Stochastic Control for Optimal Trading: State of Art and Perspectives *(an attempt of)*

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In short : I will essentially only say what can be done by using available tools from optimal control.

If you are interested, the references are given at the end of these slides.

Outline

A simple model for book liquidation : Almgren and Chriss (2001)

- Q stocks to liquidate.
- In discrete time : $t_n := n\iota_N$ with $\iota_N := T/N$.
- Price dynamics with market impact

$$S_{t_n} = S_{t_{n-1}} + \sigma \iota_N^{\frac{1}{2}} \xi_{t_n} - \iota_N g(\Delta q_{t_n}/\iota_N)$$

where

- $\Delta q_{t_n} = q_{t_n} - q_{t_{n-1}}$ is the number of stocks sold between t_{n-1} and t_n,
- $(\xi_{t_n})_n$ are iid with mean 0 and variance 1.
- g is the permanent impact function

A simple model for book liquidation

- Terminal liquidation gain with $q_{t_N} = Q$

$$V_T^q = QS_0 + \sum_{n=1}^{N} \left(\sigma \iota_N^{\frac{1}{2}} \xi_{t_n} - \iota_N g(\Delta q_{t_n}) \right) \sum_{k=n+1}^{N} \Delta q_{t_k} - \sum_{n=1}^{N} \Delta q_{t_n} h(\Delta q_{t_n}/\iota_N)$$

where h stands for the temporary impact.

- The shortfall du to volatility and market impacts is

$$QS_0 - V_T^q = \sum_{n=1}^{N} \left(-\sigma \iota_N^{\frac{1}{2}} \xi_{t_n} + \iota_N g(\Delta q_{t_n}) \right) \sum_{k=n+1}^{N} \Delta q_{t_k} + \sum_{n=1}^{N} \Delta q_{t_n} h(\Delta q_{t_n}/\iota_N)$$

Mean variance criteria and explicit resolution

- If one restricts to deterministic strategies, one can compute explicitly $\mathbb{E}\left[QS_0 - V_T^q\right]$ and $\mathrm{Var}\left[QS_0 - V_T^q\right]$ and try to solve

$$\min_{q:q_{t_N}=Q} \left(\mathbb{E}\left[QS_0 - V_T^q\right] + \lambda \mathrm{Var}\left[QS_0 - V_T^q\right]\right).$$

- In Almgren and Chriss, this is done for a specific linear setting

$$h(\Delta q_{t_n}/\iota_N) = \epsilon \mathrm{sign}(\Delta q_{t_n}) + \frac{\eta}{\iota_N}\Delta q_{t_n} \;\text{ and }\; g(\Delta q_{t_n}) = \gamma \Delta q_{t_n}$$

- Because of the dynamics and the criteria, adapted (random) strategies would provide the same result.

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- Market volume ? Intraday volatility moves ?

Possible extension

Continuous time versions of the same model :

- Forsyth et al. (2011 and 2012), and Gatheral and Schied (2011) study the continuous time setting with prices given by GBM or ABM.
- Again it is explicit : for the same reason than in discrete time.
- Both use proxies of variance (mean quadratic variation) or of VaR (expected cost + proxy of time average VaR).
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One can use general optimal control techniques to study more general settings. This is done via an impulse control approach in e.g. Kharroubi and Pham (2010).

- Less explicit but can solve pdes and find the optimal control out of it.
- More time consuming but :
  - Provides a general idea of the optimal intervention frontiers.
  - What are the important parameters.
  - Can compute *abacus* / compress the information computed off-line once for all.

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Motivation and main idea

- **Motivation :**
  - Usually follow an Almgren and Chriss type strategy and then try to optimize the real passage of order to the LOB : only focus on aggressive orders.
  - Passive orders represents most of orders passed by trading algorithms.
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- **Main idea :**
  - Propose continuously an ask quote : $S_t^a = S_t + \delta_t^a$.
  - The number of sold shares evolves according to a jump process N^a with jumps of size 1.
  - The intensity of N^a at t is $\lambda(\delta_t^a) := \lambda_0 e^{-k\delta_t^a}$.
    - If $\delta_t^a = 0$, the time to be executed is exponentially distributed with parameter λ_0.
    - The greater δ_t^a is, the longer one has to wait.

Explicit resolution

- In Guéant, Lehalle and Tapia (2010) :
  - The reference price S is an ABM.
  - The aim is to maximize an exponential utility function.
  - The HJB equation drops down to a simple system of linear ODEs.
  - Given the solution of the system of ODEs, the optimal strategy is explicit.

