Governance, strategic interactions and game theory

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Agenda

- Common Pool Resources: Static Framework
  - Tragedy of the commons
- Common Pool Resources: Dynamic Framework
  - Brief introduction to dynamic games
  - Sustainability of cooperation
- Great fish war with moratorium (Dahmouni, Parilina, Zaccour (2023))
- A Fair and Time-Consistent Sharing of the Joint Exploitation Payoff of a Fishery (Dahmouni, Vardar, Zaccour (2019))

Common Pool Resources (CPRs)

- Types of goods

	Excludable	Non-excludable
Rival	Private goods Admission to Dauphine U.	**CPRs** Ocean Fisheries
Non-rival	Club goods Wilderness Area	Public goods Air, national defense

- A good or service is a CPR if is
  - rival (consumption by one individual reduces the amount available to others)
  - non-excludable (costly or impossible to deny an individual enjoying the good)
- Fisheries, hunting grounds, forests, irrigation system, computer facility, etc.

Common Pool Resources (CPRs)

- Rivalry $\Leftrightarrow$ negative externality
- Rivalry: Fisher i harvests one more ton, this ton is not available anymore to others
- Commons are more scarce
- Fisher $j \neq i$ must work harder to maintain her catch level
  - Higher cost for j, which is not taken into account by i
- Optimizing joint payoff of all fishers allows to internalize the externality
- Socially optimal harvest

Game in strategic form

1. $M = \{1, \dots, m\}$ = set of players
2. s_i strategy of player i
   S_i set of strategies of player $i \in M$
   $s = (s_1, \dots, s_i, \dots, s_m) \in S_1 \times S_2 \dots \times S_m \triangleq S$ a strategy profile
   $s_{-i} \in S_{-i}$ strategies by all players but i
3. $\pi_i(s) \in \mathbb{R}$ payoff function of player i

- Information
  - Complete / incomplete
  - Perfect / imperfect
- Remark: π_i depends on all players' strategies. Player i strictly prefers s to s' if $\pi_i(s) > \pi_i(s')$. If $\pi_i(s) = \pi_i(s')$, then she is indifferent between s and s'

Nash equilibrium

Definition

A strategy profile $s^*$is a Nash equilibrium if

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*), \quad \forall s_i \in S_i, \quad \forall i \in M$$

- Unilateral deviation
- Best reply
- Existence and uniqueness

CPR: Simplest possible example

- Static (symmetric) game

  n fishers (players)
  u_i : harvest by fisher i
  $U = \sum u_i; \quad U_{-i} = U - u_i$
  p : price
  x : stock of the resource

$$
\begin{aligned}
\text{Harvesting cost} \quad &: \quad c_i\left(u_i, U_{-i}, x\right) = \frac{u_i\left(u_i + U_{-i}\right)}{x} \\
\text{Revenue} \quad &: \quad p u_i \\
\text{Profit} \quad &: \quad \pi_i = u_i\left(p - \frac{u_i + U_{-i}}{x}\right)
\end{aligned}
$$

CPR: Simplest possible example

- Equilibrium

$$
\begin{aligned}
\max \pi_i &= u_i \left(p - \frac{u_i + U_{-i}}{x} \right) \\
\underset{\text{Marginal revenue}}{p} &= \underset{\text{Marginal cost}}{\frac{2u_i + U_{-i}}{x}} \\
u_i \left(U_{-i} \right) &= \frac{1}{2} \left(px - U_{-i} \right) \rightarrow \text{strategic substitutability} \\
\text{Equilibrium} &: \quad u^N = \frac{px}{n+1}, \qquad U^N = \frac{npx}{n+1}
\end{aligned}
$$

CPR: Simplest possible example

- Social optimum (only profits matter)

$$\max_{(u_i)_{i=1,\dots,n}} \sum_i \pi_i = \sum_i \left(p - \frac{(u_i + U_{-i})}{x} \right) u_i$$

$$u^{SO} = \frac{px}{2n}, \qquad U^{SO} = \frac{px}{2}$$

- Comparison

$$\text{Equilibrium} : u^N = \frac{px}{n+1}, \qquad U^N = \frac{npx}{n+1}$$

- Endogenous price (market power); Consumer surplus in SO

CPR: Dealing with overexploitation

- Command-and-control policies: tell agents what to do
  - Fishing quotas, pollution permit, deforestation level, etc.

$$\text{Quota} = \text{Socially optimal appropriation (SOA)}$$

- Market instruments
  - Price the negative externality
  - Taxes such that individual optimizers choose SOA level
- Information
  - Public Disclosure Program
  - Bad guys are punished by consumers and capital markets
  - Sustainable processes, certification, blockchain, etc.
  - Naming and shaming

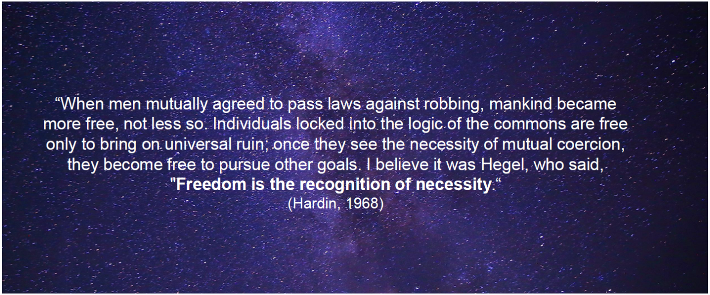
CPR: Dealing with overexploitation

- Mechanisms are harder to implement international arena
  - Climate change
  - Biodiversity
  - High-seas fisheries
- Buying cooperation (compensation)
  - Norwegian fund for Amazon forest
  - Unidirectional pollution: polluted downstream pays the upstream polluters
- Piracy!

CPR: Tragedy of the commons

Selfish (equilibrium) appropriation $>$ socially optimal appropriation

"Ruin is the destination towards which all men rush, each pursuing his own best interest in a society that believes in the freedom of the commons"



Dynamic models of CPRs

- Resource evolves overtime: birth/death rates, agents' actions, e.g., harvesting
- Framework that distinguishes between flow variables and stock variables
- Flow variables: harvest (fisheries), deforestation (forests), emissions (climate change)
- State variables: stock of biomass, forest area, stock of pollution
  - Dynamics: describe the evolution of the state

Dynamic models: One agent

- Intertemporal externality: trade-off between current and future consumptions
- Viability theory
- Dynamic optimization
  - Dynamic programming
  - Optimal control
- Optimal policy: Control function of the state; $u^* (x)$
- Optimal time path: Control function of time; $u^* (t)$

Dynamic models of CPRs

- Intertemporal externality + strategic externality
- Several (atomistic, nonatomistic) agents: CPR modelled as a dynamic game
- Time discrete or continuous
- Payoff of a player's depends not only on her own decision, but also on the decisions made by the other players: strategic interaction
- Strategic interaction: in payoff functions, in state dynamics, or in both

CPR as a dynamic game

- Time; continuous $[0, T]$; discrete $t = 0, \ldots, T$
- Set of players $M = \{1, \ldots, m\}$;
- Vector of controls $\mathbf{u}_j(t) \in U_j \subseteq \mathbb{R}^{m_j}$, $j \in M$;
- Vector of state variables $\mathbf{x}(t) \in X \subseteq \mathbb{R}^n$. State equations:

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= \frac{d\mathbf{x}}{dt}(t) = f\left(\mathbf{x}(t), \underline{\mathbf{u}}(t), t\right), \quad \mathbf{x}(0) = \mathbf{x}^0, \\
\mathbf{x}\left(t+1\right) &= f\left(\mathbf{x}(t), \underline{\mathbf{u}}(t), t\right), \quad \mathbf{x}(0) = \mathbf{x}^0,
\end{aligned}$$

where $\underline{\mathbf{u}}(t) \triangleq (\mathbf{u}_1(t), \ldots, \mathbf{u}_m(t))$;

Examples of state equations: Fish war

- Great fish war (Levhari and Mirman (1980))

$$x(t+1) = (x(t) - u_1(t) - \ldots - u_n(t))^{\alpha}, \quad t = 0, 1, \ldots,$$

$u_i(t)$: harvest of player i at time t

Examples of state equations: Interacting species

- Interacting species (Fisher and Mirman (1992, 1996), Doyen et al. (2018))
- Two species (Breton et al. (2019))

$$x_{l;t+1} = \left(x_{lt} - \sum_{i \in N_l} u_{ilt} \right)^{\alpha_l} \left(x_{mt} - \sum_{i \in N_m} u_{imt} \right)^{\beta_l}, l = 1, 2; m = 3 - l$$

$\alpha_l > 0$: regeneration capacity of species l; $\beta_l \neq 0$: indirect effect that species m exerts on species l.

