Super-hedging with partial transaction costs

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Bid-Ask spread price of Q^j in terms of Q^i: $[\pi^{ij-}, \pi^{ij+}]$

$$\pi^{ij-} := \frac{Q^j}{Q^i}(1+\lambda^{ji})^{-1} \quad \text{and} \quad \pi^{ij+} := \frac{Q^j}{Q^i}(1+\lambda^{ij})$$

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$$\begin{aligned} dP_t &= \text{diag}[P_t]\sigma_P(t, P_t)dW_t \\ dQ_t &= \text{diag}[Q_t]\sigma_Q(t, P_t, Q_t)dW_t \end{aligned}$$

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- If no exchange with the Q^i's:

$$
\begin{aligned}
X_t^0 &= x^0 + \int_0^t \phi_r \cdot dP_r \\
X_t^i &= x^i + \int_0^t \frac{X_r^i}{Q_r^i}\, dQ_r^i \quad \text{for } i > 0 \,.
\end{aligned}
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- dL_t^{ij}: amount transferred to X^j by selling units of Q^i,

$$\left(X_t^i\ ,\ X_t^j\right) \implies \left(X_t^i - (1+\lambda^{ij})dL_t^{ij}\ ,\ X_t^j + dL_t^{ij}\right)$$

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- Portfolio dynamic:

$$X_t^0 = x^0 + \int_0^t \phi_r \cdot dP_r + \sum_{j=1}^{d} \int_0^t \left[dL_r^{j0} - (1 + \lambda^{0j})dL_r^{0j}\right]$$

$$X_t^i = x^i + \int_0^t \frac{X_r^i}{Q_r^i}\, dQ_r^i + \sum_{j=0}^{d} \int_0^t \left[dL_r^{ji} - (1 + \lambda^{ij})dL_r^{ij}\right] \quad \text{for } i > 0\ .$$

Hedging Problem

- Contingent claim: $g(S_T) = (g^0(S_T), \ldots, g^d(S_T))$

- $g^i(S_T)$: amounts of Q^i to be delivered.

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- Remark: *The solvency region is*

$$K := \left\{ x \in I\!R^{1+d} \ : \ \exists\, a \in I\!M_+^{1+d},\ x^i + \sum_{j=0}^{d} (a^{ji} - (1+\lambda^{ij})a^{ij}) \ \geq \ 0 \ \forall\, 0 \leq i \leq \right.$$

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- $X_{t-} \in K \Leftrightarrow \exists \Delta L_t$ such that

$$X_t^i \ = \ X_{t-}^i + \sum_{j=0}^{d} \left[\Delta L_t^{ji} - (1+\lambda^{ij}) \Delta L_t^{ij} \right] \geq 0$$

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- X_T hedges $g(S_T)$ $\Leftrightarrow$ $\exists \Delta L_T$ such that

$$X_T^i \;=\; X_{T-}^i + \sum_{j=0}^{d} \left[\Delta L_T^{ji} - (1+\lambda^{ij}) \Delta L_T^{ij} \right] \geq g^i(S_T)$$

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- *Super-replication price*:

$$p(0, S_0) \;:=\; \inf \left\{ w \in \mathbb{R} \;:\; \exists\, (\phi, L) \in \mathcal{A}, X_T^{\phi,L} - g(S_T) \in K \text{ with } x = w\mathbf{1}_0 \right\} ,$$

where $w\mathbf{1}_0 = (w, 0, \ldots, 0) \in \mathbb{R}^{1+d}$.

Closed form solution

Theorem: (Efficient friction case) If m=0, $S = Q$, then

$$p(0, Q_0) \;=\; \min\left\{w \in I\!R \;:\; \exists\, L \in \mathcal{A}^{BH},\ X_T^L - g(Q_T) \in K \text{ with } x = w\mathbf{1}_0\right\} \;,$$

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- $p(0, Q_0) = G(Q_0)$ where G is related to the concave enveloppe of g

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Theorem: (General case) If $m \geq 1$, then

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- Similar solution: buy-and-hold on Q + dynamical trading on P.

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- Closed form solution

$$p(0, S_0) \;=\; \mathbb{E}\left[C(P_T; \hat{\Delta})\right] + \sup_{\xi \in \Lambda} \underline{\xi} \cdot \mathrm{diag}[\hat{\Delta}] Q_0$$

$$\text{where } C(P_T; \Delta) \;:=\; \sup_{z \in (0,\infty)^d} G\left(P_T, z\right) - \Delta \cdot z\;.$$

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- If we forget transaction costs at 0 and T:

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\- ϕ chosen to hedge $C(P_T; \hat{\Delta}) \geq g(P_T, Q_T) - \hat{\Delta} \cdot Q_T$ with P.

