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Invariant Measures and Scaling Limits of Discrete Integrable Systems

As a mathematically rigorous approach to providing foundations for generalized hydrodynamics, discrete integrable systems—that is, integrable systems with discrete space–time variables—have several aspects that make them particularly amenable to exact analysis. In particular, a type of cellular automaton called the box-ball system (BBS) has been extensively studied in recent years. The BBS, which exhibits solitonic behavior, has been investigated for over 30 years from various perspectives, including tropical geometry, combinatorics, and representation theory. However, research from a probabilistic viewpoint began only about a decade ago. Recently, probabilistic approaches—such as applications of the Pitman transform, analysis of invariant measures, and investigations of scaling limits—have developed rapidly.

In particular, for the box-ball system, a generalized hydrodynamic limit describing the distribution of solitons has been established rigorously. More recently, it has been proved that, under a Bernoulli product measure, a tagged soliton converges to Brownian motion under an appropriate scaling limit. Furthermore, the relationship with the Pitman transform and the characterization of i.i.d. invariant measures have been shown to extend to other discrete integrable systems, such as the discrete KdV equation and the discrete Toda equation. These developments have revealed new connections between probability theory and classical integrable systems, demonstrating that the macroscopic behavior of integrable systems exhibits a type of universality distinct from that of chaotic systems.

In this talk, I will introduce these recent developments, focusing primarily on the box-ball system, and present rigorous results obtained over the past several years, starting from the basic concepts.