

# Tragedy of Open Ecosystems

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[Dynamic Games and Applications](#)

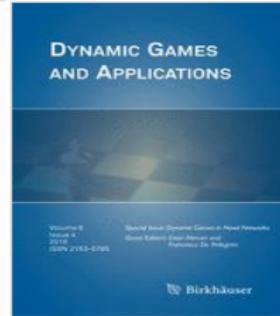
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## The Tragedy of Open Ecosystems

Authors

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# Motivations



- **Tragedy of the Commons** : Open-access detrimental
- **Game theory** : Levhari and Mirman (1980)
- But **single** stocks (Bailey et al, 2010)
- Exceptions : two-species : Fischer & Mirman (1992, 1996)

# Objectives

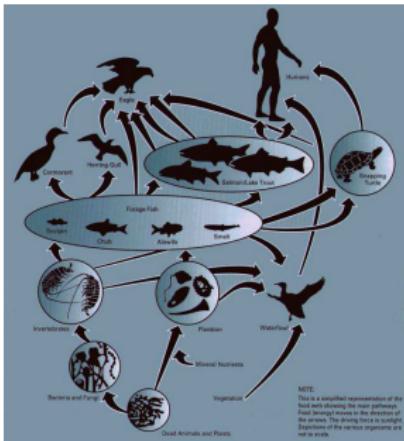


## Goals of the paper :

- Extension to larger ecosystem
- Gains in terms of biodiversity ?
- Promote models of intermediate complexity

# Ecological model

## Multi-species Gompertz dynamics in discrete time



$$x_i(t+1) = x_i(t) \exp \left( r_i + \sum_{j=1}^n s_{ij} \log (x_j(t)) \right)$$

with

- $x_i(t)$  stock of species  $i = 1, \dots, n$
- $r_i$  intrinsic growth rate
- $s_{ij}$  interspecies relationship
- $-1 \leq s_{ij} \leq 0$

- Harvest  $h_{ai}(t)$  of agents  $a = 1, \dots, A$  :

$$x_i(t+1) = \left( x_i(t) - \sum_a h_{ai}(t) \right) \exp \left\{ r_i + \sum_{j=1}^n s_{ij} \log \left( x_j(t) - \sum_a h_{aj}(t) \right) \right\}$$

- Identical users :  $a = 1, \dots, A$
- One-period utility function

$$U(h_{a1}, \dots, h_{an}) = \sum_{i=1}^n u_i \log (h_{ai}),$$

# Non-cooperative vs cooperative maximization

## Non-cooperative

For each agent  $a = 1, \dots, A$

$$\max_{h_a(\cdot)} \sum_{t=0}^{\infty} \rho^t U(h_{a1}(t), \dots, h_{an}(t)),$$

## Cooperative

$$\max_{h(\cdot)} \sum_{t=0}^{\infty} \rho^t \left( \sum_{\text{agents } a} U(h_{a1}(t), \dots, h_{an}(t)) \right)$$

with  $0 < \rho < 1$  discount factor

Method : Dynamic programming + Nash solution

$$V_a(x) = \max_{h_{a.}} \left\{ u' \log(h_{a.}) + \rho V_a(G(x - h_{a.} - h_{-a.})) \right\}.$$

## Proposition

*The noncooperative optimal catch for all species  $i$  is linear*

$$h_i^{nc} = f_i^{nc} x_i \quad \text{with} \quad f_i^{nc} = \frac{A u_i}{A u_i + \rho ((I + S)' w)_i}$$

with  $w = (I - \rho (I + S)')^{-1} u$

and assuming that  $w$  well-defined and  $(I + S)' w > 0$ .

Remark : non cooperative with  $A = 1$

## Proposition

*Same assumptions. The cooperative harvest for all species  $i$  is linear :*

$$h_i^c = f_i^c x_i \quad \text{with} \quad f_i^c = \frac{u_i}{u_i + \rho ((I + S)' w)_i}$$

## Proposition

*The non cooperative catch rate is larger than the cooperative mortality catch rate for all species  $i$  :*

$$\text{number of agents} \quad A > 1 \implies f_i^{nc} > f_i^c$$

## Sketch of the proof

- Control : harvest rate  $f_{ai}(t) = \frac{h_{ai}(t)}{x_i(t)}$
- A linear problem with  $y(t) = \log(x(t))$

$$y(t+1) = r + (I + S) \log(1 - f(t)) + (I + S) y(t)$$

- Form of the value function  $V_a(y) = v_a + w'y$
- Optimality conditions

$$\frac{u_i}{f_{ai}} = \frac{\rho ((I + S)' w)_{ai}}{1 - f_{ai} - f_{(-a)i}} \Leftrightarrow f_{ai}^{nc} = \frac{u_i}{Au_i + \rho ((I + S)' w)_{ai}}.$$

# How cooperation promotes biodiversity with many agents

- Many agents  $\approx A \rightarrow +\infty$
- Extinction in the non cooperative case :

$$\lim_{A \rightarrow +\infty} f_i^{nc}(t) = \lim_{A \rightarrow +\infty} \frac{Au_i}{Au_i + \rho ((I + S)' w)_i} = 1$$

