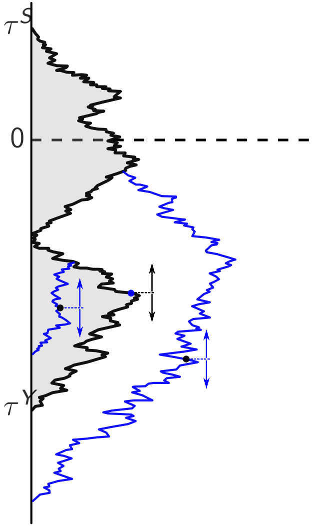
$$S_x := \mathcal{S}_{r,x}(a) = a + \int_r^x \frac{2}{\mathcal{S}_{r,u}(a)^\alpha} \int_{\mathbb{R}_+} s^\alpha \mathbf{1}_{\{s\leq \mathcal{S}_{r,u}(a)\}} \mathcal{W}(\mathrm{d}s, \mathrm{d}u) + \frac{x-r}{2\alpha+1}, \;\; x \geq r,$$

$$Y_{-x} := \mathcal{S}_{r,-x}(a) = a - \int_{-r}^x \frac{2}{\mathcal{S}_{r,-u}(a)^\alpha} \int_{\mathbb{R}_+} s^\alpha \mathbf{1}_{\{s\leq \mathcal{S}_{r,-u}(a)\}} \mathcal{W}^*(\mathrm{d}s, \mathrm{d}u) + \frac{y+r}{2\alpha+1}, \;\; x \geq -r,$$

with the corresponding absorption conditions. Here, $\mathcal{W}$ is a white noise on $\mathbb{H} := \mathbb{R}_+ \times \mathbb{R}$, and $\mathcal{W}^*$ is its image under $(a, r) \mapsto (a, -r)$

The polynomially self-repelling motion (PSRM) is constructed following the strategy used for the "true" self-repelling motion.

Properties of PSRM



Spatial Markov property

Fix $(a_0, r_0) \in \mathbb{H}$. Conditionally on $\mathcal{S}_{r_0,\cdot}(a_0)$, the families $\mathcal{S}_-$ and $\mathcal{S}_+$ are independent.

Markov Property

The process $(X_t, L_t(\cdot))_{t\geq 0}$ is Markovian.

Properties of PSRM

- **Symmetry.** $(X_t,\ t \geq 0)$ and $(-X_t,\ t \geq 0)$ are identical in law.
- **Continuity.** Almost surely, the sample paths $(X_t,\ t \geq 0)$ are continuous and $X_0 = 0$
- **Recurrence.** Almost surely, for any $x \in \mathbb{R}$, the set $\{t \geq 0 :\ X_t = x\}$ is unbounded.
- **Scaling.** For any $a > 0$, $(X_{at},\ t \geq 0)$ and $(a^{1/2}X_t,\ t \geq 0)$ are identical in law.
- **No "points of increase".** Almost surely, there do not exist $t, \varepsilon > 0$ such that $L(t, X_t) > 0$, $X_s \leq X_t$ for all $s \in (t - \varepsilon, t)$, and $X_u \geq X_t$ for all $u \in (t, t + \varepsilon)$.

Thank you