

# MAFRAN Days 2021

Christmas session of the Cambridge Kinetic Group ! Organized at Paris-Dauphine from December 15 to December 17, 2021, by Emeric Bouin, Jessica Guerand and Clément Mouhot.

	<i>Wednesday</i> <i>Room A701</i>	<i>Thursday</i> <i>Room A701 - A709</i>	<i>Friday</i> <i>Room A709</i>
<i>8:50-9:00</i>	<i>Welcome</i>		
<i>9:00-9:45</i>	<i>Jacques Smulevici</i>	<i>Stéphane Mischler</i>	<i>Renato Velozo</i>
<i>9:50-10:35</i>	<i>Helge Dietert</i>	<i>Kleber Carrapatoso</i>	<i>Giovanni Brigati</i>
<i>10:35-10:55</i>	<b>BREAK</b>	<b>BREAK</b>	<b>BREAK</b>
<i>10:55-11:40</i>	<i>Fatima-Ezzahra Jabiri</i>	<i>Thierry Bodineau</i>	<i>Frederic Rousset</i>
<i>11:45-12:30</i>	<i>Lucas Ertzbischoff</i>	<i>Marc Briant</i>	<i>Ludovic Cesbron</i>
<i>12:30-14:00</i>	<b>LUNCH</b>	<b>LUNCH</b>	<b>LUNCH</b>
<i>14:00-14:45</i>	<i>Daniel Han-Kwan</i>	<i>François Golse</i>	<i>Léo Bigorgne</i>
<i>14:50-15:35</i>	<i>Dominic Wynter</i>	<i>José Cañizo</i>	<i>Iván Moyano</i>
<i>15:35-15:55</i>	<b>BREAK</b>	<b>BREAK</b>	<b>BREAK</b>
<i>15:55-16:40</i>	<i>Laurent Lafleche</i>	<i>Francesca Anceschi</i>	



*Francesca Anceschi: "Mission Kolmogorov: fundamental solution protocol"*

The Kolmogorov equation is a strongly degenerate second order pde, that was firstly introduced in 1934 as a fundamental ingredient for the description of the density of a system of  $n$  particles of gas in the phase space. Later on, Hörmander considered it as a prototype for the family of hypoelliptic operators that can be written as a sum of squares. Nowadays, the research community aims to study the weak regularity theory for this class of equations, by extending to this hypoelliptic framework the results that hold true in the elliptic and parabolic setting. However, due to the strong degeneracy of the equation, various precautions are required. In this talk, we prove the existence of a fundamental solution for the Kolmogorov equation, alongside with Gaussian upper and lower bound for it.

*Léo Bigorgne: "Decay for Vlasov on black holes : the massless menace"*

A long time ago in a mathematics department far, far away, the vector field method has been developed in order to prove decay estimates for solutions to wave equations. Recently, the approach has been adapted to Vlasov equations. It is based on the geometric properties of these equations and has been used in order to study several nonlinear problem.

We will see how to adapt it to Vlasov equations modeling the evolution of spaceships travelling at the speed of light in the exterior of a Kerr black hole. Compared to the case of Minkowski spacetime, which is a flat universe, the difficulties are related to the smaller number of symmetries, the event horizon and the trapped trajectories.

*Thierry Bodineau: "Boltzmann v Kac : Dawn of Justice"*

*Esther Bou-Dagher: "All I Want For Coercive Inequalities Is U-Bounds"*

In the setting of step-two Carnot groups, we prove Poincaré and  $\beta$ -Logarithmic Sobolev inequalities for probability measures as a function of various homogeneous norms. To do that, the key idea is to obtain an intermediate inequality called the U-Bound inequality (based on joint work with B. Zegarliński). Using this U-Bound inequality, we show that certain infinite dimensional Gibbs measures- with unbounded interaction potentials as a function of homogeneous norms- on an infinite product of Carnot groups satisfy the Poincaré inequality (based on joint work with Y. Qiu, B. Zegarliński, and M. Zhang).

We also enlarge the class of measures as a function of the Carnot-Carathéodory distance that gives us the  $q$ -Logarithmic Sobolev inequality in the setting of Carnot groups. As an application, we use the Hamilton-Jacobi equation in that setting to prove the  $p$ -Talagrand inequality and hypercontractivity.

*Marc Briant: "Transporting"*

How can you stay regular when you are on a bouncing trip ? This is an ongoing joint work with Ariane Trescases and Ludovic Cesbron.