- Biodiversity metrics : Species Richness  $SR(x) = \sum_i 1_{\mathbb{R}_+^*}(x_j)$

## Proposition

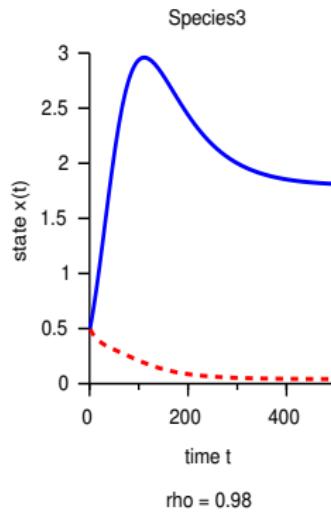
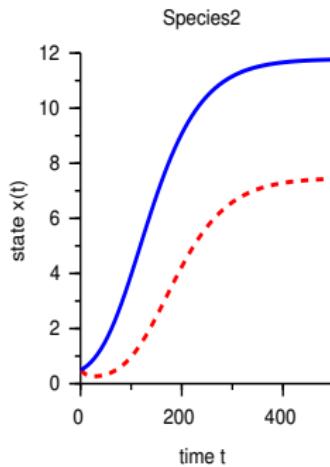
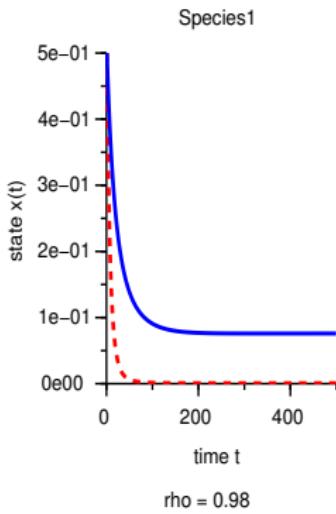
*When numerous agents cooperate, the ecosystem is more diverse*

$$SR \left( \lim_{A \rightarrow +\infty} x^c(t) \right) > SR \left( \lim_{A \rightarrow +\infty} x^{nc}(t) \right)$$

# Biodiversity gains of cooperation with few agents

$$r = \begin{pmatrix} -0.0026 \\ 0.0392 \\ 0.0644 \end{pmatrix} \quad S = \begin{pmatrix} -0.0218 & 0.0005 & 0.0001 \\ -0.0143 & -0.0153 & 0.0003 \\ -0.0003 & -0.0085 & -0.0161 \end{pmatrix} \quad u = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$
$$\rho = 0.98,$$

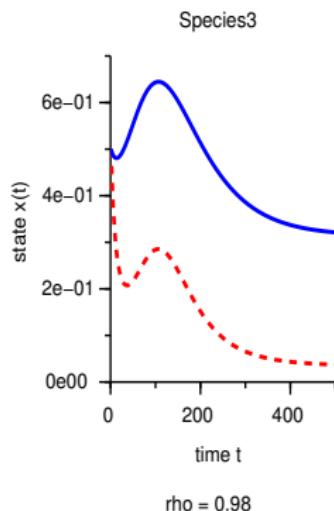
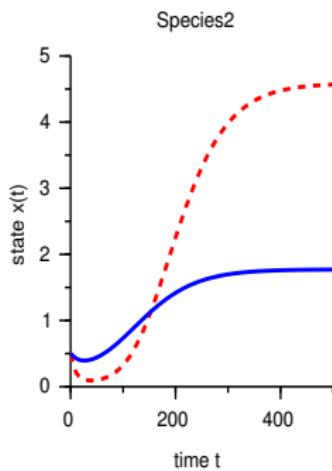
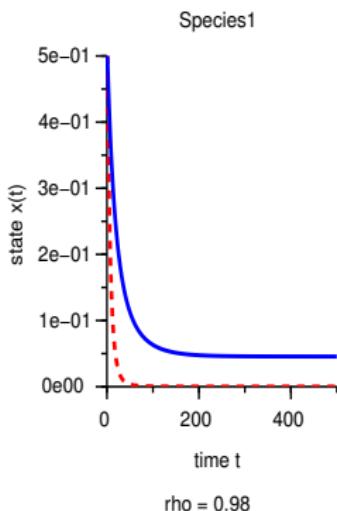
harvest fractions (%)	Species 1	Species 2	Species 3
cooperative $f^c$	5.45	3.91	3.52
non cooperative $f^{nc}$ $m = 3$	14.79	10.88	9.87



# But Gains of NON cooperation with few agents

$$r = \begin{pmatrix} -0.00002 \\ 0.00018 \\ 0.00027 \end{pmatrix} \quad S = \begin{pmatrix} -0.01902 & 0.00072 & 0.00030 \\ -0.01819 & -0.01766 & 0.00054 \\ -0.00757 & -0.01364 & -0.01254 \end{pmatrix} \quad u = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

