Mathematical modeling of natural resource management

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Modeling of fisheries systems

$$\begin{cases} \dfrac{dn}{dt} = f(n) - h(n,E), \\[2ex] \dfrac{dE}{dt} = \phi\Big(ph(n,E) - cE\Big) \end{cases}$$

- Smith V.L. (1968). Economics of Production from Natural Resources. American Economic Review, 409-431
- Smith V.L. (1969). On models of commercial fishing. Journal of Political Economy, 77, 181-198

Fishery modelling

$$\begin{cases} \dfrac{dn}{dt} = f(n) - h(n, E), \\[2ex] \dfrac{dE}{dt} = \phi\Big(ph(n, E) - cE\Big) \end{cases}$$

Fishery modelling

$$\begin{cases} \dfrac{dn}{dt} = \boxed{f(n)} - h(n, E) \\[2ex] \dfrac{dE}{dt} = \phi\Big(ph(n, E) - cE\Big) \end{cases}$$

Fishery modelling

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Fishery modelling

$$\begin{cases} \dfrac{dn}{dt} = f(n) - h(n,E) \\[2ex] \boxed{\dfrac{dE}{dt} = \phi\Big(ph(n,E) - cE\Big)} \end{cases}$$

Fishery modelling

$$
\begin{cases}
\dfrac{dn}{dt} = f(n) - h(n, E), \\[2ex]
\dfrac{dE}{dt} = \phi\Big(ph(n, E) - cE\Big)
\end{cases}
$$

Fishery modelling

- $f(n) = rn(1 - \frac{n}{K})$

$$\begin{cases} \dfrac{dn}{dt} = \boxed{f(n)} - \boxed{h(n,E)} \\ \\ \dfrac{dE}{dt} = \phi\Big(ph(n,E) - cE\Big) \end{cases}$$

- $h(n, E) = qnE$

Fishery modelling

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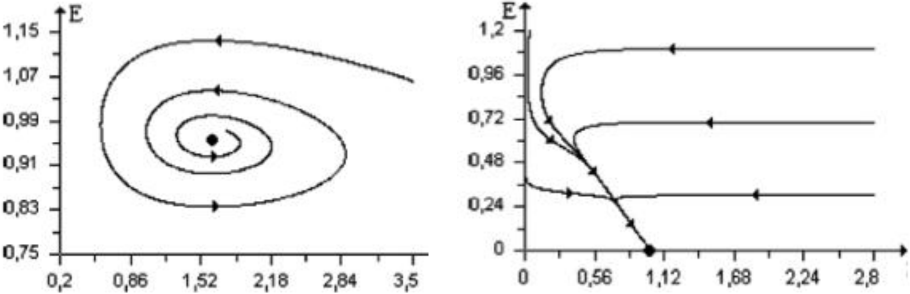
Fishery modelling

- $f(n) = rn(1 - \frac{n}{K})$
- $h(n, E) = qnE$

$$\begin{cases} \dfrac{dn}{dt} = rn(1 - \dfrac{n}{K}) - qnE \\[2ex] \dfrac{dE}{dt} = \phi\Big(pqnE - cE\Big) \end{cases}$$

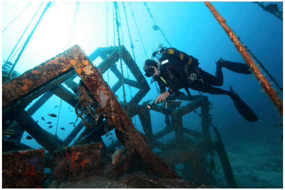
p= constant, q= constant, $\phi = 1$

Condition for fishery persistence



$pqK > c$ $\qquad$ $pqK < c$

Artificial reefs



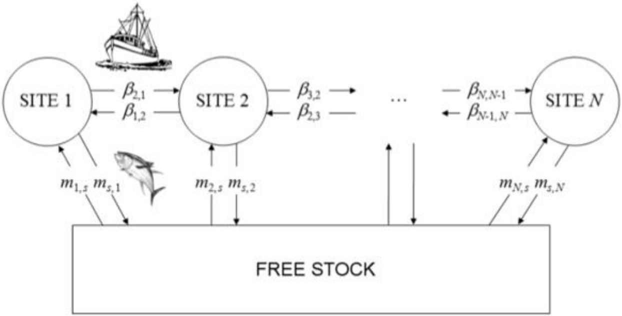
Artificial Fish habitats

Fish Aggregating Devices, FAD

Fish aggregating devices (FADs)

- Fish aggregating devices (FADs) are floating objects that allow to attract fishes in the open sea.
- Fishermen regularly visit FADs to catch fishes.
- FADs are often moored along lines that are parallel to coasts or reefs or around islands (Dagorn et al. 2007).
- It is clear that fishermen move frequently between FADs.
- There is evidence that tuna stay around a specific FAD for a relatively short time (a few days).
- It is reasonable to assume that fish movements occur at a fast time scale compared to the scale at which the fish population grows.