- Relationship
  - *symbiotic:* $\beta_1, \beta_2 > 0$
  - *competitive:* $\beta_1, \beta_2 < 0$
  - *prey-predator:* β_1 and β_2 have opposite signs

Examples of state equations: Habitat

- Importance of protecting the habitat (Barbier (2000), Fluharty (2000), Kaiser and de Groot (2000), Botsford et al. (1997)).
- Directives of the European Parliament.
- Magnuson-Stevens Fishery Act (US) defines "essential fish habitat" as "those waters and substrate necessary for spawning, breeding, feeding, or growth to maturity."

Examples of state equations: Habitat

- Two-way relationship between commercial fishing and fish habitat
  - Mobile fishing gear can remove biogenic and sedimentary structures, and organisms creating these structures.
  - Damage to habitat leads to loss of spawning $\longrightarrow$ reduction in growth rate of fish stock.
- No formal modelling of this dynamic two-way relationship.
- Long, Tidball, Zaccour (2019): model this dynamic two-way relationship

Examples of state equations: Habitat

- Foley et al. (2012):

$$F(x, H) = r(H)x\left(1 - \frac{x}{K(H)}\right)$$

x : stock of fish; H : habitat; $K(H)$: carrying capacity; $r(H)$: growth rate

- Long Tidball, Zaccour (2019)

$e(t)$: Fishing effort at t; $u_i(t) = qe_i(t)x(t)$
State dynamics:

$$\dot{x}(t) = bH(t)x(t)\left(1 - \frac{x(t)}{gH(t)}\right) - \sum_i u_i(t), \quad x(0) = x_0,$$
$$\dot{H}(t) = H(t)(1 - H(t)) - \sum_i u_i(t), \qquad H(0) = H_0.$$

CPR as a dynamic game

- Payoff for Player $j \in M$,

$$J_j \triangleq \int_0^T g_j(\mathbf{x}(t), \underline{\mathbf{u}}(t), t)\, dt + S_j(\mathbf{x}(T))$$

$$J_j \triangleq \sum_{t=0}^{T} g_j(\mathbf{x}(t), \underline{\mathbf{u}}(t), t) + S_j(\mathbf{x}(T))$$

  g_j : instantaneous payoff; S_j : terminal payoff
- Information structure, i.e., information available to Player j when he selects $\mathbf{u}_j(t)$ at t;
- Strategy set Γ_j, where $\gamma_j \in \Gamma_j$ is a decision rule that defines the control $\mathbf{u}_j(t) \in U_j$ as a function of the information available at time t.

CPR as a dynamic game

Time horizon:

- T can be finite or infinite;
- T can be prespecified or endogenous (e.g., resource depletion, patent-race game).

Control set: $\mathbf{u}_j(t) \in U_j$, with U_j set of admissible controls (or control set).

- Time-invariant and independent of the state;
- Depends on the *position of the game* $(t, \mathbf{x}(t))$, i.e., $\mathbf{u}_j(t) \in U_j(t, \mathbf{x}(t)))$.
- Depends also on controls of other players (coupled constraints, Rosen equilibrium)

CPR as a dynamic game

Information structure and strategies:

Open loop: Decisions based on time and initial condition

$$\text{Open-loop strategy} : \mathbf{u}_j(t) = \mu_j(\mathbf{x}^0, t)$$

Feedback or Markovian: Decisions based on the *position of the game* $(t, \mathbf{x}(t))$

$$\text{Feedback strategy} : \mathbf{u}_j(t) = \sigma_j\left(t, \mathbf{x}(t)\right)$$

Non-Markovian: Decisions are history-dependent

$$\text{Non-Markovian strategy} : \mathbf{u}_j(t) = \sigma_j\left(H(t)\right)$$

Open-Loop Nash Equilibrium

Definition

The control m-tuple $\underline{\mathbf{u}}^*(\cdot) = (\mathbf{u}_1^*(\cdot), \ldots, \mathbf{u}_m^*(\cdot))$ is an **open-loop Nash equilibrium** (OLNE) at $(t^0, \mathbf{x}^0)$ if the following holds:

$$J_j(t^0, \mathbf{x}^0; \underline{\mathbf{u}}^*(\cdot)) \geq J_j(t^0, \mathbf{x}^0; [\mathbf{u}_j(\cdot), \underline{\mathbf{u}}_{-j}^*(\cdot)]), \quad \forall \mathbf{u}_j(\cdot), j \in M,$$

Definition

The feedback m-tuple $\underline{\sigma}^*(\cdot) = (\sigma_1^*(\cdot), \ldots, \sigma_m^*(\cdot))$ is a **feedback or Markovian-Nash equilibrium** (MNE) on $[0, T] \times X$ if for each $(t^0, \mathbf{x}^0)$ in $[0, T] \times X$, the following holds:

$$J_j(t^0, \mathbf{x}^0; \underline{\sigma}^*(\cdot)) \geq J_j(t^0, \mathbf{x}^0; [\sigma_j(\cdot), \boldsymbol{\sigma}^*_{-j}(\cdot)]), \quad \forall \sigma_j(\cdot), j \in M,$$

Cooperative games

- Coalition: $K \subseteq M$;
- Strategic force of a coalition (characteristic function $v(K) : \mathcal{P}(M) \rightarrow \mathbb{R}$)
- Cooperative game theory: grand coalition M will form
- Noncooperative approach to coalition formation
  - Climate change

Cooperation

Static game

- Determine a collectively optimal solution (joint optimization)
- Allocate the collective outcome to the players
  - Solutions of a cooperative game (core, Shapley value, etc.)
  - Axiomatic approach (stability, fairness, etc.)

Dynamic game

- Sustainability of cooperation over time

Motivation

- Long-term contracts are common
  - Marriage
  - Union and management
  - International agreements (trade, environments, etc.)
- Why to commit?
  - Negotiation and contracting cost
  - Future's outcomes depend on today's decisions
- Breakdowns before maturity; *Time inconsistency*
  - *Time consistency: Sustainability, dynamic individual rationality, dynamic stability, durability, agreeable solution*, etc.
- Design mechanisms, schemes, side payments, etc.

Sustaining Cooperation

Cooperative Equilibria

- Trigger strategies: based on past actions;
  - Threat to punish credibly cheating on the agreement
  - Dutta (1995), Parilina and Zaccour (2015), Tolwinski, Haurie & Leitmann (1986), (Haurie, Hämäläinen, Pohjola (1980s)), DoJøLoSo (2000), HKZ (2012)
- Incentive strategies:, Ehtamo and Hämäläinen (1986, 1989, 1993). Two players (u_1^*, u_2^*)

$$S_j\left(u_{3-j}\right) = u_j^* + p_j\left(u_{3-j}^* - u_{3-j}\right), \quad p_j(0) = 0, \quad j = 1, 2;$$

  - Main issue: credibility

Sustaining Cooperation

- Time consistency:

$$\text{Coop. payoff-to-go} \geq \text{noncoop. payoff-to-go}, \quad \forall j, \quad \forall t$$

  - Comparison along the optimal state trajectory
  - Starr and Ho (1973), Haurie (1976), Petrosjan (1979)
  - Large literature with various applications (Yeung and Petrosjan (2005, 2018), Zaccour (2008, 2017), Petrosjan and Zaccour (2018))

- Agreeability Kaitala and Pohjola (1990)
  - Payoff dominance along any state trajectory

$$\text{Agreeability} \quad \Rightarrow \quad \text{Time consistency}$$

Great fish war with moratorium

Ilyass Dahmouni, Elena Parilina, Georges Zaccour

Introduction

- Fishery exploited by n firms
- Regulator implements a moratorium on harvesting any time the stock $x \leq \underline{x}$.
- Questions:
  1. Can a moratorium be avoided in equilibrium?
  2. If a moratorium is not avoidable, what should be its optimal duration?
  3. Can the players design a coordinated harvesting profile such that
     3.1 the corresponding steady-state value is $\underline{x}$; and
     3.2 the outcome is a Nash equilibrium?

Introduction

- Forms of a moratorium
  - Seasonal fishing moratorium (Pearl River estuary (Wang et al. (2015));
  - Partial moratorium (American shad in Virginia riverine fisheries (Olney and Hoenig (2001));
  - Transshipment in high seas(Ewell et al. (2017));
  - Administrative form (Indonesian government moratorium on fishing licenses for foreign vessels (Khan et al. (2018));
  - Operational form (United Nations ban on all driftnets over 2.5 km in length (Hewison (1994)).

