Exact Hydrodynamics of Integrable Systems: Mathematics and Physics

9–10 March 2026 Université Paris-Dauphine, PSL

Monday, 9 March

09:00–09:15	*Welcome and opening remarks*		
09:15–10:30	**Herbert Spohn** Université technique de Munich	*Generalized hydrodynamics of the Toda fluid*	[L]
10:30–11:00	*Break and discussion*		
11:00–12:15	**Alice Guionnet** ENS de Lyon	*Large deviations of the periodic Toda chain*	[L]
12:30–14:00	*Lunch*		
14:00–15:15	**Abhishek Dhar** Tata Institute Of Fundamental Research	*Quasiparticle dynamics in the hard rod gas and the Toda chain*	[L]
15:15–16:00	*Break and discussion*		
16:00–16:30	**Pablo Ferrari** Universidad de Buenos Aires	*Initial distributions for hard rod hydrodynamics*	[T]
16:30–17:00	**Jacopo De Nardis** Cergy Paris Université	*Anomalous and KPZ spin dynamics in integrable spin chains from a hard rods gas*	[T]
17:00–17:30	**Takato Yoshimura** King's College London	*Ballistic macroscopic fluctuation theory: formalism and applications*	[T]
17:30–20:00	*Discussion & Conference Cocktail*		

Tuesday, 10 March

09:00–10:15	**Makiko Sasada** Université de Tokyo	*Invariant Measures and Scaling Limits of Discrete Integrable Systems*	[L]
10:15–11:00	*Break and discussion*		
11:00–12:15	**Tamara Grava** University of Bristol	*Soliton gas in integrable PDEs*	[L]
12:30–14:00	*Lunch*		
14:00–15:15	**Benjamin Doyon** King's College London	*Where solitons are in a KdV soliton gas*	[L]
15:15–16:00	*Break and discussion*		
16:00–16:30	**Gennady El** Northumbria University	*Soliton gas at the interface of dispersive and generalised hydrodynamics*	[T]
16:30–17:00	**Pierre Suret** Université de Lille	*Soliton gas and generalized hydrodynamics in photonics*	[T]
17:00–17:30	**Katja Klobas** University of Birmingham	*Microscopic origin of the quantum Mpemba effect in integrable systems*	[T]
17:30–17:45	*Concluding remarks*		

[L]: Lecture **[T]**: Talk

Abstracts

Jacopo De Nardis (Cergy Paris Université) jacopo.de-nardis@cyu.fr

Anomalous and KPZ spin dynamics in integrable spin chains from a hard rods gas

We introduce a multi-species generalization of the hard-rod gas in which each species has a distinct effective length, and the repulsive scattering shift is set by the minimal size of the colliding rods. This model has similar quasiparticle and scattering data to the XXZ model. We show that fixing only the functional decay of the bare velocities with rod length is sufficient for the model to reproduce the XXZ spin-transport phase diagram: spin diffusion with anomalous fluctuations in the anisotropic regime and superdiffusion at the isotropic point. We show that the statistics of the charge transfer is entirely different from that of particle trajectories. For long rods, the latter is Gaussian in the diffusive regime and KPZ at the isotropic point. This provides the first direct signature of KPZ physics in the microscopic motion of integrable quasiparticles. On the other hand, the statistics of charge transfer is anomalous in the anisotropic regime, while instead it becomes Gaussian at late times at the isotropic point, resolving the paradox between the presence of KPZ fluctuations and Gaussian statistics. Our results establish classical hard-rod dynamics as a minimal, yet quantitatively faithful, framework for anomalous spin and charge transport in integrable systems, and offer new insight into the origin of KPZ fluctuations in isotropic integrable models.

Abhishek Dhar (Tata Institute Of Fundamental Research) abhishek.dhar@icts.res.in

Quasiparticle dynamics in the hard rod gas and the Toda chain

It is well known that the hydrodynamics of integrable models can be understood by viewing the system as a gas of quasiparticles. The lecture will discuss the dynamics of individual quasiparticles. In the first part, I will discuss the hard rod gas and present a simple microscopic derivation of quasiparticle statistics. Next, the Dean–Kawasaki formalism will be used to give a phenomenological derivation of fluctuating hydrodynamics of the homogeneous hard-rod gas. In the second part, I will discuss quasiparticle statistics of the Toda chain.

