Stochastic Differential Equations with Normal Constraints in Law and associated PDEs

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In this talk, we are concerned with reflected stochastic differential equations in the case where the constraint is on the law of the solution rather than on its paths. Such equation is motivated by the super-hedging of claims under running risk management constraint.

More precisely, we consider the following reflected SDE:

$$\begin{cases} X_t = X_0 + \int_0^t b(s, X_s)\, ds + \int_0^t \sigma(s, X_s)\, dB_s + \int_0^t \mathrm{D}_\mu H([X_s])(X_s)\, dK_s, \quad t \geq 0, \\ H([X_t]) \geq 0, \quad \int_0^t H([X_s])\, dK_s = 0, \quad t \geq 0, \end{cases} \tag{1}$$

where H is a map from $\mathcal{P}(\mathbb{R}^n)$ to $\mathbb{R}$.

We first study the existence and uniqueness of solution to such equation. Then we study the approximation of the solution by a particle system, i.e. the mean-field limit. Finally, we describe the PDE associated with the reflected process. In fact, we can show that the reflected process X is linked with the following PDE with Neumann boundary condition:

$$\begin{cases} (i) \quad -\partial_t U(t,\mu) - \frac{1}{2}\int_{\mathbb{R}^n} \mathrm{Tr}\left(a(t,y)\partial_y \mathrm{D}_\mu U(t,\mu)(y)\right)\mu(dy) - \int_{\mathbb{R}^n} \mathrm{D}_\mu U(t,\mu)(y)\cdot b(t,y)\mu(dy) = 0 \\ \qquad\qquad \text{in } (0,T)\times\mathcal{O}, \\ (ii) \quad \int_{\mathbb{R}^n} \mathrm{D}_\mu U(t,\mu)(y)\cdot \mathrm{D}_\mu H(\mu)(y)\mu(dy) = 0 \text{ in } (0,T)\times\partial\mathcal{O}, \\ (iii)\ U(T,\mu) = G(\mu) \text{ in } \mathcal{O}. \end{cases} \tag{2}$$

where $a_i = \sigma_i \sigma_i^T$.

This is a joint work in progress with Philippe Briand, Pierre Cardaliaguet and Paul-Éric Chaudru de Raynal.