ANR MEAN FIELD GAMES

March 15—16, 2018 — Tours

PRACTICAL INFORMATION

BUS — To get to the university from/to the center of Tours, use **bus number 2** (also called *tempo*). You have to buy a card from the driver (cardboard card, 0.10 euros), then the drive can charge two trips (round trip) for 2.80 euros. The next day you have to charge another time two trips on the same card.

Bus stop on the campus is **Faculté de Grandmont**.

You can find a map of bus 2 stops and timetable here: `http://www.math.univ-tours.fr/bus2.pdf`

ROOM — All the conferences and coffee breaks will take place in **building E1 - room 2100** (second floor, turn right). Note: the maths department is in building E2, which is connected to building E1 (see campus map below).

WIFI — There are two wifi networks available: (*i*) eduroam ; (*ii*) eduspot (french universities only). There is no special wifi for the conference, if you can't use one of these wifi networks, let us know and we will find a solution for you.

LUNCH AND DINNER — We will have the lunch breaks on the campus. The dinner will take place at the restaurant «*O petit Paris*», 5 rue Marceau, Tours. We will give you more information during the conference.

SCHEDULE & ABSTRACTS

THURSDAY, MARCH 15

09:30-10:00 *Welcome with coffee*

10:00-10:15 *Opening*

- 10:15-11:00 Alessio Porretta — *Long time behviour for mean-field games*
- 11:00-11:45 Annalisa Cesaroni — *Concentration phenomena for Mean Field Games systems with aggregation*
- 11:45-12:00 Charles Bertucci — *Optimal stopping and impulse control in MFG*

12:00-14:00 *Lunch Break*

- 14:00-14:45 Marco Cirant — *Oscillating solutions in non-monotone Mean-Field Games*
- 14:45-15:30 Filippo Satambrogio — *Minimal time MFG*
- 15:30-15:45 Guilherme Mazanti — *Some examples of minimal time MFGs*

15:45-16:15 *Coffee Break*

- 16:15-17:00 Ying Hu — *Stochastic Differential Equations with Normal Constraints in Law and associated PDEs*
- 17:00-17:15 Marco Masoero — *Weak KAM theory for potential mean field games*
- 17:15-17:30 Adriano Festa — *MFG and Traffic: a multilane management problem*

FRIDAY, MARCH 16

- 09:30-09:45 Manh Khang Dao — *Second order Mean field games on networks*
- 09:45-10:30 Cyril Imbert — *Effective boundary conditions for Hamilton-Jacobi equations*

10:30-11:00 *Coffee Break*

- 11:00-11:45 Nicolas Forcadel — *Justification of macroscopic traffic flow model by specified homogenization of microscopic models*
- 11:45-12:00 Jessica Guérand — *Effective boundary conditions for Hamilton-Jacobi equations*

12:00-13:45 *Lunch Break*

- 13:45-14:30 Francisco Silva — *Finite mean field games: fictitious play and convergence analysis*
- 14:30-15:15 François Delarue — *CLT and LDP for mean field games*

Alessio Porretta — Rome Tor Vergata

Long time behavior of mean field games

In this talk, based on my collaborations with P. Cardaliaguet, J-M. Lasry and P.-L. Lions, I will discuss the long time behavior of mean field games systems in the stable case(monotone couplings) when the dynamic takes place in the flat torus. I will explain the main features that appeared in the study of the long time limit: the effects of the forward-backward coupling, the ergodic behavior and the turnpike property of the underlying control problems, the long time convergence of the master equation. This latter step is crucial in order to fully characterize the limit of the value function compared to what happens for a single Hamilton-Jacobi equation

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Analisa Cesaroni — Padova

Concentration phenomena for Mean Field Games systems with aggregation

I will present some results about ergodic mean field game systems in the whole space $\mathbb{R}^n$, without periodicity conditions, describing equilibrium configurations for a population of rational individuals subjected to a coercive potential and an aggregation force. I will describe the variational framework for the analysis of these problems, and discuss concentration phenomena arising in the semiclassical limit. These results are obtained in collaboration with Marco Cirant (Padova).