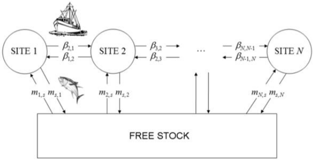
Effect of the number of FADs on the capture



$$\beta(n_i) = \frac{1}{\beta n_i + \beta_0}$$

We assume that the movement rates for the fishing vessels, $\beta(n_i)$ depend on the fish stock in the particular patch : When n_i increases, then $\beta(n_i)$ decreases. We can explain these rates of migration by the fact that the aim of the fleets owners is to increase their revenues. So, the fishing vessels try to operate in the most abundant patch. Consequently, the tendency of each fleet to leave a patch must increase when the stock is locally small.

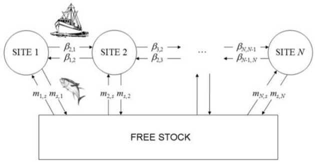
Effect of stock-dependent boat dispersal



$$
\begin{cases}
\dfrac{dn_s}{d\tau} = \displaystyle\sum_{i=1}^{N} m_{si} n_i - \sum_{i=1}^{N} m_{is} n_s + \varepsilon r_s n_s \left(1 - \frac{n_s}{k_s}\right) \\[2ex]
\dfrac{dn_i}{d\tau} = m_{is} n_s - m_{si} n_i + \varepsilon \left(r n_i \left(1 - \dfrac{n_i}{k_i}\right) - q n_i E_i \right) \\[2ex]
\dfrac{dE_i}{d\tau} = \beta_{i,i-1}(n_{i-1}) E_{i-1} + \beta_{i,i+1}(n_{i+1}) E_{i+1} - \left(\beta_{i-1,i}(n_i) + \beta_{i+1,i}(n_i)\right) E_i \\
\qquad + \varepsilon (pqn_i - c) E_i
\end{cases}
$$

$$
k_s = \alpha K, \quad \sum_{i=1}^{N} k_i = (1-\alpha) K, \qquad m_s = \frac{\delta}{k_s}, \quad m_i = \frac{\delta}{k_i}, \quad \beta(n_i) = \frac{1}{\beta n_i + \beta_0}
$$

Effect of stock-dependent boat dispersal



$$
\left\{
\begin{aligned}
\frac{dn_s}{d\tau} &= \sum_{i=1}^{N} m_{si} n_i - \sum_{i=1}^{N} m_{is} n_s + \varepsilon r_s n_s \left(1 - \frac{n_s}{k_s}\right) \\
\frac{dn_i}{d\tau} &= m_{is} n_s - m_{si} n_i + \varepsilon \left(r_1 n_i \left(1 - \frac{n_i}{k_i}\right) - q n_i E_i \right) \\
\frac{dE_i}{d\tau} &= \beta_{i,i-1}(n_{i-1}) E_{i-1} + \beta_{i,i+1}(n_{i+1}) E_{i+1} - \left(\beta_{i-1,i}(n_i) + \beta_{i+1,i}(n_i)\right) E_i \\
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\right.
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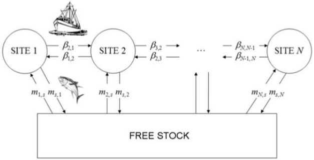
Effect of stock-dependent boat dispersal



$$
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\begin{aligned}
\frac{dn_s}{d\tau} &= \sum_{i=1}^{N} m_{si} n_i - \sum_{i=1}^{N} m_{is} n_s + \varepsilon\, r_s n_s \left(1 - \frac{n_s}{k_s}\right) \\
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Effect of stock-dependent boat dispersal

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\frac{dn_s}{d\tau} &= \sum_{i=1}^{N} m_{si} n_i - \sum_{i=1}^{N} m_{is} n_s + \varepsilon r_s n_s (1 - \frac{n_s}{k_s}) \\
\frac{dn_i}{d\tau} &= m_{is} n_s - m_{si} n_i + \varepsilon \left(r_1 n_i (1 - \frac{n_i}{k_i}) - q n_i E_i \right) \\
\frac{dE_i}{d\tau} &= \beta_{i,i-1}(n_{i-1}) E_{i-1} + \beta_{i,i+1}(n_{i+1}) E_{i+1} - (\beta_{i-1,i}(n_i) + \beta_{i+1,i}(n_i)) E_i \\
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Effect of stock-dependent boat dispersal