"mesopredator release" hypothesis CrooksSoule:1999



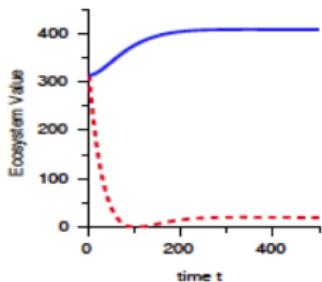
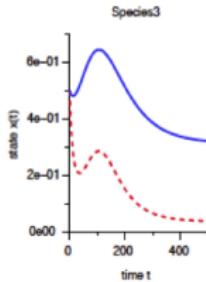
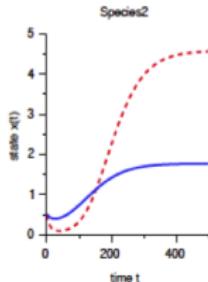
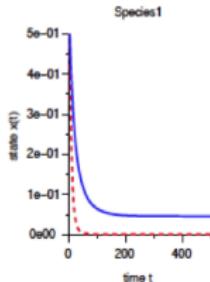
# Gains for the ecosystem in every case

$$\text{Ecos}(x) = w' \log(x) = \sum_j w_j \log(x_j)$$

## Proposition

Assume that  $(I + S')' w > 0$ . The cooperative ecosystem is larger than the noncooperative ecosystem at every time  $t$

$$\text{Ecos}(x^c(t)) \geq \text{Ecos}(x^{nc}(t))$$



# Contribution value of species

- A marginal value :  $w = \frac{\partial V}{\partial y}(y)$
- $w \geq u \geq 0$
- Example :  $w \approx (54 \quad 51 \quad 28)'$
- 2 species : The marginal prices

$$w_1 = \frac{u_1 (1 - \rho (1 + s_{22})) + \rho s_{21} u_2}{\Delta}, \quad w_2 = \frac{u_2 (1 - \rho (1 + s_{11})) + \rho s_{12} u_1}{\Delta}$$

where  $\Delta = \det(I - \rho(I + S'))$

Account for interactions and species without direct utility !!

# Gains at equilibrium in every case

At equilibrium

$$x_* = \exp(-S^{-1}(r + (I + S) \log(1 - F)))$$

Idea :

$$\log(x_*^c) - \log(x_*^{nc}) = -(I + S^{-1}) \log\left(\frac{1 - F^c}{1 - F^{nc}}\right)$$

Define the ecosystem value

$$E(x) = \exp(-(1 \dots 1)(I + S^{-1})^{-1} \log(x))$$

Proposition

*At equilibrium the cooperative ecosystem performs better*

$$E(x_*^c) \geq E(x_*^{nc})$$



## Gains at equilibrium : example 2D

Looks like a trophic index (Pauly)

Example 2D predator-prey :  $S = \begin{pmatrix} 0 & s_1 \\ s_2 (< 0) & 0 \end{pmatrix}$

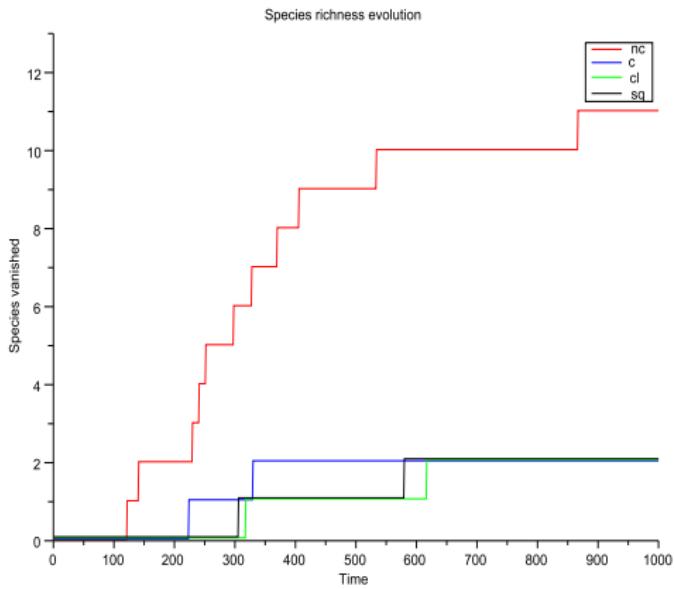
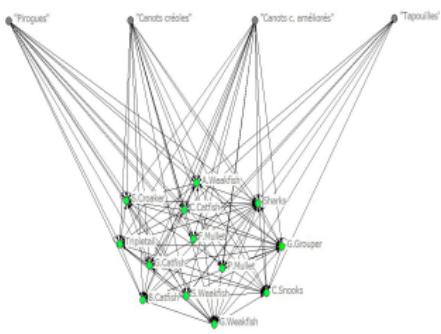
$$-(1 \ 1)(I + S^{-1})^{-1} \approx \frac{1}{\Delta}(-s_2 - 1 \ -s_1 - 1)$$

Put more weight on the predators !!!

## French Guiana

Cissé et al., *Environmental Development Economics*, 2013

Cissé et al., Ecological Economics, 2015



# Conclusion and perspectives

- Tragedy of the 'commons' revisited for many species
- Clear gains of cooperation in terms of pressure
- Clear gains of cooperation in terms of biodiversity when high number of players
- When few players, more ambiguous
- Contribution value of every species to ecosystem services
- Interest of models of intermediate complexity models MICE