Charles Bertucci — Paris Dauphine

Optimal stopping and impulse control in MFG

We present here the forward-backward models which describe the Nash equilibria associated with MFG where players face an optimal stopping or impulse control problem. The main issue here is to characterize the density of players, as the classical Fokker-Planck approach collapse in this case. We also present assumptions under which there exists a unique Nash equilibrium of the MFG.

Marco Cirant — Padova

Oscillating solutions in non-monotone Mean-Field Games

For non-monotone time-dependent Mean-Field Game systems, we will discuss the existence of solutions that exhibit an oscillatory behaviour in time. We will show how local and global bifurcation methods lead to the existence of such solutions, with a focus on the analysis of the space-time Fourier coefficients of the linearized problem. We will show some numerical experiments and discuss further possible generalisations in the direction of truly time-periodic solutions.

Filippo Santambrogio — Paris-Sud

Minimal-Time MFG

I will present a new class of MFG that we have recently been studying with G. Mazanti, S. Dweik, and C. Jimenez. They consist in the following problem : suppose that agents want to reach a certain target (for simplicity, let's say they want to exit a given domain Ω, the target being $\partial\Omega$), but their speed at each time t is bounded by a quantity K(t,x) which depends on their position x and on the global distribution of players around x at time t. The typical case should be $|x'(t)| \leq 1 - \rho(t, x(t))$ where ρ is the density of players, as it happens in the Hughes model for pedestrian motion, the difference with such a model being the rational behavior of the agents which anticipate that ρ will evolve in time, considering that each other agent wilh also anticipate it. This local case being of course very difficult, most of our results are devoted to a non-local case involving a convolution. Many nontrivial questions also arise when studying a minimal-time control problem with fixed $K(t, x)$, as soon as it is non-autonomous, in particular if one needs to find the optimal assumptions on the regularity of K in t and x separately.

In the talk I'll present the model, the main questions and difficulties, and the main results we obtained in different settings. Most of the work is ongoing.

This talk is complemented by the talk by G. Mazanti, who will present in more details some interesting examples, with numerical simulations.

Guilherme Mazanti — Paris Sud

Some examples of minimal time MFGs

This talk considers a MFG model motivated by crowd motion where each agent wants to reach the boundary of a given bounded domain in minimal time. Each agent is free to move in any direction, but their maximal speed is assumed to be bounded in terms of the distribution of other agents in order to take into account congestion phenomena. After briefly recalling the setting and some of the main results for this MFG presented in Filippo Santambrogio's talk, I will present simulations in three examples highlighting some interesting characteristics of our model.

In the first case, we consider that the domain is a segment and agents are initially concentrated in a single point, the main goal being to understand the behavior of the equilibrium, and in particular the exit time of the agents, as the initial position is changed.

The second example concerns the case where agents evolve in a simple graph and shows that one may obtain a Braess-type paradox in this setting. We finally consider a MFG in the two-dimensional Euclidean ball with all agents starting from the same point, showing how one can simulate the evolution of the distribution of agents in this case.

This talk is based on an ongoing work with Filippo Santambrogio.

Ying Hu — Rennes

Stochastic Differential Equations with Normal Constraints in Law and associated PDEs

In this talk, we are concerned with reflected stochastic differential equations in the case where the constraint is on the law of the solution rather than on its paths. Such equation is motivated by the super-hedging of claims under running risk management constraint. More precisely, we consider the following reflected SDE:

$$\begin{cases} X_t = X_0 + \int_0^t b(s, X_s)\, ds + \int_0^t \sigma(s, X_s)\, dB_s + \int_0^t \mathrm{D}_\mu H([X_s])(X_s)\, dK_s, \quad t \geq 0, \\ H([X_t]) \geq 0, \quad \int_0^t H([X_s])\, dK_s = 0, \quad t \geq 0, \end{cases} \tag{1}$$

where H is a map from $\mathcal{P}(\mathbb{R}^n)$ to $\mathbb{R}$.