- Moratorium on cod harvesting imposed by Canada in 1992 (Frank et al. (2005) and Rose and Row (2015)).
  - Stock severely impoverished under open-access in 1960s and 1970s.
  - In 1968, 85% of harvest is by foreigners (Baird et al. (1991))
  - In 1977 foreign vessels were banned from Canada's Exclusive Economic Zone (EEZ)
  - Full recovery of the stock expected by 2030 (Castaneda et al. (2020)).

Introduction

- Literature recommends preventive actions over regulations.
- Cooperation among fishers may fail if no legal framework is put in place (Hardin (1968)).
- Coalition stability achieved by a small subset of players (Breton and Keoula (2014) and Kwon (2006)).
- Support cooperation (2 players) using incentive strategies (Mazalov and Rettieva (2010)).
- Game theory literature (Bailey et al. (2010); Hannesson (2011), Sumaila (2013) and Gronbæk et al. (2018)).
- Regime switching (Gromov and Gromova (2017)).

Model

- Fish war model à la Levhari and Mirman (1980)
  - $N = \{1, 2, \dots, n\}$: Set of players.
  - $u_i(t) \in U_i$: harvest effort at $t = 0, 1, \dots,$
  - $u(t) = (u_1(t), \dots, u_n(t)) \in \prod_{i \in N} U_i$: Strategy profile at time t
  - $x(t) \in X = [0, 1]$: Fish stock (state variable); $x(0) = x_0 \in X$.
- State dynamics:

$$\begin{aligned} x(t+1) &= (x(t) - u_1(t) - \dots - u_n(t))^\alpha, \\ t &= 0, 1, \dots, \\ \text{Reproduction rate} &: 0 < \alpha < 1. \end{aligned}$$

Model

- Moratorium starts if $x \leq \underline{x}$ (common knowledge)
- State dynamics
  - *Normal regime*:

$$x(t+1) = (x(t) - u_1(t) - \ldots - u_n(t))^{\alpha}, \quad t = 0, 1, \ldots,$$

  - *Moratorium regime:* If there exists T such that $x(T-1) > \underline{x}$ and $x(T) \leq \underline{x}$, then the moratorium starts at T :

$$\begin{aligned} x(t+1) &= (x(t))^{\alpha}, \quad t = T, \ldots, T + t' - 1, \\ t' &= \text{moratorium duration} \end{aligned}$$

Remark: In the absence of human activities, $x_{\infty} = 1, \forall\ x_0$. We take $X = [0, 1]$, but the upper bound could be larger than one.

Model

- Duration of the moratorium:

1. t' periods, i.e., moratorium periods are $T, T+1, \ldots, T+t'-1$. At time $T+t'$, the system switches to the normal regime.
2. Revert to normal regime when x reaches $\overline{x} > \underline{x}$.
   - Moratorium lasts until period $T+t'-1$ inclusively. Time $T+t'$ can be found given that $x(T)$ is known.
   - $\overline{x}$ can be set, e.g., equal to x_0.

- Direct relationship between t' and $\overline{x}$.
- Regulator's payoff represented by the moratorium policy.
- Time-based moratorium; South China Sea (May 1 to August 16).
- Biomass-based decision; Canadian cod moratorium.

Model

- Player i maximizes

$$
\begin{aligned}
J_i(x_0, u) &= \sum_{t=0}^{\infty} \rho^t \phi_i(t, x(t), u(t)), \qquad \rho \in (0,1), \\
&\quad \text{subject to state dynamics;} \\
\phi_i(\cdot) &= \begin{cases} \ln(\tilde{d}\ u_i(t)) = \ln \tilde{d} + \ln u_i(t), & \text{normal regime,} \\ 0, & \text{moratorium regime,} \end{cases}
\end{aligned}
$$

where $\tilde{d} > 1$ is a scaling parameter. Let $d = \ln \tilde{d}$.

Assumption 1: In a normal regime, $u_i\,(t) \in \left(e^{-d}, 1\right]$ for all t.
d can be arbitrary large.

Model

In a great fish war model, players use linear strategies $u_i(t) = \gamma_i x(t)$, with $\gamma_i \in (0,1)$.

Proposition

Let the players' strategies be linear in the stock, i.e., $u_i(t) = \gamma_i x(t)$, with $\gamma_i \in (0,1)$, for any $t > 0$. If

$$\gamma_i > \frac{1}{e^d \underline{x}},$$

then Assumption 1 is satisfied.

Model

Feedback information structure;

- $\psi_i = \psi_i(t, x)$: Player i's strategy
- $\psi(t, x) = (\psi_1(t, x), \dots, \psi_n(t, x))$: Profile of feedback strategies at t
- $\psi_{-i}(t, x) = (\psi_1(t, x), \dots, \psi_{i-1}(t, x), \psi_{i+1}(t, x), \dots, \psi_n(t, x))$
- $\phi_i\,(t, x(t), u(t)) \equiv 0$ for any $i \in N$ for all periods t in the moratorium regime

Model

Definition

A Nash equilibrium in a fish war game with moratorium is the profile of feedback strategies $\psi^{nc}(t,x) = (\psi_1^{nc}(t,x), \ldots, \psi_n^{nc}(t,x))$ if

$$J_i(x_0, \psi^{nc}(\cdot)) \geq J_i(x_0, \psi_i(\cdot), \psi_{-i}^{nc}(\cdot)),$$

for any admissible feedback strategy $\psi_i(\cdot)$ of player $i \in N$.

Model

Scenario 1. A moratorium is not needed, if $x_0 > \underline{x}$, and $u(t)$ is such that $x(t) > \underline{x}$ for all $t = 1, 2, \ldots$, and $\lim_{t \to \infty} x(t) \geq \underline{x}$.

Scenario 2. A moratorium is implemented if there exists a time T at which $x(t) > \underline{x}$ for any $t < T - 1$ and $x(T) = \underline{x}$. Then, at T the moratorium starts and lasts for t' periods. Next, the normal regime starts, and so on.

Moratorium could happen an infinite (but countable) number of times and we apply the "same pattern of strategies," the players' behavior will be "periodical".

Always normal regime

- Conditions under which a moratorium is not needed.
- Class of linear-feedback strategies in the great fish war game.
- Player i's strategy: harvest a positive share $\gamma_i \in [0, 1]$ of the stock x,
$$u_i(t) = \gamma_i x(t), \quad \text{for all } t \geq 0.$$

Always normal regime

Proposition

When the players' strategies are of the form $\gamma_i x(t)$,the trajectory of the state variable is given by

$$x(t) = x_0^{\alpha^t} \left(1 - \sum_{i \in N} \gamma_i\right)^{\left[\frac{\alpha(1-\alpha^t)}{1-\alpha}\right]},$$

and the steady-state value by

$$x_\infty = \left(1 - \sum_{i \in N} \gamma_i\right)^{\left[\frac{\alpha}{1-\alpha}\right]}.$$

Always normal regime

- Find a feedback Nash equilibrium (FNE) such that $\underline{x}$ is never reached.
- $V_i(x)$: Value function of player i
- FNE strategies are derived by solving the following Hamilton-Jacobi-Bellman (HJB) equation:

$$V_i(x) = \max_{u_i \geq 0} \left(d + \ln u_i + \rho V_i \left((x - \sum_{i \in N} u_i)^\alpha \right) \right).$$

Always normal regime

Proposition

The unique symmetric FNE is given by

$$\gamma_i^{nc} = \frac{1 - \alpha\rho}{n(1 - \alpha\rho) + \alpha\rho}, \quad \forall i \in N,$$

and the value function by

$$\begin{aligned} V_i^{nc}(x) &= A_i^{nc} \ln x + B_i^{nc}, \qquad \forall i \in N, \\ A_i^{nc} &= \frac{1}{1 - \rho\alpha}, \\ B_i^{nc} &= \frac{\rho\alpha \ln(1 - n\gamma_i^{nc}) + (1 - \rho\alpha)(d + \ln \gamma_i^{nc})}{(1 - \rho\alpha)(1 - \rho)}. \end{aligned}$$

Always normal regime

- A moratorium is not needed, if $x_0 > \underline{x}$, and $u(t)$ is such that $x(t) > \underline{x}$ for all $t = 1, 2, \ldots$, and $\lim_{t\to\infty} x(t) \geq \underline{x}$.

Proposition

When symmetric players adopt the unique FNE harvesting strategies, the moratorium is never applied if

$$n \leq \frac{\alpha\rho(1 - \underline{x}^{\frac{1-\alpha}{\alpha}})}{(1 - \alpha\rho)\underline{x}^{\frac{1-\alpha}{\alpha}}} \triangleq Z$$

where $\underline{x}$ is the moratorium level.