Idea of the proof (forgetting transaction costs at 0 and T)

- 1. Show that $p(0, S_0) \geq \sup_{\mu \in \mathcal{U}} \mathbb{E}\left[g(P_T, Z_T^\mu)\right] =: v(0, P_0, Q_0)$

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- 2. Show that v is concave in Q_0

$$\inf_{\mu} \; -v_t - \frac{1}{2}p^2(\sigma_P)^2 v_{pp} - \frac{1}{2}z^2(\mu)^2 v_{zz} - pz\sigma_P \mu v_{pz} \geq 0$$

$$v(T, s, z) \geq g(s, z) \;\Rightarrow\; v(T, s, z) \geq G(s, z)$$

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- 5. $p(0, S_0) \geq \sup_z \sup_{\mu \in \mathcal{U}} \mathbb{E}\left[G(P_T, Z_T^{z,\mu})\right] - \Delta \cdot (z - Q_0)$

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- 7. $p(0, S_0) \geq \mathbb{E}\left[\sup_z\ G(P_T, z) - \Delta \cdot z\right] + \Delta \cdot Q_0 \quad \diamond$

Closed form solution

- If we forget transaction costs at 0 and T:

$$\begin{aligned} p(0, S_0) &= \mathbb{E}\left[C(P_T; \hat{\Delta})\right] + \hat{\Delta} \cdot Q_0 \\ \text{where } C(P_T; \Delta) &:= \sup_{z \in (0,\infty)^d} G\left(P_T, z\right) - \Delta \cdot z \ . \end{aligned}$$

and G is the concave enveloppe of g with respect to Q.

Example

- $m = d = 1$, $\sigma(t,s) = \sigma \in \mathbb{M}^2$ invertible, $g^0(P_T, Q_T) = \Big([P_T - K]^+ \mathbf{1}_{\{Q_T > \widehat{K}\}}\Big)$

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- Hedging price : $\quad \min_{\Delta \geq 0} \mathbb{E}\left[[P_T - K - \Delta\tilde{K}]^+\right] + (1+\lambda^{01})\Delta Q_0$

with $\tilde{K} := \widehat{K}/(1+\lambda^{10})$.

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- Hedging strategy:

1. If $\mathbb{P}[P_T - K \geq 0] \leq (1+\lambda^{01})Q_0/\tilde{K}$ then $\hat{\Delta} = 0$

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$\Rightarrow$ hedges $[P_T - K]^+$: $\phi_t = \mathbb{E}\left[P_T \mathbf{1}_{\{P_T \geq K\}} \mid \mathcal{F}_t\right]/P_t$, $L = 0$.

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- Hedging strategy:

2. If $\mathbb{P}\left[P_T - K \geq 0\right] > (1 + \lambda^{01})Q_0/\tilde{K}$ then $\hat{\Delta}$ solves

$$-\tilde{K}\,\mathbb{E}\left[\mathbf{1}_{\{P_T - K \geq \Delta\tilde{K}\}}\right] + (1 + \lambda^{01})Q_0 = 0$$

Example

- $m = d = 1$, $\sigma(t,s) = \sigma \in I\!M^2$ invertible, $g^0(P_T, Q_T) = \left([P_T - K]^+ \mathbf{1}_{\{Q_T > \widehat{K}\}}\right)$

- Hedging price : $\min_{\Delta \geq 0} \mathbb{E}\left[[P_T - K - \Delta\tilde{K}]^+\right] + (1+\lambda^{01})\Delta Q_0$

with $\tilde{K} := \widehat{K}/(1+\lambda^{10})$.

- Hedging strategy:

2. If $\mathbb{P}\left[P_T - K \geq 0\right] > (1+\lambda^{01})Q_0/\tilde{K}$ then $\hat{\Delta}$ solves

$-\tilde{K}\,\mathbb{E}\left[\mathbf{1}_{\{P_T - K \geq \Delta\tilde{K}\}}\right] + (1+\lambda^{01})Q_0 = 0$

$\Rightarrow$ hedges $[P_T - K - \hat{\Delta}\tilde{K}]^+$: $\phi_t = \mathbb{E}\left[P_T \mathbf{1}_{\{P_T \geq K + \hat{\Delta}\tilde{K}\}} \mid \mathcal{F}_t\right]/P_t$, $L = \hat{\Delta}$.