Singular Mean-field Control via Singular Mean-field Games
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We study a mean-field control (MFC) problem with singular controls over a finite horizon, allowing for general dependence on the measure argument. To analyze the search for an optimal MFC strategy, we associate to it a mean-field game (MFG), which we refer to as a potential MFG of singular controls. We show that, under suitable convexity assumptions, any solution to this potential MFG yields a solution to the original MFC problem. Under additional conditions, we further demonstrate that the solution to the potential MFG can be used to approximate the central planner's optimum in the underlying N-agent system. Finally, we apply our results to a mean-field control version of the classical Monotone Follower problem of Karatzas and Shreve (SIAM J. Control Optim., 1984). The scalar mean-field interaction term is modulated by an interaction-strength parameter, leading to either strategic complementarity or strategic substitutability. The associated potential MFG with singular controls is solved by relying on the Bank–El Karoui representation theorem for the optimization step, and on two distinct fixed-point theorems to handle the two strategic regimes. This talk is based on a joint ongoing work with Andrea Amato and Federico Cannerozzi.