- Z is defined and nonnegative for any $\underline{x} \in (0, 1]$
- Z is decreasing in $\underline{x}$ and α; increasing in ρ (patience helps).
- If $\underline{x} = 1$, then $n \leq 0$.

Normal and moratorium regimes

- If $n > Z$, then a moratorium will be imposed
- Suppose the regulator wants to bring back the stock to x_0 (could be any other value).

Moratorium starts at $t = T$, when $x(T) = \underline{x}$, and lasts for t' periods (from $t = T$ till $t = T + t' - 1$ inclusively), where t' is obtained by solving

$$\begin{aligned} x(T + t') &= x_0, \\ x(t + 1) &= (x(t))^{\alpha}, \quad t = T, \ldots, T + t' - 1. \end{aligned}$$

Normal and moratorium regimes

Lemma

Given x_0 and $\underline{x}$, the duration of the moratorium is given by

$$t' = \frac{\ln\left[\frac{\ln x_0}{\ln \underline{x}}\right]}{\ln \alpha}.$$

- If time $t' \notin \mathbb{N}$, then set $t' := \lceil t' \rceil$.

If $\underline{x} = \theta x_0$, where $\theta \in (0, 1)$, then

$$t' = -\frac{\ln\left[1 + \frac{\ln \theta}{\ln x_0}\right]}{\ln \alpha}, \qquad \frac{\partial t'}{\partial \theta} < 0.$$

Normal and moratorium regimes

Sequence of events:

- At $t = 0, \ldots, T-1$, player i harvests $u_i^*(t)$, $i \in N$. At T, $x(T) = \underline{x}$, and a moratorium is implemented in periods $t = T, \ldots, T+t'-1$; players get zero payoffs.

- At $t = T + t'$, the stock level is back to the desired level x_0, and the players can again harvest.

Normal and moratorium regimes

Player i's payoff when a moratorium is firstly applied at $t = T$

$$J_i^T(x_0, u) = \sum_{t=0}^{T-1} \rho^t (d + \ln u_i(t, x(t))) + \rho^{T+t'} J_i^T(x_0, u), \qquad (1)$$

from which we get

$$J_i^T(x_0, u) = \frac{1}{1 - \rho^{T+t'}} \sum_{t=0}^{T-1} \rho^t (d + \ln u_i(t, x(t))),$$

with $x(T + t') = x_0$, and with-harvest dynamics for $t = [0, T - 1]$ and no-harvest dynamics for $t = [T, \ldots, T + t' - 1]$. Player $i \in N$ maximizes (1) with respect to $u_i \geq 0$ and $T \geq 0$.

Normal and moratorium regimes

Proposition

The symmetric NE in closed-loop strategies in a T-stage game, with $x(0) = x_0$ and $x(T) = \underline{x}$, is given as a unique solution of Bellman equation:

$$V_i(t, x(t)) = d + \max_{u_i(t,x(t)) \in [0, x(t)]} \Big\{ \ln u_i(t, x(t)) + \rho V_i\Big(t+1, \big(x(t) - u_i(t, x(t)) - \sum_{j \in N, j \neq i} u_j(t, x(t))\big)^{\alpha}\Big)$$

with

$$\begin{aligned} \textit{terminal condition } V_i(T-1, x(T-1)) &= d + \ln\left(\frac{x(T-1) - \underline{x}^{\frac{1}{\alpha}}}{n}\right), \\ \textit{such that } u(T-1, x(T-1)) &= \frac{x(T-1) - \underline{x}^{\frac{1}{\alpha}}}{n}. \end{aligned}$$

Normal and moratorium regimes

- Difficult to write down a solution to the Bellman equation in an explicit form
- Approximation

$$u_i(t, x(t)) = \frac{x(t) - \underline{x}^{1/\alpha^{T-t}}}{n + \sum\limits_{k=1}^{T-t-1} (\alpha\rho)^k},$$

Works well for low α or/and large n.

Normal and moratorium regimes

Finding T

Step 1: Set $T = 1$. Using previous Proposition, find any player's equilibrium payoff

$$V(x_0, T) = \sum_{t=0}^{T-1} \rho^t (d + \ln u_i(t, x(t)))$$

Next, compute $J_i^1(x_0, u)$ using

$$J_i^T(x_0, u) = \frac{1}{1 - \rho^{T+t'}} \sum_{t=0}^{T-1} \rho^t (d + \ln u_i(t, x(t))), \tag{2}$$

for a given moratorium duration t'.

Step 2: Set $T := T + 1$ and do Step 1 for a given moratorium duration t'.

Step 3: Find $\max_{T>0} J_i^T(x_0, u)$.

Does cooperation allow to avoid moratorium?

Proposition

The cooperative strategy in the game without moratorium is given by $u_i^*(x) = \gamma_i^* x$*, where*

$$\gamma_i^* = \frac{1-\alpha\rho}{n}, \text{ for any } i \in N.$$

Steady-state stock:

$$x_\infty^* = (\alpha\rho)^{\frac{\alpha}{1-\alpha}},$$

and $x_\infty^* > x_\infty^{nc}$, where x_∞^{nc} is the steady state stock value in Nash equilibrium.

Noncooperative harvest

$$\gamma_i^{nc} = \frac{1-\alpha\rho}{n-\alpha\rho(n-1)}, \quad \forall i \in N.$$

Does cooperation allow to avoid moratorium?

A moratorium is never implemented if

$$n \leq \frac{\alpha\rho(1-\underline{x}^{\frac{1-\alpha}{\alpha}})}{(1-\alpha\rho)\underline{x}^{\frac{1-\alpha}{\alpha}}} \triangleq Z. \tag{3}$$

1. $\underline{x} < x_\infty^{nc} < x_\infty^*$ hold true under (3). Never a moratorium in both coop and noncoop games. (3) is $\Leftrightarrow$

$$\rho > \frac{n\underline{x}^{\frac{1-\alpha}{\alpha}}}{\alpha(1+(n-1)\underline{x}^{\frac{1-\alpha}{\alpha}})} \triangleq Y.$$

2. $x_\infty^{nc} \leq \underline{x} < x_\infty^*$ satisfied if $\frac{\underline{x}^{\frac{1-\alpha}{\alpha}}}{\alpha} < \rho \leq Y$. Moratorium under noncoop, but no if players cooperate.
3. $x_\infty^{nc} < x_\infty^* \leq \underline{x}$ satisfied if $\rho \leq \frac{\underline{x}^{\frac{1-\alpha}{\alpha}}}{\alpha}$. Noncoop and coop strategy profiles do not allow to avoid the moratorium regime.

How to avoid moratorium regimes

- Can players agree on harvesting levels that result in avoiding a moratorium throughout the entire duration of the game?
- Assuming $u_i(x) = \gamma_i x$, we seek a γ_i^c, for all $i \in N$, such that the steady state computed with

$$x_\infty = (1 - \sum_{i \in N} \gamma_i)^{\left[\frac{\alpha}{1-\alpha}\right]}. \tag{4}$$

  satisfies the condition $x_\infty = \underline{x}$.
- If it exists, the constructed harvesting profile $(u_1^c(x), \ldots, u_n^c(x))$ will be referred to as *coordinated* profile.

How to avoid moratorium regimes

Proposition

For $i \in N$, the coordinated strategy is given by $u_i^c(x) = \gamma_i^c x$, where

$$\gamma_i^c = \frac{1}{n}\left(1 - \underline{x}^{\frac{1-\alpha}{\alpha}}\right),$$

and the corresponding fish stock by

$$x^c(t) = x_0^{\alpha^t}\underline{x}^{1-\alpha^t}, \quad t = 1, 2, \ldots,$$

with initial stock $x^c(0) = x_0$.

Note that $\lim_{t\to\infty} x^c(t) = \underline{x}$.