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\frac{dn_s}{d\tau} &= \sum_{i=1}^{N} m_{si} n_i - \sum_{i=1}^{N} m_{is} n_s \;+ \varepsilon\, r_s n_s (1 - \frac{n_s}{k_s}) \\
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\end{aligned}
\right.
$$

Fast dynamic

$$\begin{cases} \dfrac{dn_s}{d\tau} = \displaystyle\sum_{i=1}^{N} m_{si} n_i - \sum_{i=1}^{N} m_{is} n_s \\[2ex] \dfrac{dn_i}{d\tau} = m_{is} n_s - m_{si} n_i \\[2ex] \dfrac{dE_i}{d\tau} = \beta_{i,i-1}(n_{i-1}) E_{i-1} + \beta_{i,i+1}(n_{i+1}) E_{i+1} - \Big(\beta_{i-1,i}(n_i) + \beta_{i+1,i}(n_i) \Big) E_i \end{cases}$$

Fast equilibrium

$$\begin{cases} m_{si} n_i = m_{is} n_s \\ \\ \beta_{i,i-1}(n_{i-1}) E_{i-1} + \beta_{i,i+1}(n_{i+1}) E_{i+1} = \Big(\beta_{i-1,i}(n_i) + \beta_{i+1,i}(n_i) \Big) E_i \end{cases}$$

$$n = n_s + \sum_{i=1}^{N} n_i$$

$$E = \sum_{i=1}^{N} E_i$$

Aggregated model

$$\begin{cases} n_s^* = v_s^* n \\ n_i^* = v_i^* n \\ E_i^* = \mu_i^*(n) E \end{cases}$$

$$\begin{cases} v_s^* = \dfrac{k_s}{K} \\ v_i^* = \dfrac{k_i}{K} \\ \mu_i^*(n) = \dfrac{\beta v_i^* n + \beta_0}{\beta(1-\alpha) v_i^* n + N\beta_0} \end{cases}$$

$$\begin{cases} \dfrac{dn}{dt} = rn\left(1 - \dfrac{n}{K}\right) - Q(n)nE \\ \dfrac{dE}{dt} = (pQ(n)n - c)E \end{cases}$$

$$\begin{cases} r = \alpha r_s + (1-\alpha) r_1 \\ Q(n) = q \displaystyle\sum_{i=1}^{N} v_i^* \mu_i^*(n) \\ \qquad = q \dfrac{\tau_1 n + (1-\alpha)\beta_0}{\beta(1-\alpha)n + N\beta_0} \\ \text{where } \tau_1 = \beta \displaystyle\sum_{i=1}^{N} {v_i^*}^2 \end{cases}$$

Local and global models are different

$$
\begin{cases}
\dfrac{dn_s}{d\tau} = \displaystyle\sum_{i=1}^{N} m_{si} n_i - \sum_{i=1}^{N} m_{is} n_s + \varepsilon r_s n_s (1 - \frac{n_s}{k_s}) \\[2ex]
\dfrac{dn_i}{d\tau} = m_{is} n_s - m_{si} n_i + \varepsilon \left(r n_i (1 - \dfrac{n_i}{k_i}) - q n_i E_i \right) \\[2ex]
\dfrac{dE_i}{d\tau} = \beta_{i,i-1}(n_{i-1}) E_{i-1} + \beta_{i,i+1}(n_{i+1}) E_{i+1} - (\beta_{i-1,i} n_i + \beta_{i+1,i})(n_i) E_i \\[1ex]
\qquad\qquad + \varepsilon (p q n_i - c) E_i
\end{cases}
\tag{1}
$$

$$
\begin{cases}
\dfrac{dn}{dt} = rn \left(1 - \dfrac{n}{K} \right) - Q(n) n E \\[2ex]
\dfrac{dE}{dt} = (p Q(n) n - c) E
\end{cases}
\tag{2}
$$

Emergence

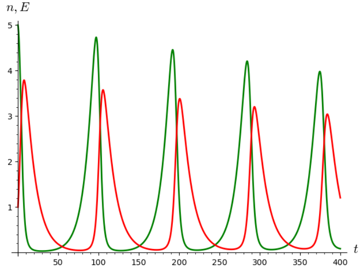
- If $L\tau_1 > \beta(1-\alpha)^2$ and $n^* > K$ then (n^*, E^*) does not belongs to the positive quadrant and $(K, 0)$ is a stable node.

