Portfolio Management

TD6. Factor Models and APT

We consider N risky assets $S^1, \dots, S^N$ and a risk-free asset S^0 of return r^0.
We note R^i the return of the risky asset i over $[0, T]$ and $\pi_N \in \mathbb{R}^N$ a vector whose component π^i corresponds to the $\%$ of the initial wealth invested in the risky asset S^i.
We consider a K-factors model : $R^i = a^i + b_1^i f^1 + \cdots + b_K^i f^K + \varepsilon^i$
with $\mathbf{E}[f^i] = 0$, $\mathbf{E}[\varepsilon^i] = 0$, $cov(\varepsilon^i, f^j) = 0$ and $cov(\varepsilon^i, \varepsilon^j) = 0$ for $i \neq j$
We also note matricially the factor model as : $R = A + BF + E$

Exercise 1 Let π_N be the investment in the risky assets of an investment portfolio
1. Express the investment in the risk-free asset π_N^0 as a function of π_N
2. Express R_{π_N} as a function of π_N, A, B, F, E and r^0
3. Express $\mathbf{E}[R_{\pi_N}]$ as a function of π_N, A, and r^0
4. Show that we can write $R_{\pi_N} = a^{\pi_N} + b_1^{\pi_N} f^1 + \cdots + b_K^{\pi_N} f^K + \varepsilon^{\pi_N}$
5. We assume that the $\varepsilon^i$are i.i.d of variance σ^2 and that $\pi_N^i = \frac{1}{N}$. Shows that ε^{π_N} tends to 0 when $N \to \infty$

Exercise 2 We assume here that $K = 2$, and that there are three assets with the following characteristics :

Assets i	$\mathbf{E}[R^i]$	b_1^i	b_2^i
$i = A$	$12,0\%$	1	$0,5$
$i = B$	$13,4\%$	3	$0,2$
$i = C$	$12,0\%$	3	$-0,5$

1. Show that the fundamental relation of APT is satisfied
2. What is the expected return of asset D such that $b_1^D = 1$ et $b_2^D = -1$?
3. We consider an asset I such that $b_1^I = 1$ and $b_2^I = 2$. Build an investment portfolio π with assets A, B and C having the same sensitivities to the factors as I.
4. what do you find for the expected return of asset I based on its factor decomposition and on the portfolio replicating it ?

Exercise 3 We assume that $K = 2$, and that there are two risky assets satisfying :

Asset i	$\mathbf{E}[R^i]$	b_1^i	b_2^i
$i = A$	16%	$1,2$	$0,4$
$i = B$	26%	$0,8$	1.6

1. A third asset C satisfies $\mathbf{E}[R^C] = 12\%$ with $b_1^C = 1$ et $b_2^C = 0,5$. is this possible if the fundamental theorem of APT is satisfied ?