How to avoid moratorium regimes

Proposition

Let inequality $n \leq \frac{\alpha\rho(1-\underline{x}^{\frac{1-\alpha}{\alpha}})}{(1-\alpha\rho)\underline{x}^{\frac{1-\alpha}{\alpha}}} \triangleq Z$ *be not satisfied. The coordinated profile* $u_i^c(x) = \gamma_i^c x$, *and* $\gamma_i^c = \frac{1}{n}\left(1 - \underline{x}^{\frac{1-\alpha}{\alpha}}\right)$, $i \in N$, *is the NE in the game with moratorium if* $J_i^c \geq J_i'$,*where* J_i^c *is the coordinated payoff, and*

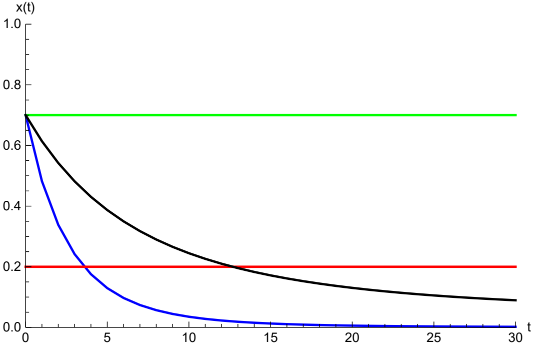
$$J_i' = \max_T \left\{ \frac{1-\rho^T}{(1-\rho)(1-\rho^{T+t'})} \left(d + \ln \gamma_i' + \frac{1}{1-\alpha^T} \ln \underline{x} - \frac{\alpha^T}{1-\alpha^T} \ln x_0 \right) \right. \tag{5}$$

$$\left. + \frac{1-(\alpha\rho)^T}{(1-\alpha^T)(1-\alpha\rho)(1-\rho^{T+t'})} \ln\left(\frac{x_0}{\underline{x}}\right) \right\},$$

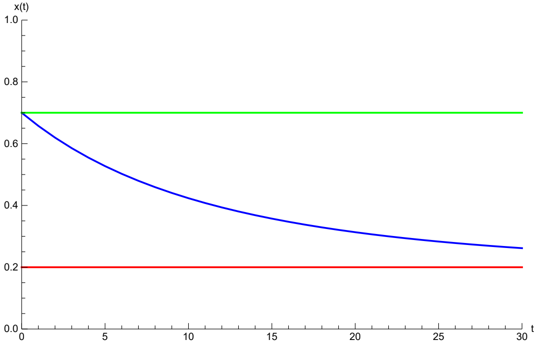
and $\gamma_i' = \frac{1}{n} + \frac{n-1}{n}\underline{x}^{\frac{1-\alpha}{\alpha}} - \left(\frac{\underline{x}}{x_0^{\alpha^T}}\right)^{\frac{1-\alpha}{\alpha(1-\alpha^T)}}$. (6)

Example

- 3-player game
- $x_0 = 0.7$
- $\underline{x} = 0.2$
- $\rho = 0.9$
- $\alpha = 0.95$
- $d = 6$.



(a) The fish stock trajectories with the Nash and cooperative equilibrium (green — x_0, black — $x^*(t)$, blue — $x^{nc}(t)$, red — $\underline{x}$)



(b) The fish stock trajectories with moratorium-free strategy profile (green — x_0, blue — $x^{mf}(t)$, red — $\underline{x}$).

Figure 1: The game with $\alpha = 0.95$, $\rho = 0.9$.

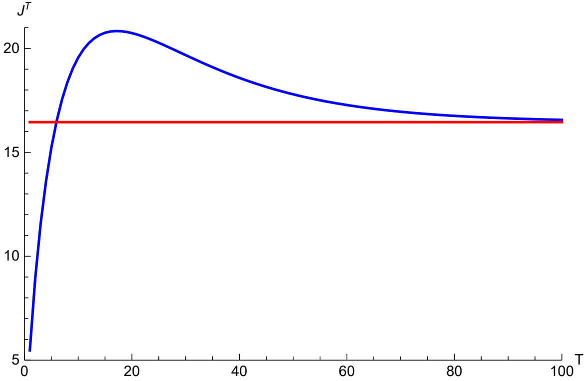
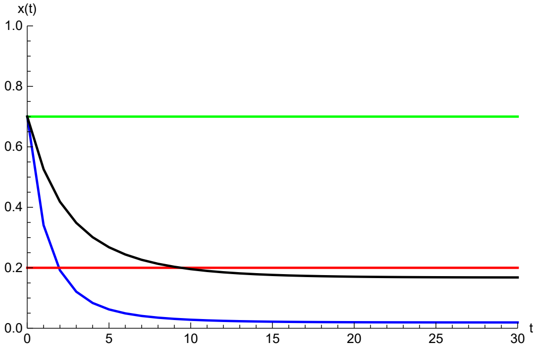
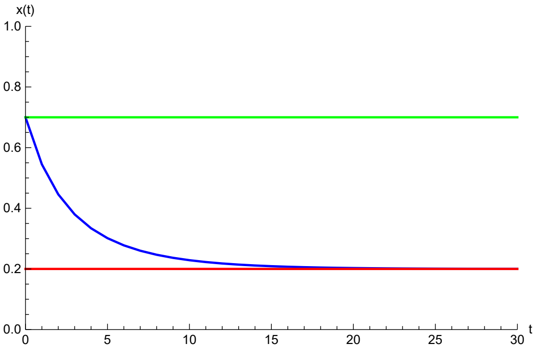


Figure 2: Blue: profit of a deviating player J_i^T as a function of T; red: profit in the moratorium-free strategy profile J_i^{mf}, $\alpha = 0.95$, $\rho = 0.9$.



(a) The fish stock trajectories with the Nash and cooperative equilibrium (green — x_0, black — $x^*(t)$, blue — $x^{nc}(t)$, red — $\underline{x}$)



(b) The fish stock trajectories with moratorium-free strategy profile (green — x_0, blue — $x^{mf}(t)$, red — $\underline{x}$)

Figure 3: The game with $\alpha = 0.8$, $\rho = 0.8$.

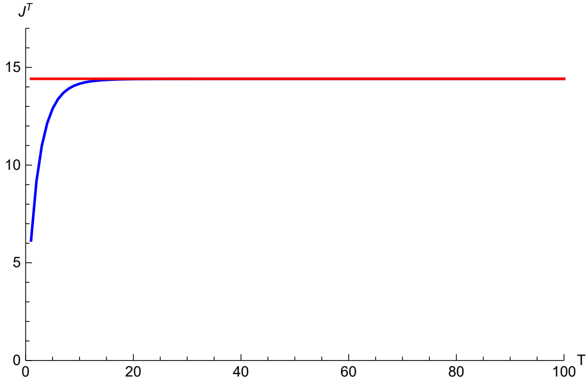


Figure 4: Blue: profit of a deviating player J_i^T as a function of T ; red: profit in the moratorium-free strategy profile J_i^{mf}, $\alpha = 0.8$, $\rho = 0.8$.

Extensions

- Fishery with multiple species biologically interacting with each other.
- Other objectives for the central planner.
- Asymmetric players.

Research paper

A Fair and Time-Consistent Sharing of the Joint Exploitation Payoff of a Fishery

Ilyass Dahmouni, Baris Vardar, Georges Zaccour

How to sustain cooperation in a fishery?

- Introduction
- Model
- Coocooperative and cooperative solutions
- A fair and time-consistent allocation
- Conclusion

How to sustain cooperation in a fishery?

- Open access (competition) leads to harvesting at levels that are higher than the resource's rate of reproduction, which may cause its depletion (Hardin (1968))
- Mechanisms to prevent such a tragedy
- Cooperation (joint optimization): Sumaila (2002), Kaitala and Lindroos (2004), Lindroos et al. (2005), Trisak (2005), Munro (2006), Sumaila and Armstrong (2006), Kronbak and Lindroos (2007), Pintassilgo and Lindroos (2008), Mazalov and Rettieva (2010) and Breton et al. (2019).
- A main issue here is how to build and sustain over time a cooperative agreement

How to sustain cooperation in a fishery?

1. What are the harvesting policies under a noncooperative and cooperative mode of play?
2. How to share the total dividend of cooperation among the players?
3. How to insure that the cooperative agreement, established at the game's initial date, remains in force as time goes by?

Model

- n symmetric players exploiting a fishery over an infinite time horizon.
- $e_i(t)$: fishing effort at time t.
- Linear production function (harvest): $q_i(e_i(t)) = ae_i(t)$
- Cost of effort: $g_i(e_i) = be_i^2$
- Stock of fish: x
- Stock of pollution: Z
- Environmental damage: $D_i(Z) = \frac{\phi}{2}Z^2$
- a , b and ϕ are positive parameters.

