Intergenerational justice and time-inconsistency

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The Gordon-Schaefer model

An optimisation problem

$$\max \int_0^\infty e^{-\delta t} \left(p - c\left(x\left(t\right)\right)\right) h\left(t\right) dt \tag{1}$$

$$\frac{dx}{dt} = f\left(x\right) - h\left(t\right), \quad x\left(0\right) = x_0 \tag{2}$$

$$0 \leq h\left(t\right) \leq h_{\max} \tag{3}$$

where t is time, $e^{\delta f}$ is the (psichiological) discount rate, $h\left(t\right)$ is the catch, $x\left(t\right)$ is the population, $c\left(x\right)$ is the unit cost of catching, p is price, and $f\left(x\right)$ is the demographics.

The solution

Consider the equation:

$$f'(x) - \frac{c(x)}{p - c(x)} f(x) = \delta \tag{4}$$

- if it has no positive solution, the optimal solution consists of bringing the population $x(t)$ to zero as quickly as possible: $h(t) = h_{\text{max}}$
- if it has a positive solution x_{opt} the optimal solution consists of bringing $x(t)$ to x_{opt} as quickly as possible and keeping the population at that level

$$h(t) = h_{\text{max}} \text{ until } x(t) = x_{\text{opt}} \tag{5}$$

$$h(t) = f(x_{\text{opt}}) \text{ afterwards} \tag{6}$$

Who is the optimizer ?

$$\max \int_0^\infty e^{-\delta t} \left(p - c\left(x\left(t\right)\right)\right) h\left(t\right) dt$$

- An infinite-lived monopolist who discounts future profits at the rate $e^{-\delta t}$,
- Usually understood as a proxy: society, like individuals, discount future gains at some exponential rate
- But there should be two rates:
  - δ for gains accruing to oneself (the present generation)
  - $\sigma > \delta$ for gains accruing to other (future generations)

Intergenerational equity

The present generation is concerned with its own utility and also with the utility of future generations.
Its own lifetime utility is

$$\int_0^{\infty} e^{-\delta t} \left(p - c\left(x\left(t\right)\right)\right) h\left(t\right) dt \tag{7}$$

All individuals born at time t have lifetime utility at birth of

$$\int_t^{\infty} e^{(s-t)} \left(p - c\left(x\left(s\right)\right)\right) h\left(s\right) ds \tag{8}$$

The present generation will discount it the rate σ and sum it over all generations. It is assumed that the population is constant with a renewal rate of n

Non-exponential discount rates

The final criterion is

$$\int_0^\infty e^{-\delta t} \left(p - c\left(x\left(t\right)\right)\right) h\left(t\right) dt + n \int_0^\infty e^{-\sigma t} \left(\int_t^\infty e^{(s-t)} \left(p - c\left(x\left(s\right)\right)\right) h\left(s \right. \right. \tag{9}$$

Integrating by parts, we find the criterion

$$I\left(h\right) = \int_0^\infty R\left(t\right) \left(p - c\left(x\left(t\right)\right)\right) h\left(t\right) dt \tag{10}$$

$$R\left(t\right) = \left(1 + \frac{n}{\sigma - \delta}\right) e^{-\delta t} - \frac{n}{\sigma - \delta} e^{\sigma t} \tag{11}$$

Correspond to a non-constant discount rate $r\left(t\right) = -R'\left(t\right) / R\left(t\right)$:

$$r\left(t\right) \to \delta - n \quad \text{when} \quad t \to 0 \tag{12}$$

$$r\left(t\right) \to \delta \quad \text{when } t \to \infty \tag{13}$$

Time inconsistency

At time 0 I am asked to commit between two streams of benefits $u_1(t)$ and $u_2(t)$, which will start at time T. I find that

$$\int_0^\infty R(t)\, u_1(t)\, dt > \int_0^\infty R(t)\, u_2(t)\, dt \tag{14}$$

So naturally I commit to u_1. When time T comes, and it is time to act, I find

$$\int_T^\infty R(t-T)\, u_1(t)\, dt < \int_T^\infty R(t-T)\, u_2(t)\, dt \tag{15}$$

What do I do ? Note that this cannot occur with exponential utilities, because $R(t-T) = R(t)\, R(-T)$. But in other cases, it does happen, and optimization then, though mathematically possible and correct, becomes meaningless. Something else is needed.

Intergenerational equilibrium

Suppose the equation

$$f'(x) - \frac{c(x)}{p - c(x)} f(x) = \delta - n \tag{16}$$

has a positive solution x_{eq}. Then the strategy consisting of bringing $x(t)$ to x_{eq} as quickly as possible penalizes all unilateral deviations. The generation born at time t, and holding power between t and $t + dt$, given that all previous generations have applied that strategy, and assuming that all future generations will apply it as well, will find that it has no incentive to apply a different one.

Conclusion

- Non-exponential discounting is not a particular quirk of fisheries management. It is a standard fact of human psychology
- Nor is intergenerational equity particular to fisheries management: bringing this concern to growth models à la Ramsey-Solow gives rise to a multiplicity of equilibria
- The particular feature of fisheries management is its robustness: the degree of concern for future generations does not affect the equilibrium, it is sufficient that it exists, however small

Bibliography

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