*José Canizo: "Fine asymptotic bounds of the spatial distribution of cargo vessel staff under xenomorph threats"*

*We give new asymptotic bounds of the Dirichlet heat kernel outside a hole in any dimension  $d$ , with an application to the life expectancy of staff in interplanetary vessels which unfortunately often carry unknown threats on board. Previous upper and lower bounds are known in the recent literature, and we show the result that a self-similar behavior much like the one for the usual heat equation also takes place, with explicit rates which are new. An adaptation of common entropy methods in kinetic theory is used for this, which seems not to have been considered in this context. As far as we know, the application to hazardous interplanetary working conditions also has not been previously considered.*

*Kleber Carrapatoso: "Convergin' to the Rain"*

*Ludovic Cesbron: "Ions flew over the Cuckoo's domain"*

*Helge Dietert: "The return of Bogovski  $\bar{\imath}$ "*

*Lucas Ertzbischoff: "The unbearable lightness of particles in a fluid"*

*In this talk, I will introduce the Vlasov-Navier-Stokes system, which is a fluid-kinetic model describing a cloud of particles sedimenting in an incompressible viscous fluid. I will present some recent developments about the large time behavior of global (weak) solutions to this system, when one considers the absorption of the particles at the physical boundary. I will explain how the combined effect of the absorption and the gravity (through an exit geometric condition) leads to polynomial decay in time estimates for the solutions.*

*François Golse: "When Quantum Wasserstein met Observability"*

*Daniel Han-Kwan: "Benney: ill?"*

*Fatima-Ezzahra Jabiri: "Stationary axisymmetric Einstein-Vlasov bifurcations of the Kerr spacetime"*

*In this talk, I am going to discuss a novel approach of the construction of stationary axisymmetric black hole solutions to the EV system. These solutions have the property that the spatial support of the matter is a finite, axially symmetric shell located away from the black hole. To this end, I will start by reviewing some of the progress made in the context of solutions to the Einstein-Vlasov system. Then, I will explain how the study of trapped timelike geodesics in a perturbed Kerr spacetime allowed us to provide a one-parameter family of solutions.*

*Laurent Lafleche: "Classix and Quantix : Mission singular potential."*

*It is still unknown whether one can derive the Vlasov-Poisson equation by taking the large particles limit from a system of particles interacting through the Coulomb or gravitational interactions. In collaboration*

with Chiara Saffirio and Jacky Chong, we were able to derive for the first time the Vlasov equation in the case of unbounded but square integrable singular potentials by doing a joint mean-field and the semiclassical limit from the many-body Schrödinger equation to the Vlasov equation.

The derivation is done in the case of Fermions and uses the Hartree-Fock equation as an intermediate equation. The strategy depends on the following points:

- A weak-strong stability estimate for the Vlasov equation
- Propagation of semiclassical regularity
- Semiclassical inequalities for commutators
- Bogoliubov transformations for mixed states

**Matthieu Léautaud:** "Fifty shades of control (in the vanishing viscosity limit)"

We consider a transport equation by a gradient vector field with a small viscous perturbation. We study uniform observability properties from a small subset in the (singular) vanishing viscosity limit. We prove with a series of examples that in general, the minimal time for uniform observability may be much larger than the minimal time needed for the observability of the limit equation. We also prove that the two minimal times coincide for positive solutions. This is a joint work with Camille Laurent.

**Stéphane Mischler:** "When Harris met Krein (and Rutman)"

**Iván Moyano:** "Shall I compare thee to your time derivative?"

*Abstract:* We consider kinetic Fokker-Planck (or Vlasov-Fokker-Planck) equations on the torus with Maxwellian or fat tail local equilibria. Results based on weak norms have recently been achieved by S. Armstrong and J.-C. Mourrat in the case of Maxwellian local equilibria. Using adapted Poincaré and Lions-type inequalities, we develop an explicit and constructive method for estimating the decay rate of time averages of norms of the solutions, which covers various regimes corresponding to subexponential, exponential and superexponential (including Maxwellian) local equilibria.

**Thomas Rey:** "The social network of plant and pollinators" (You don't get to 500 species without killing a few)

**Frédéric Rousset:** "Pendulum"

**Jacques Smulevici:** "Happiness is in the field"

I will give an overview of vector field methods for kinetic equations. After an introduction to these methods, I will focus on their applications to various wave-kinetic equations, in particular the Einstein-Vlasov system. This talk is based on several collaborations with Jérémie Joudioux, David Fajman, Léo Bigorgne and Maximilian Thaller.

*Renato Velozo: "Interstellar: A trip around my favourite black hole."*

*The Einstein--Vlasov system is a relevant model in the study of collisionless many particle systems in general relativity. Motivated by the black hole stability problem, I will present dispersive estimates for the linear Vlasov equation in the exterior of my favourite black hole - the celebrated Schwarzschild solution to the Einstein vacuum equations. The results rely on the precise behaviour of free-falling particles in spacetime. In particular, the estimates for derivatives of the energy momentum tensor require bounds for the tidal forces experienced by free-falling particles. I will conclude by discussing a stability result for Schwarzschild as a solution of the spherically symmetric Einstein--massless Vlasov system.*

*Dominic Wynter: "Twister: Approximation of 2D Navier Stokes by Stochastic Point Vortices"*

*We outline a new result by the author on approximation of the 2D Navier Stokes system by point vortices with Brownian noise, at an optimal rate. Previous mean field results of this type have been non quantitative or required that vortices all turn in the same direction. These results have usually proceeded by proving propagation of chaos - that is, the statement that independent initial conditions for the stochastic particle system are propagated in time under the many particle limit. We show how to adapt this method to systems of non-identical particles by introducing an artificial pairing of positive and negative vorticities and conclude our result.*