Model: Stock dynamics

- Resource

$$\dot{x}\left(t\right)=F(x(t),Z\left(t\right))-\sum_{i=1}^{n}q_i(e_i(t)),\qquad S(0)=S^0>0,$$

- Pollution

$$\dot{Z}\left(t\right)=\sum_{i=1}^{n}\omega e_i(t)-kZ(t),\quad Z(0)=Z^0\geq 0,$$

Model: Growth function

- Literature: $F(x, Z) \rightarrow F(x)$
- $F(x)$: concave; max at $x = \bar{x}$,where $\bar{x}$ is the carrying capacity, beyond which the resource growth rate is negative and
- $F(\bar{x})$: max sustainable yield of the resource
- Benchekroun (2003)

$$F(x, Z) = \begin{cases} \delta x - \lambda Z, & \text{for } x \leq \frac{\bar{x}}{2}, \\ \delta\left(\bar{x} - x\right) - \lambda Z, & \text{for } S > \frac{\bar{x}}{2}, \end{cases}$$

$\lambda > 0$: parameter capturing the negative externality of pollution

Model

- Optimization problem

$$\max J_i \quad = \quad \int_0^{\infty} e^{-\rho t} \left(a e_i(t) - b e_i^2(t) - \frac{\phi}{2} Z^2(t) \right) dt,$$

subject to state dynamics

- $V_i^N(x, Z)$ and $V^C(x, Z)$ are noncoop and coop value function, resp.
- Effort

$$\begin{aligned} a - 2be_i^N &= a\frac{\partial V_i^N(x, Z)}{\partial x} - \omega \frac{\partial V_i^N(x, Z)}{\partial Z} \\ a - 2be_i^C &= a\frac{\partial V^C(x, Z)}{\partial x} - \omega \frac{\partial V^C(x, Z)}{\partial Z} \end{aligned}$$

- Instantaneous revenue = opportunity cost of a unit of harvesting effort

Model

For $j \in \{N, C\}$

Region of no economic activity $\mathcal{R}_0^j$:

$$e_i^j\,(x, Z) = 0 \;\; \text{and} \;\; a \leq a\frac{\partial V_i^j(x, Z)}{\partial S} - \omega\frac{\partial V_i^j(x, Z)}{\partial Z}.$$

Region of scarce resource $\mathcal{R}_x^j$:

$$\begin{aligned} e_i^j(x, Z) &> 0, \quad a > a\frac{\partial V_i^j(x, Z)}{\partial x} - \omega\frac{\partial V_i^j(x, Z)}{\partial Z} \\ \text{and} \;\; \frac{\partial V_i^j(x, Z)}{\partial x} &> 0. \end{aligned}$$

Region of abundant resource $\mathcal{R}_A^j$:

$$\begin{aligned} e_i^j(x, Z) &> 0, \quad a > a\frac{\partial V_i^j(x, Z)}{\partial x} - \omega\frac{\partial V_i^j(x, Z)}{\partial Z} \\ \text{and} \;\; \frac{\partial V_i^j(x, Z)}{\partial x} &= 0. \end{aligned}$$

Solutions

Proposition

For $i = 1, \ldots, n$, the unique symmetric feedback-Nash equilibriumharvesting-effort is given by

$$e^N(x, Z) = \frac{1}{2b}\left(a + \theta^N + \eta^N x + \zeta^N Z\right),$$

where θ^N, η^N, and ζ^N are constants.

Proposition

The unique optimal harvesting-effort policy $e^C(x, Z), \forall i$ is given by

$$e^C(x, Z) = \frac{1}{2b}(a + \theta^C + \eta^C x + \zeta^C Z),$$

where θ^C, η^C, and ζ^C are constants.

Fair sharing

Individual rationality:

$$J_i^* \geq J_i^N, \;\; i = 1, \ldots, n.$$

Collective rationality:

$$J^C = \sum_{i=1}^{n} J_i^*.$$

Egalitarian sharing:

$$D = J^C - \sum_{i=1}^{n} J_i^N = V^C\left(x^0, Z^0\right) - \sum_{i=1}^{n} V_i^N\left(x^0, Z^0\right),$$

Player i obtains

$$J_i^* = J_i^N + \frac{D}{n}, \quad i = 1, \ldots, n.$$

Time consistency

Definition

At any intermediate instant of time, no player finds it individually rational to abandon the agreement and switch to the noncooperative strategies.

Algorithm

- Step 1: Compute the total payoff to be shared by the players.
- Step 2: Determine the individual payoffs in the absence of an agreement.
- Step 3: Share the total cooperative payoff.
- Step 4: Define a time-consistent solution.

Time consistency: Imputation distribution procedure

Definition

A vector $\beta(t) = (\beta_1(t), ..., \beta_n(t))$ of time functions is an IDP if, for all $i = 1, ..., n$, it satisfies

$$J_i^* \left(S^0, Z^0\right) = \int_0^\infty e^{-\rho t} \beta_i(t) dt.$$

- An IDP decomposes over time the imputation that player i is entitled to receive under the agreement.
- There is an infinite number of time functions that satisfy the above equality.
- Time consistent IDP.

Time consistency: Imputation distribution procedure

Definition

The IDP $\beta(t)$ is time consistent, if

$$J_i^*\left(S^0, Z^0\right) = \int_0^\tau e^{-\rho t}\beta_i(t)dt + e^{-r\tau}J_i^*\left(S^C(\tau), Z^C(\tau)\right),$$

where $J_i^*\left(S^C(\tau), Z^C(\tau)\right)$ is player i's payoff-to-go in the subgame starting at time τ, along the cooperative state trajectory.

- Time-consistent IDP

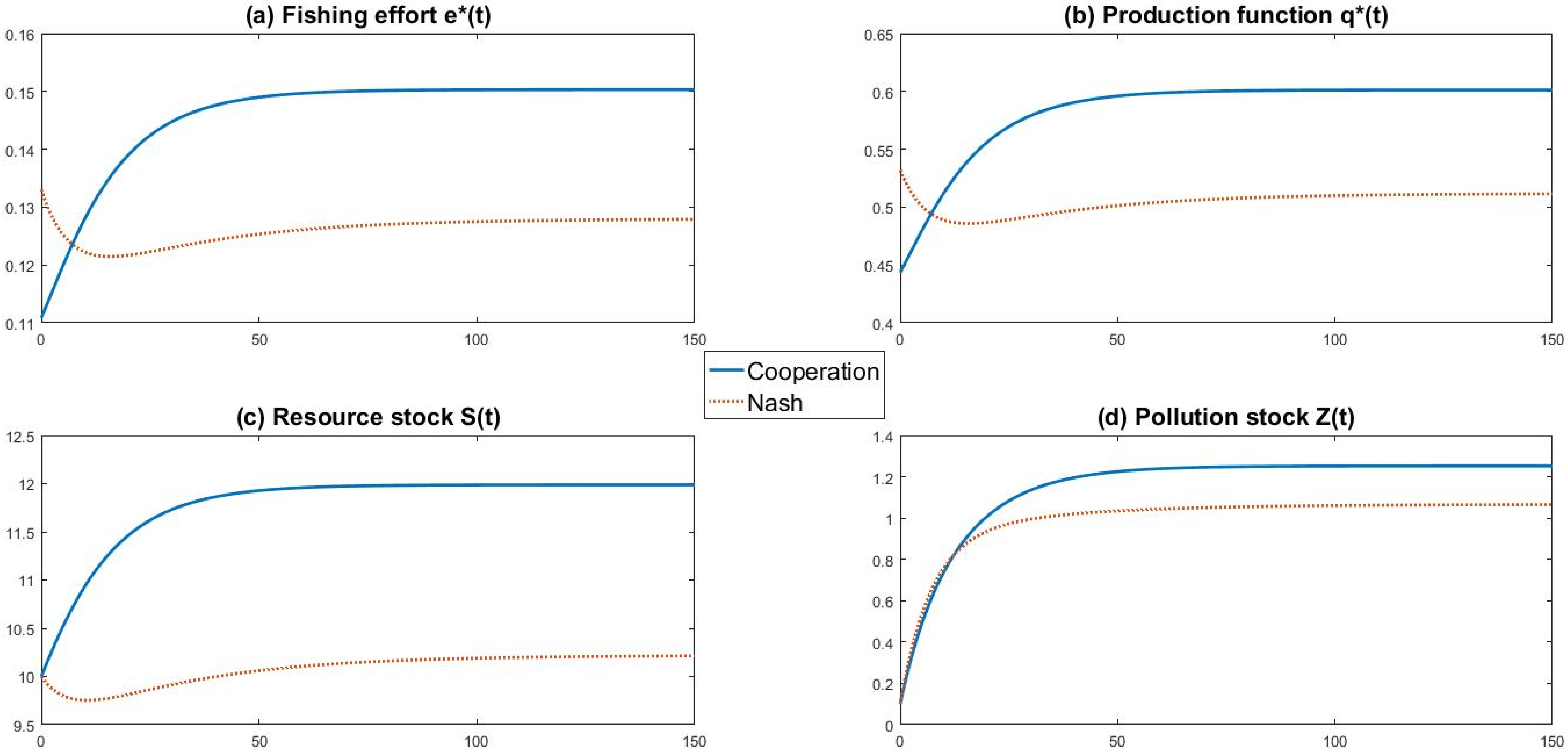
$$\beta_i(t) = \frac{1}{4b}\left(a^2 - 2b\phi\left(Z^C(t)\right)^2 - \left(\theta^C + \eta^C x^C(t) + \zeta^C Z^C(t)\right)^2\right)$$

