

Master 280 - Mido : 26 June 2020

Pierre Brugière ¹: Credit Risk 1h30

[12pts]Exercise 1:

1. which of these models are the more likely to be used by a financial analyst
 - (a) Altman's model
 - (b) Ohlson's model
 - (c) Intensity models
2. which of these models is the more likely to be used by a derivatives trader
 - (a) Altman's model
 - (b) Structural models
 - (c) Intensity models
3. In a structural model of default a stochastic process is used to model the evolution of
 - (a) The asset side of the balance sheet
 - (b) The liability side of the balance sheet
 - (c) Both the asset side and the liability side of the balance sheet
4. For a company with EUR 2 billion of assets and EUR 1.6 billion of liabilities and with a volatility for the assets of 20% what is the distance to default at a horizon time of one year.
 - (a) 0.8926
 - (b) 0.9234
 - (c) 1.1236
5. Which equation(s) correspond(s) to a Cox Process for τ
 - (a) $P(\tau > t) = E(\exp(-\int_0^t \lambda(s, X_s) ds))$
 - (b) $P(\tau < t) = E(\exp(-\int_0^t \lambda(s, X_s) ds))$
 - (c) $P(t < \tau < t + dt | \tau > t, \{X_s, s \in [0, t]\}) = \lambda(t, X_t) dt$
6. What are the possibilities offered by Cox processes

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- (a) to model an intensity of default depending on time
 - (b) to model an intensity of default depending on some macro economic variables
 - (c) to model a correlation of defaults between bonds
7. In an intensity model, if the intensity of default of a 1 year zero coupon bond is 1% per annum and if the bond has a recovery rate of 20% then the spread of the bond should be:
- (a) 0.2%
 - (b) 0.8%
 - (c) 1%
8. In a CDO, priced with a constant correlation ρ and probability of default p for the constituents, then when ρ decreases:
- (a) the price of the senior tranche is always increasing
 - (b) the price of the equity tranche is always increasing
 - (c) the price of the junior tranche is always increasing
 - (d) none of the above
9. In a CDO when the correlation decreases
- (a) the "Diversity Score" increases
 - (b) the "Diversity Score" decreases
 - (c) the "Diversity Score" stays the same
10. In the default model $X_i = 1 \iff Z_i < \tilde{p}$ what can be said about $\frac{1}{n} \sum_{i=1}^n X_i$ when n tends to $+\infty$
- (a) it converges in law towards $\mathcal{L}(\tilde{p})$
 - (b) it converges almost surely towards $E(\tilde{p})$
 - (c) it converges towards a normal law
11. Which of the following assertions are true. The copula of a Gaussian vector is determined in a unique way by
- (a) the matrix of correlation of its components
 - (b) the matrix of variance-covariance of its components
 - (c) the means and standard deviations of its components.
12. Let (U_1, U_2) be a random vector, which of the following random vector has the same copula as (U_1, U_2)
- (a) $(\exp(U_1), \exp(U_2))$

- (b) $(2U_1 + 1, 3U_2 - 4)$
(c) $(1 - U_1, U_2)$.

[4pts] Exercise 2 :

Let $\mathcal{E}(\mu)$ be the exponential law of parameter $\mu > 0$. Such a law is defined by $\forall t \in [0, +\infty[, P(\mathcal{E}(\mu) > t) = \exp(-\mu t)$.

Let $(X_s)_{s \in [0, +\infty[}$ be a random process and $(\mathcal{E}_i)_{i \in \llbracket 1, d \rrbracket}$ be a random variables of exponential law of parameter 1 independent of the process $(X_s)_{s \in [0, +\infty[}$ and independent between them.

Let c be a strictly positive number and λ be a continuous function such that $\forall x \in \mathbf{R}, \forall s \in \mathbf{R}^+, \lambda(s, x) > c$.

Let $(X_i)_{i \in \llbracket 1, d \rrbracket}$ be the Bernouilli random variables defined by

$$X_i = 1 \iff \mathcal{E}_i < \int_0^T \lambda(s, X_s) ds$$

1. [1pt] Calculate the expression of $P(X_i = 1)$
2. [2pt] Show that for $i \neq j$ $Cov(X_i, X_j) = Var \left[\exp \left(- \int_0^T \lambda(s, X_s) ds \right) \right]$
3. [1pt] What is sufficient and necessary condition on $\lambda(., .)$ for the X_i, X_j to have a correlation of zero when $i \neq j$. Is the correlation zero when λ is purely deterministic ?

[4pts] Exercise 3 :

Let $(X_i, Y_i)_{i \in \llbracket 1, n \rrbracket}$ be a sample of i.i.d random variables of the same law as (X, Y) which has a density.

Let $K_n = \frac{2}{n(n-1)} \sum_{i < j} sgn(X_i - X_j) sgn(Y_i - Y_j)$ where $sgn(x) = 1$ if $x > 0$ and -1 otherwise.

1. [2pts] Show that

$$E[K_n] = P((X_1 - X_2)(Y_1 - Y_2) > 0) - P((X_1 - X_2)(Y_1 - Y_2) < 0)$$

2. [2pts] Show that $Var[K_n]$ tends to zero when n tends to ∞ .