Benjamin Doyon (King's College London) benjamin.doyon@kcl.ac.uk

Where solitons are in a KdV soliton gas

The Korteweg–De Vries (KdV) equation is a paradigmatic model of integrable classical fields, with localised single-soliton profiles that make their positions easily identifiable. However, when many solitons are near to each other, soliton shapes are modified, and it is not manifest, from the KdV field, where they are. This is a key problem in the analysis of a soliton gas, where its main object, the spectral density of states, is a density of solitons per unit length: Where are the solitons in a dense, macroscopic soliton gas? Besides results in very special systems and recent results based on Lax eigenvectors in the Toda model in thermal states, this is an open problem for KdV and more generally. A sensible criterium comes from the operation of fluid-cell projection on which the spectral density of states is based: a good notion of solitons' positions should be such that keeping only those inside a mesoscopic region, the KdV field is unchanged in this region. In this talk I will discuss new mathematically rigorous results that do this. I will define solitons' positions, along with an explicit fluid-cell projection that does not introduce radiative corrections and that leaves the KdV field unchanged in the mesoscopic region on which solitons are projected. The solitons' positions satisfy an equation that accounts for the two-body scattering shift and encodes factorised scattering. The equation is related to the flea-gas or semi-classical Bethe equations recently introduced in the context of Generalised Hydrodynamics and representing quasi-particles' trajectories in many-body integrable systems. The results hold for a wide family of distributions of spectral and impact parameters, which do not need to be random, and the proof introduces new methods based on a novel tau function for the multi-soliton KdV field. A simple non-rigorous calculation from this reproduces the kinetic equation of the KdV soliton gas, first proposed by Gennady El in 2003 using Witham modulation theory from finite-gap solutions. Thus, this is the first derivation of the KdV kinetic equation from its multi-soliton solutions.

Gennady El (Northumbria University) gennady.el@northumbria.ac.uk

Soliton gas at the interface of dispersive and generalised hydrodynamics

It has recently been realised that the equations describing soliton gases in nonlinear dispersive wave systems are strikingly similar to those arising in the framework of generalised hydrodynamics (GHD) for integrable quantum

and classical many-body systems. These deep structural parallels have been recognised from both perspectives, leading to a number of important advances at the interface between the kinetic theory of soliton gases and GHD. In my talk, I will outline the key ideas underlying this connection, emphasising the common mathematical structures and physical principles that unify the two approaches.

Pablo Ferrari (Universidad de Buenos Aires) pferrari@dm.uba.ar

Initial distributions for hard rod hydrodynamics

In the recent literature for hard rod hydrodynamics we can find different approaches to introduce initial distributions matching a macroscopic profile. I will discuss their properties.

Tamara Grava (University of Bristol) tamara.grava@bristol.ac.uk

Soliton gas in integrable PDEs

The concept of a soliton gas was introduced by Zakharov in 1971 and further developed by El in 2003, modeling solitons as interacting particle-like structures. In recent years, rigorous analytical results have been established that provide confirmation of the qualitative theory. In this talk, I will describe some of these advances, including (1) a rigorous derivation of kinetic equations governing soliton gases in KdV-type systems without randomness, as well as (2) the analysis of random collections of solitons, in which both mean behavior and fluctuation results are established. This is joint work with several teams, including Manuela Girotti, Robert Jenkins, Ken McLaughlin, Guido Mazzuca, and Oleksandr Minakov.

Alice Guionnet (ENS de Lyon) alice.guionnet@ens-lyon.fr

Large deviations of the periodic Toda chain

I will discuss convergence and large deviations for the spectral measure of the Lax matrix associated to the periodic Toda chain of N particles, subject to a generalised Gibbs measure.

Katja Klobas (University of Birmingham) k.klobas@bham.ac.uk

Microscopic origin of the quantum Mpemba effect in integrable systems

The highly complicated nature of far from equilibrium systems can lead to a complete breakdown of the physical intuition developed in equilibrium. A famous example of this is the Mpemba effect, which states that nonequilibrium states may relax faster when they are further from equilibrium or, put another way, hot water can freeze faster than warm water. Despite possessing a storied history, the precise criteria and mechanisms underpinning this phenomenon are still not known. I will present a quantum version of the Mpemba effect that takes place in closed many-body systems with a U(1) conserved charge: in certain cases a more asymmetric initial configuration relaxes and restores the symmetry faster than a more symmetric one. I will outline the criteria for this to occur in arbitrary integrable quantum systems, relating it to the properties of the initial state.