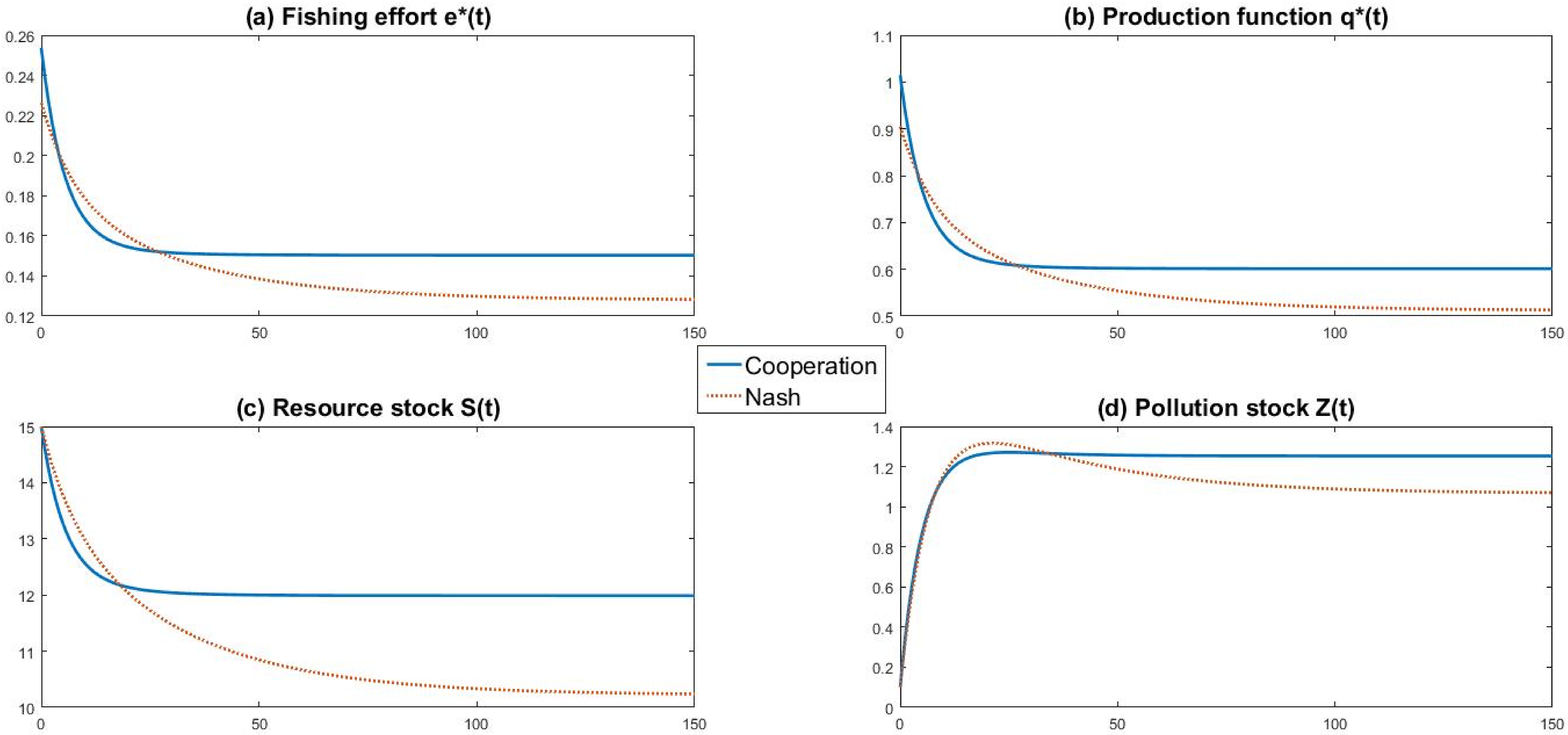
Numerical Illustration

$$\begin{aligned}
\text{Welfare function parameters} &: \quad a = 4, \quad b = 10, \quad \phi = 0.1, \\
\text{Resource dynamics parameters} &: \quad x^0 = 10, \quad \delta = 0.1, \quad \lambda = 0.1, \\
\text{Pollution dynamics parameters} &: \quad Z^0 = 0.1, \quad \omega = 0.5, \quad k = 0.12, \\
\text{Other parameters} &: \quad \rho = 0.002, \quad n = 2.
\end{aligned}$$

Steady-state values:

$$x_\infty^N = 10,2135, \quad x_\infty^C = 11,9889, \quad Z_\infty^N = 1,0655, \quad Z_\infty^C = 1,2528,$$


(a) Fishing effort e*(t)
(b) Production function q*(t)
(c) Resource stock S(t)
(d) Pollution stock Z(t)
Cooperation
Nash
0.16
0.15
0.14
0.13
0.12
0.11
0.65
0.6
0.55
0.5
0.45
0.4
12.5
12
11.5
11
10.5
10
9.5
1.4
1.2
1
0.8
0.6
0.4
0.2
0
50
100
150



