Solving Heterogeneous Agent Models with the Master Equation

Adrien Bilal (Stanford University)

This paper proposes an analytic representation of perturbations in heterogeneous agent economies with aggregate shocks up to any order. Treating the underlying distribution as an explicit state variable, the value function defined on an infinite-dimensional state space summarizes the entire economy and satisfies the 'Master Equation' introduced in the mathematics mean field games literature. I show that analytic perturbations of the Master Equation deliver dramatic simplifications. The First-order Approximation to the Master Equation (FAME) reduces to a standard Bellman equation for the directional derivatives of the value function with respect to the distribution and aggregate shocks. The FAME has six main advantages: (i) finite dimension; (ii) closed-form mapping to steady-state objects; (iii) applicability when many distributional moments or prices enter individuals' decision such as dynamic trade, urban or job ladder settings; (iv) block-recursivity by-passing further fixed points; (v) characterization of stability and of the stochastic steady-state; (vi) fast implementation using standard numerical methods. I develop the Second-order Approximation to the Master Equation (SAME) and show that it shares these properties, making it suitable to handle nonlinearities, aggregate risk, and asset pricing. I illustrate the FAME and the SAME with two applications. First, I show that the cost of business cycles is several orders of magnitude larger in incomplete market models. Second, I show that dynamic spatial migration models display near linearity at conventional aggregate shocks.