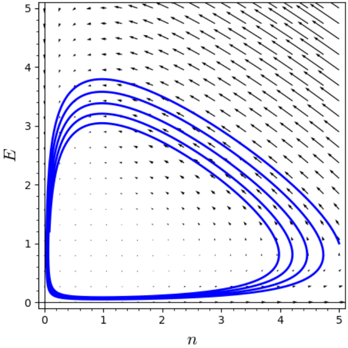
- If $L\tau_1 > \beta(1-\alpha)^2$and $n^* < K$ then (n^*, E^*) belong to the positive quadrant and is globally asymptotically stable while $(K, 0)$ is a saddle.

- If $L\tau_1 < \beta(1-\alpha)^2$ and $n^* > K$ then (n^*, E^*) does not belongs to the positive and $(K, 0)$ is a stable node.

- If $L\tau_1 < \beta(1-\alpha)^2$ and $\overline{n} < n^* < K$ then (n^*, E^*) belong to the positive quadrai globally asymptotically stable while $(K, 0)$ is a saddle.

- If $L\tau_1 < \beta(1-\alpha)^2$ and $n^* < \overline{n} < K$ then (n^*, E^*) belong to the positive quadrai unstable. $(K, 0)$ is a stable node, In this case, there exists a limit cycle. see Fig. 7.





Optimal number of sites

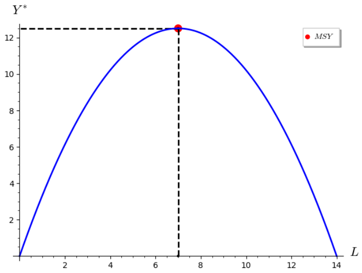
Total catch at equilibrium

$$n^* = \frac{cL}{pq(1-\alpha)}$$

$$Y^* = Q(n^*)n^*E^* = rn^*(1 - n^*/K) = F(L)$$

Identically sites :

$$k_i = \frac{(1-\alpha)K}{L} \quad \text{we obtain} \quad L_{opt} = \frac{pq(1-\alpha)K}{2c}$$



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Increase of maximum sustainable yield for fishery in two patches with fast migration

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[b] Faculty of Science, Vrije Universiteit, Amsterdam, The Netherlands
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A two-patch population model with logistic growth and constant fast migrations

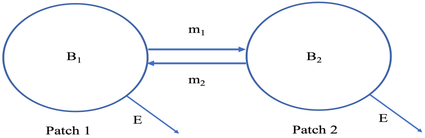


Figure – System of two connected patches with fast migration and harvesting

We still consider a system of two patches connected by fast migrations where fish sub-populations grow logistically. We still assume that the exchanges between the sites are fast in comparison to local growth and fishing. The only change we consider is to assume that the fish population is harvested in both sites. The complete model reads as follows :

$$\frac{dB_1}{d\tau} = m_2 B_2 - m_1 B_1 + \epsilon\Big(r_1 B_1(1 - \frac{B_1}{K_1}) - qEB_1\Big) \tag{3}$$

$$\frac{dB_2}{d\tau} = m_1 B_1 - m_2 B_2 + \epsilon\Big(r_2 B_2(1 - \frac{B_2}{K_2}) - qEB_2\Big) \tag{4}$$

A two-patch population model with logistic growth and constant fast migrations

The complete model can be reduced. As a first step, we consider the fast system which is obtained by setting $\epsilon = 0$ in the complete system. In our case, the fast equilibrium can be easily calculated and is given by :

$$B_1^* = uB = \frac{m_2}{m_1 + m_2} B \tag{5}$$

$$B_2^* = (1-u)B = \frac{m_1}{m_1 + m_2} B \tag{6}$$

Where $u = \dfrac{m_2}{m_1 + m_2}$ represents the constant proportion of fish Biomass in patch 1 and $(1-u)$ in patch 2 at the fast equilibrium. We obtain an aggregated model which reads as follows :