- In Bayraktar and Ludkovski (2012) :
  - The reference price S is just a martingale.
  - The aim is to maximize an expectation.
  - The solution is explicit.

Other works are dealing with similar but more complex models : one can only obtain HJB equations that have to be solved numerically or analyzed for small inventory expansion (e.g. Avellaneda and Stoikov 2008, and Guilbaud and Pham 2012).

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  - Once a liquidation strategy is determined, orders are usually executed by trading robots.
  - Trading robots are optimized according to a given criteria.
  - The trader chooses the robot according to market conditions.
  - The different slices are executed by different robots, with different sets of parameters.

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  - The trader chooses the robot according to market conditions.
  - The different slices are executed by different robots, with different sets of parameters.
  - How can one optimize this use given a set of parameterized robots ?

- Bouchard, Lehalle and Dang (2011) propose to write down the associated optimal control problem.

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- **Numerical resolution :**
  - This is a relatively standard impulse control problem.
  - Leads to HJB equations which can be solved numerically.
  - Can deduce an abacus on how/when/how long to launch robots within a given strategy.

Outline

Searching for the good market : a stochastic algorithm approach

• Works around Laruelle, Lehalle and Pagès : test the different markets to know which one will be the most efficient given the strategy one has in mind.

Previous talk....

Outline

An example of un-hedgeable derivative

- Guaranteed VWAP :
  - Offer a protection against execution price.
  - Payoff= [Guaranteed mean price − Effective mean price $]^+$.
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  - A perfect hedge is obviously not possible !
  - What is the price of the guarantee ?

The inverse problem point of view

- The abstract model :
  - ϕ = liquidation strategy.
  - V_T^ϕ the mean price obtained at T when following ϕ.
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- Risk minimization problem :
  - If the guarantee is sold at a unit price/share p, the terminal loss/unit is $[\kappa M_T - V_T^\phi - p]^+$
  - Fix ℓ a loss function (depending on the number of shares to liquidate).
  - One tries to minimize

$$v(p) := \min_\phi \mathbb{E}\left[\ell\left(\left[\kappa M_T^\phi - V_T^\phi - p\right]^+\right)\right].$$

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  - Given a threshold γ on the risk, compute the price

$$\hat{p}(\gamma) := \inf\{p : v(p) \leq \gamma\}.$$

A direct approach

- Assume a Markovian structure : $X^{t,x,\phi}$ drives the markets (prices, volumes, volatility, etc...), $M_T^{t,x,\phi} := M(X_T^{t,x,\phi})$ and $V^{t,x,p,\phi}$.

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- PDEs driving the evolution of $\hat{p}$ can be derived directly !
  - Avoids the numerical inversion of the previous approach.
  - Provides a dynamics for $\Gamma^{t,\gamma,\alpha}$ which amounts for the evolution of the conditional expected loss.

A direct approach

- It has been investigated within the framework of Guaranteed VWAP pricing by Bouchard and Dang (2010).

- Instead of a loss function, one can put several P&L constraints :

$$\mathbb{P}\left[V_T^{t,x,p,\phi} \geq \kappa M(X_T^{t,x,\phi}) - c_i\right] \geq q_i \text{ for } i = 1, \ldots, I,$$

- ... or more generally several constraints (see Bouchard and Vu 2012).

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- The price is now

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- Can be made time consistent as in the previous case, PDEs can be derived (see Bouchard, Moreau and Nutz 2012).

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- Example : Use as a proxy of mean-variance problem

$$\min\left\{\mathbb{E}\left[(V_T^{t,x,v,\phi} - \gamma_0)^2\right],\ \phi \text{ s.t. } \mathbb{E}\left[V_T^{t,x,v,\phi}\right] \geq \gamma_0\right\}$$

Reduction to a time consistent optimization problem with path constraint

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$\Rightarrow$ back to a standard optimization problem with state constraints :

  - Domain given by ϖ which can be pre-computed.
  - Leads to standard HJB equations with constraint : numerical resolution in general.

- First suggested in Bouchard, Elie and Imbert (2009), and then further developed by Bouchard and Nutz (2012).

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- Do more on uncertainty or models with aggressive players ?

- Optimal control techniques require to postulate some a-priori laws/distributions : more model free strategies ? (cf previous talk of G. Pagès and the next talk on Reinforcement Learning by Yuriy Nevmyvaka).

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