(a) Fishing effort e*(t)
(b) Production function q*(t)
(c) Resource stock S(t)
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Cooperation
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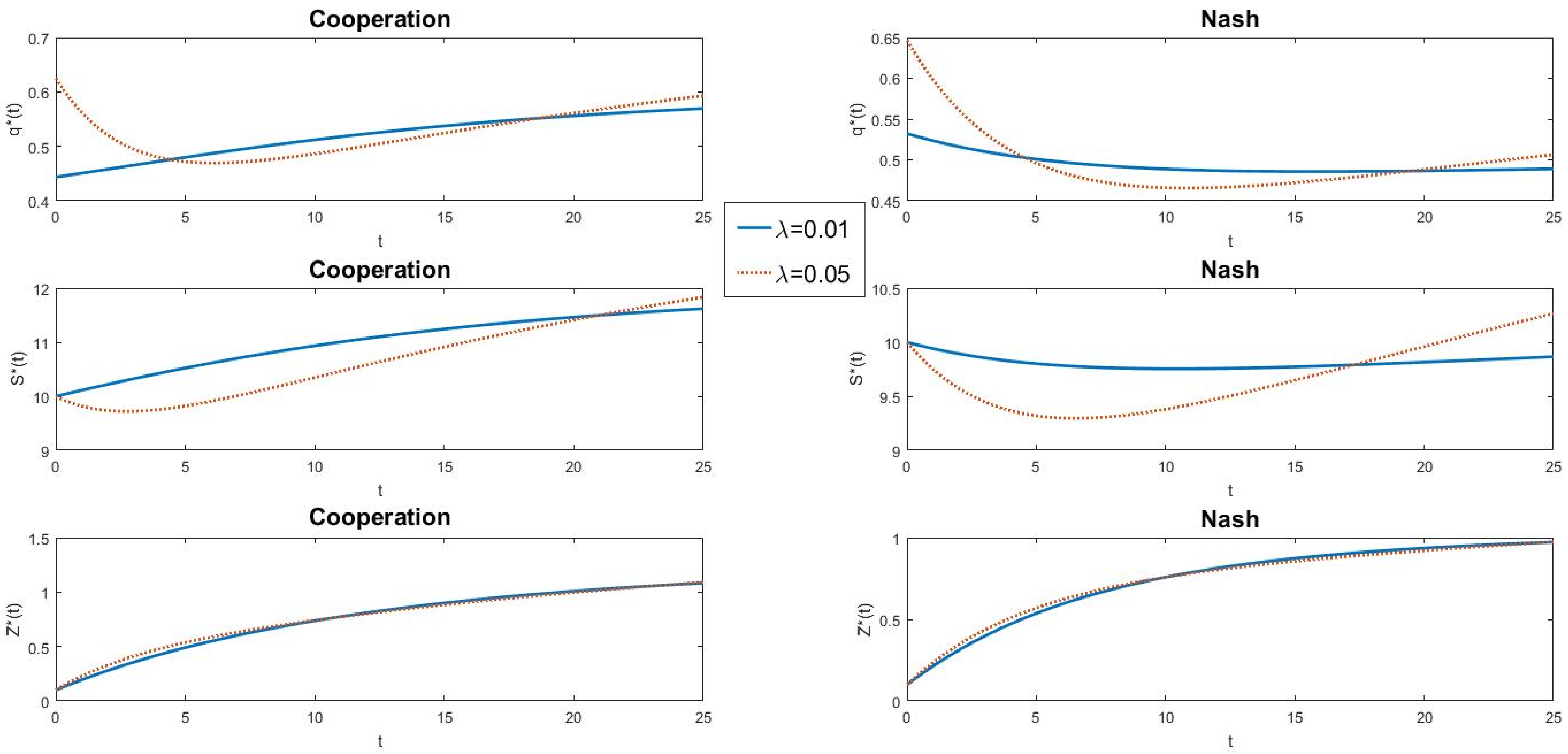
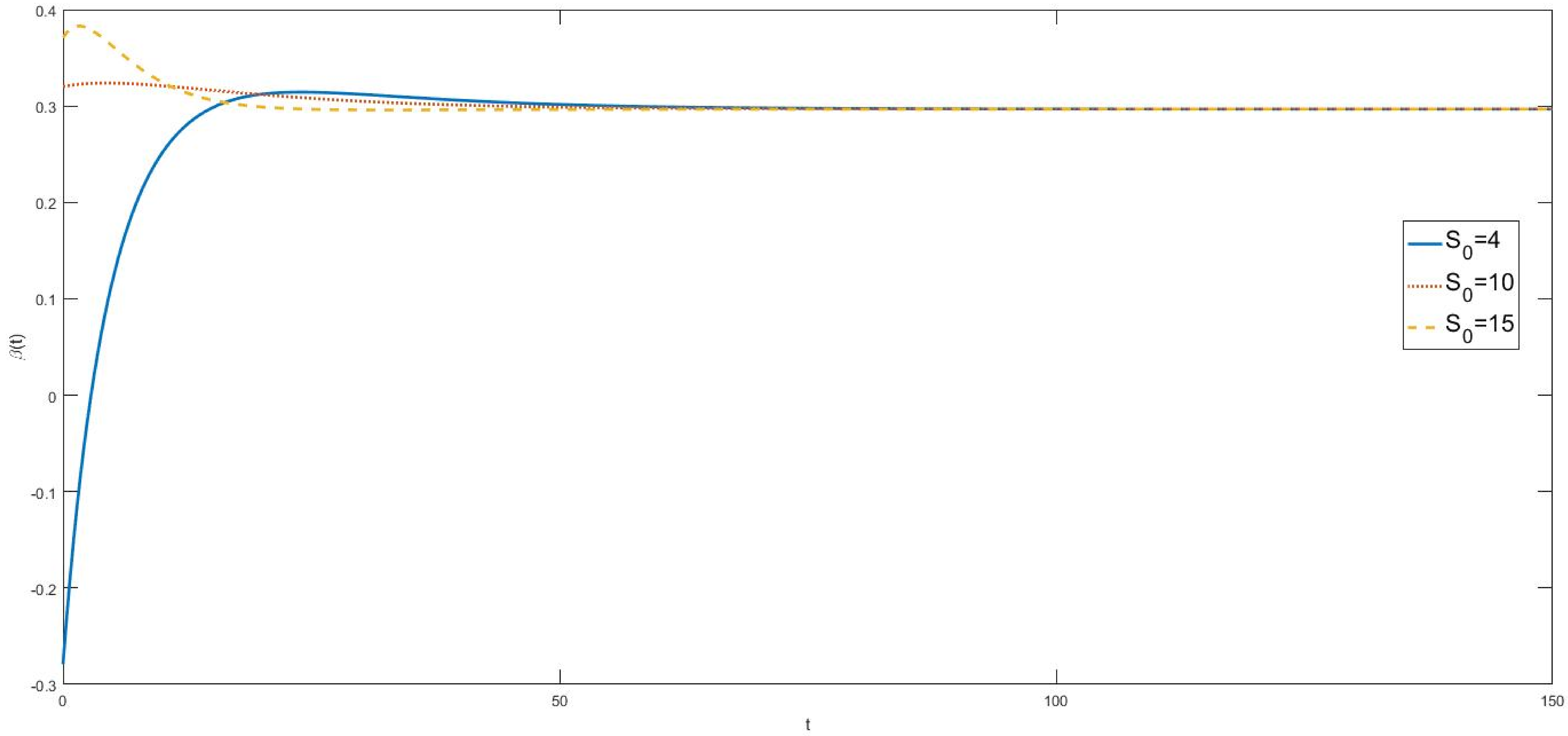
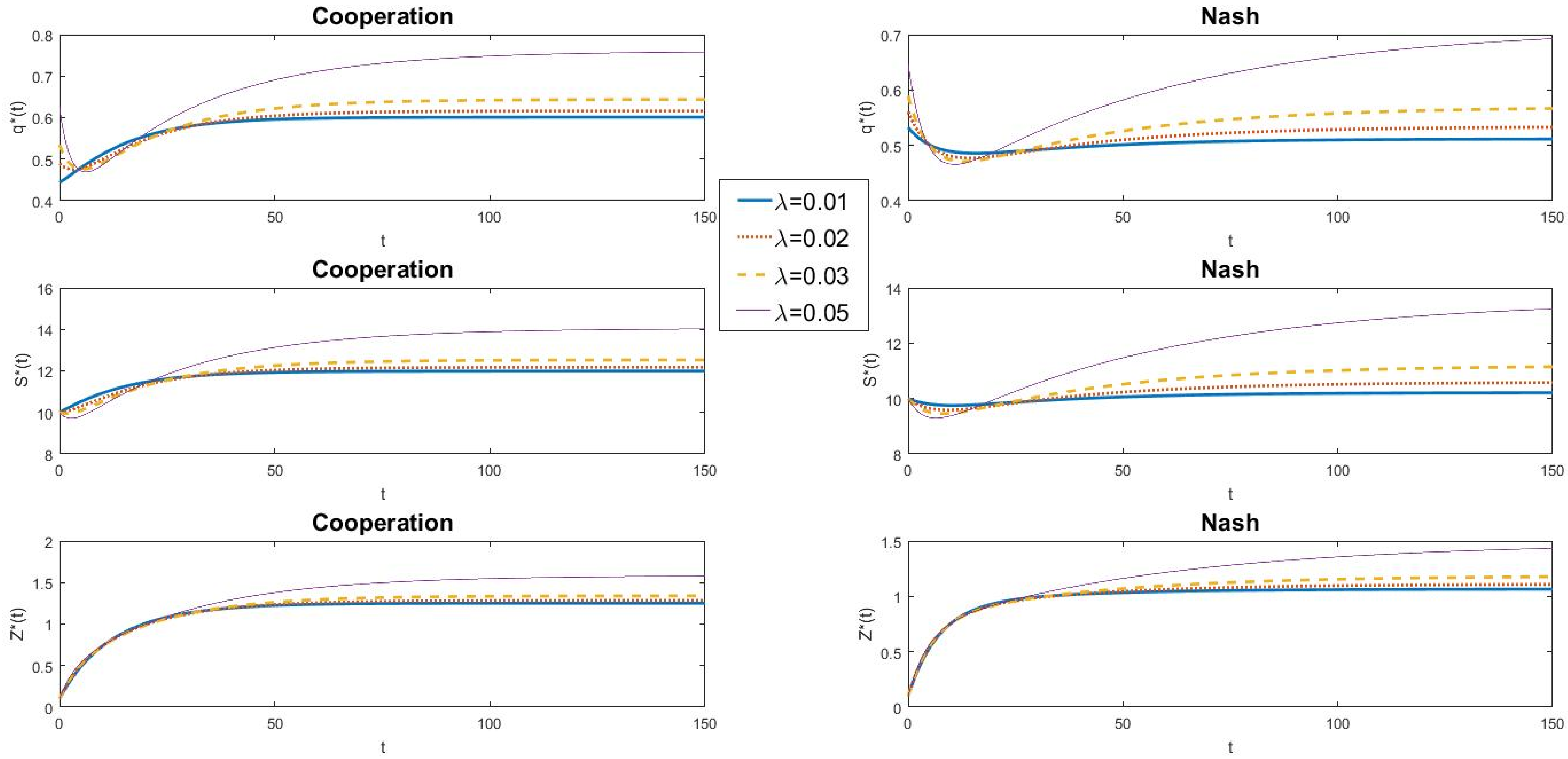


Cooperation
Nash
q*(t)
S*(t)
Z*(t)
t
λ=0.01
λ=0.05



0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
β(t)
0
50
100
150
t
S_0=4
S_0=10
S_0=15



Cooperation
Nash
q*(t)
S*(t)
Z*(t)
t
λ=0.01
λ=0.02
λ=0.03
λ=0.05

Extensions

- Explore other regions (model with extinction)
- Asymmetric players

General conclusion

- Dynamic games offer a nice paradigm for CPRs
- How to move from simple (cartoon) models to large-scale models?
- Empirical estimation