Benjamin Doyon (King's College London) benjamin.doyon@kcl.ac.uk

Where solitons are in a KdV soliton gas

The Korteweg-De Vries (KdV) equation is a paradigmatic model of integrable classical fields, with localised single-soliton profiles that make their positions easily identifiable. However, when many solitons are near to each other, soliton shapes are modified, and it is not manifest, from the KdV field, where they are. This is a key problem in the analysis of a soliton gas, where its main object, the spectral density of states, is a density of solitons per unit length: Where are the solitons in a dense, macroscopic soliton gas? Besides results in very special systems and recent results based on Lax eigenvectors in the Toda model in thermal states, this is an open problem for KdV and more generally. A sensible criterium comes from the operation of fluid-cell projection on which the spectral density of states is based: a good notion of solitons' positions should be such that keeping only those inside a mesoscopic region, the KdV field is unchanged in this region. In this talk I will discuss new mathematically rigorous results that do this. I will define solitons' positions, along with an explicit fluid-cell projection that does not introduce radiative corrections and that leaves the KdV field unchanged in the mesoscopic region on which solitons are projected. The solitons' positions satisfy an equation that accounts for the two-body scattering shift and encodes factorised scattering. The equation is related to the flea-gas or semi-classical Bethe equations recently introduced in the context of Generalised Hydrodynamics and representing quasi-particles' trajectories in many-body integrable systems. The results hold for a wide family of distributions of spectral and impact parameters, which do not need to be random, and the proof introduces new methods based on a novel tau function for the multi-soliton KdV field. A simple non-rigorous calculation from this reproduces the kinetic equation of the KdV soliton gas, first proposed by Gennady El in 2003 using Witham modulation theory from finite-gap solutions. Thus, this is the first derivation of the KdV kinetic equation from its multi-soliton solutions.