Viscosity solutions for time-dependent pb

$$(HJ)\ \begin{cases} -\partial_t u(t,x) + H(t,x,\nabla u(t,x), D^2 u(t,x)) = 0 \\ \qquad \text{in } [0,T) \times \mathbb{R}^d \\ u(T,x) = g(x) \text{ in } \mathbb{R}^d. \end{cases}$$

Def: We say that a continuous map $u : [0,T] \times \mathbb{R}^d \to \mathbb{R}$ is a visco. subsolution (supersol) of (HJ) if (i) for any test function $\varphi \in C^{1,2}$ such that $u - \varphi$ has a maximum (min) at $(t,x) \in [0,T) \times \mathbb{R}^d$ one has $-\partial_t \varphi(t,x) + H(t,x,\nabla\varphi(t,x), D^2\varphi(t,x)) \leq 0$. ($\geq 0$)

(ii) $u(T,x) \leq g(x)$ ($\geq$) $\forall x \in \mathbb{R}^d$.

- Sol = sub + super sol.

Comparison principle:

- Assume $H(t,x,p,A) =$

$$\sup_{a \in A} \left\{ -f(t,x,a) - b(t,x,a)\cdot p - \frac{1}{2} \mathrm{Tr}(\sigma\sigma^T(t,x,a) A) \right\}$$

where f, b, σ satisfy our standing assumptions and that g is Lipschitz and bd. If u & v are uniformly continuous and bd, u sub sol and v super sol

L $\quad u \leq v$ in $[0,T] \times \mathbb{R}^d$.