Makiko Sasada (Université de Tokyo) sasada@ms.u-tokyo.ac.jp

Invariant Measures and Scaling Limits of Discrete Integrable Systems

As a mathematically rigorous approach to providing foundations for generalized hydrodynamics, discrete integrable systems—that is, integrable systems with discrete space–time variables—have several aspects that make them particularly amenable to exact analysis. In particular, a type of cellular automaton called the box-ball system (BBS) has been extensively studied in recent years. The BBS, which exhibits solitonic behavior, has been investigated for over 30 years from various perspectives, including tropical geometry, combinatorics, and representation theory. However, research from a probabilistic viewpoint began only about a decade ago. Recently, probabilistic approaches—such as applications of the Pitman transform, analysis of invariant measures, and investigations of scaling limits—have developed rapidly. In particular, for the box-ball system, a generalized hydrodynamic limit describing the distribution of solitons has been established rigorously. More recently, it has been proved that, under a Bernoulli product measure, a tagged soliton converges to Brownian motion under an appropriate scaling limit. Furthermore, the relationship with the Pitman transform and the characterization of i.i.d. invariant measures have been shown to extend to other discrete integrable systems, such as the discrete KdV equation and the discrete Toda equation. These developments have revealed new connections between probability theory and classical integrable systems, demonstrating that the macroscopic behavior of integrable systems exhibits a type of universality distinct from that of chaotic systems. In this talk, I will introduce these recent developments, focusing primarily on the box-ball system, and present rigorous results obtained over the past several years, starting from the basic concepts.

Herbert Spohn (Université technique de Munich) spohn@ma.tum.de

Generalized hydrodynamics of the Toda fluid

Toda invented his chain in 1967 as an integrable discrete nonlinear wave equation in one dimension. Since then many studies followed, both on the mathematical and theoretical physics side. In my presentation I will focus on aspects of the Toda chain related to GHD, in particularly covering generalized Gibbs ensembles, the respective random Lax matrix, its density of states, the dynamics of eigenvectors, and GHD either based on particles or solitons.

Pierre Suret (Université de Lille) pierre.suret@univ-lille.fr

Soliton gas and generalized hydrodynamics in photonics

I will review recent experimental and theoretical advances on soliton gases and generalized hydrodynamics (GHD) in optical fibers, highlighting quantitative agreement between GHD predictions and experiments for intensity statistics in the defocusing regime and unequal-time correlations in the focusing regime. The concept of a soliton gas was first introduced by V. Zakharov in 1971 as an infinite ensemble of weakly interacting solitons within the framework of the Korteweg–de Vries (KdV) equation. In this theoretical model of a rarefied soliton gas, solitons with randomly distributed amplitudes and phases interact minimally and remain mostly non-overlapping. More recently, this concept has been extended to dense soliton gases, where strong and continuous interactions occur between solitons. Soliton gases are inherently linked to integrable wave systems described by nonlinear partial differential equations, such as the KdV equation or the one-dimensional nonlinear Schrödinger (1DNLS) equation, which can be solved using the inverse scattering transform (IST). Profound connections between soliton gas theory and generalized hydrodynamics (GHD) have been recently established. In this talk, I will review the latest theoretical and experimental advancements in the study of soliton gases. I will first introduce key conceptual tools (IST, Generalized Gibbs Ensemble—GGE—. . .). I will then focus on two recent comparisons between optical fiber experiments and GHD theory.

Takato Yoshimura (King's College London) takato.yoshimura@physics.ox.ac.uk

Ballistic macroscopic fluctuation theory: formalism and applications

Recently, ballistic macroscopic fluctuation theory (BMFT) was invented as a generalization of macroscopic fluctuation theory (MFT) to study large fluctuations in systems that support ballistic transport, such as integrable systems. In this talk, I will overview the main idea of BMFT and show how it allows us to compute objects like the current cumulant generating function. I will then review an important application of BMFT, which is to describe the hydrodynamic origin of the so-called anomalous fluctuations.