

Strategic Complementarity = Persistence in Linear-Quadratic Mean Field Games

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We analyze linear-quadratic Mean Field Games (LQ MFGs), which are games involving a continuum of small agents, each solving a non-stationary, infinite-horizon, linearquadratic regulator problem. In a LQ MFG, each agent's payoff is influenced by her own state, her action, and the aggregate state, while being subject to both idiosyncratic and common shocks. We say that there are strategic complementarities, captured by a positive (definite) cross derivative of the period return function with respect to the own state and the average state, and strategic substitutability when it is negative (definite). A MFG equilibrium exists and is unique provided strategic complementarity is below a given threshold; above that threshold, no equilibrium exists. This threshold is increasing in the concavity of the agent's problem with respect to its own state and controls, and in the discount rate. For small discount factors, the steady state is unique and dynamically stable. A central finding is that the strength of strategic complementarities directly translates into the degree of persistence in the equilibrium dynamics of the average state. In particular, when agents' states exhibit strategic complementarity, the aggregate state converges more slowly to its steady state. Moreover, we show that a planner's solution is even more persistent under strategic complementarities than the decentralized equilibrium. We add aggregate shocks (i.e., common noise) to the system to study the identification, or lack thereof, of the degree of strategic complementarity for an econometrician with access to panel data generated by the model. Finally, we extend the main results to the case of coupling with higher moments.