$$\frac{dB}{dt} = rB(1 - \frac{B}{K}) - EB \tag{7}$$

where

$$r = r_1 u + (1-u) r_2$$

$$K = \frac{K_1 K_2 (r_1 u + r_2 (1-u))}{(K_2 r_1 u^2 + K_1 r_2 (1-u)^2)}$$

A two-patch population model with logistic growth and constant fast migrations

$$B^* = K > K_1 + K_2$$

$$\frac{rK}{4} \leq \frac{r_1 K_1}{4} + \frac{r_2 K_2}{4}$$

$$r = r_1 u + (1-u) r_2$$

$$K = \frac{K_1 K_2 (r_1 u + r_2 (1-u))}{(K_2 r_1 u^2 + K_1 r_2 (1-u)^2)}$$

The 2-patch Holling type II predator-prey model with fast migrations

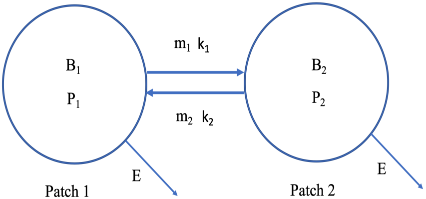
We consider a system of two fishing areas connected by migrations. We consider the prey-predator model with a Holling type II functional response at each patch. The prey and its predator can move from one fishing area to another. The complete model reads as follows :

$$\frac{dB_1}{d\tau} = m_2 B_2 - m_1 B_1 + \epsilon\Big(r_1 B_1 (1 - \frac{B_1}{K_1})\Big) - \epsilon\Big(\frac{aB_1 P_1}{B_1 + D}\Big) \tag{8}$$

$$\frac{dB_2}{d\tau} = m_1 B_1 - m_2 B_2 + \epsilon\Big(r_2 B_2 (1 - \frac{B_2}{K_2})\Big) - \epsilon\Big(\frac{aB_2 P_2}{B_2 + D}\Big) \tag{9}$$

$$\frac{dP_1}{d\tau} = k_2 P_2 - k_1 P_1 + \epsilon\Big(\frac{eaB_1 P_1}{B_1 + D} - dP_1 - EP_1)\Big) \tag{10}$$

$$\frac{dP_2}{d\tau} = k_1 P_1 - k_2 P_2 + \epsilon\Big(\frac{eaB_2 P_2}{B_2 + D} - dP_2 - EP_2)\Big) \tag{11}$$

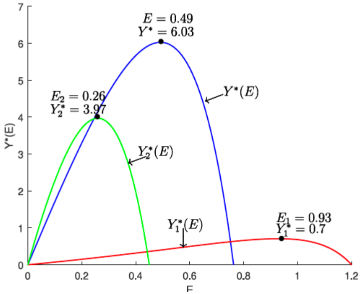


After reduction, the aggregated model reads as follows :

$$\frac{dB}{dt} = rB(1 - \frac{B}{K}) - \frac{auvBP}{uB + D} - \frac{a(1-u)(1-v)BP}{(1-u)B + D} \tag{12}$$

$$\frac{dP}{dt} = \frac{eauvBP}{uB + D} + \frac{ea(1-u)(1-v)BP}{(1-u)B + D} - dP - EP \tag{13}$$

$$u = \frac{m_2}{m_1 + m_2} \qquad v = \frac{k_2}{k_1 + k_2} \tag{14}$$



$$B^* = \frac{-eaD(uv + (1-u)(1-v)) + D(d+E) + \sqrt{\Delta}}{2(ea - (d+E))u(1-u)} \tag{15}$$

where the discriminant Δ is given by

$$\Delta = [eaD(uv + (1-u)(1-v)) - D(d+E)]^2 + 4(d+E)D^2(ea - (d+E))u(1-u)$$

The yield $Y^*(E) = EP^*$ reads as follows :

$$Y^*(E) = EP^* = re\frac{E}{d+E}B^*\left(1 - \frac{B^*}{K}\right) \tag{16}$$

$$Y^*_{MSY} > Y^*_{MSY1} + Y^*_{MSY2} \tag{17}$$

This result is obtained in the case of a Lotka–Volterra (type I) or Holling (type II) functional response for the prey–predator model.

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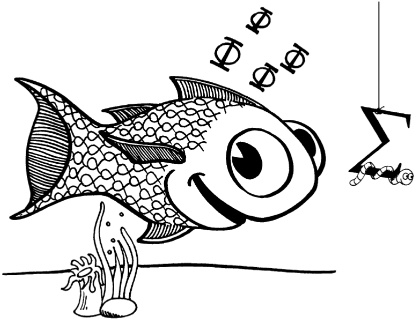
A. Moussaoui, Modélisation des systèmes de pêcheries. Gazette de la Société Mathématique de France. 172, 1-10, 2022. 53, 359–370.

Thank you for your attention



Un poisson mathématique – par Theo Engell-Nielsen.