We first study the existence and uniqueness of solution to such equation. Then we study the approximation of the solution by a particle system, i.e. the mean-field limit. Finally, we describe the PDE associated with the reflected process. In fact, we can show that the reflected process X is linked with the following PDE with Neumann boundary condition:

$$\begin{cases} (i) \quad -\partial_t U(t,\mu) - \dfrac{1}{2}\displaystyle\int_{\mathbb{R}^n} \mathrm{Tr}\left(a(t,y)\partial_y \mathrm{D}_\mu U(t,\mu)(y)\right)\mu(dy) - \int_{\mathbb{R}^n} \mathrm{D}_\mu U(t,\mu)(y)\cdot b(t,y)\mu(dy) = 0 \\ \qquad\qquad\qquad \text{in } (0,T)\times\mathcal{O}, \\ (ii) \quad \displaystyle\int_{\mathbb{R}^n} \mathrm{D}_\mu U(t,\mu)(y)\cdot \mathrm{D}_\mu H(\mu)(y)\mu(dy) = 0 \text{ in } (0,T)\times\partial\mathcal{O}, \\ (iii) \; U(T,\mu) = G(\mu) \text{ in } \mathcal{O}. \end{cases} \tag{2}$$

where $a_i = \sigma_i \sigma_i^T$.

This is a joint work in progress with Philippe Briand, Pierre Cardaliaguet and Paul-Éric Chaudru de Raynal.

Marco Masoero — Dauphine

Weak KAM theory for potential mean field games

We study the long time behavior of second order potential mean field games. For this we use tools coming from weak KAM theory. We show that, in general, the limit is not described by the so-called ergodic mean field game system.

Adriano Festa — INSA Rouen

MFG and Traffic: a multilane management problem

In this talk we discuss an Mean Field Games approach to traffic management on multi-lane roads. Such approach is particularly indicated to model self driven vehicles with perfect information of the domain. The mathematical interest of the problem is related to the fact that the system of partial differential equations obtained in this case is not in the classic form, but it consists of two continuity equations (one for each lane) and a variational inequality, coming from the Hamilton-Jacobi theory of the hybrid control.

Manh Khang Dao — Rennes

Second order Mean field games on networks

We consider stationary and non-stationary mean field game systems constrained on a network following the previous work of Camilli and Marchi (2016). At the cross point, there are some special conditions such as Kirchhoff conditions for the value function and conservation flux and "jump" conditions for the density of states. We prove the well- posedness for each of the two equations composing the system in the class of Sobolev solutions and then establish the regularity. The last step is to prove existence and uniqueness for the mean field games system.

Cyril Imbert — ENS

Effective boundary conditions for Hamilton-Jacobi equations

It has been known for a long time how to handle boundary conditions for Hamilton-Jacobi equations, even if they can be lost or in competition with the equation (in particular if both the equation and the boundary condition are of order 1). The viscosity solution theory enables one to address this problem by imposing the boundary condition in a weak sense, the so-called viscosity sense. Solutions are then stable and existence follows at once by Perron's method. It is next important to study (possibly strong) uniqueness. We will discuss other questions:
1. What are the boundary conditions that are satisfied in a strong sense, and not in the viscosity sense?
2. If a boundary condition is satisfied in a weak sense, is there a (so-called effective) boundary condition satisfied in a strong sense?

The talk is based on joint works with R. Monneau and V. Nguyen.

Nicolas Forcadel — INSA Rouen

Justification of macroscopic traffic flow model by specified homogenization of microscopic models

The goal of this talk is to present and to justify Hamilton-Jacobi formulation for macroscopic traffic flow model. The idea is to show how it is possible to deduce macroscopic models of traffic flow from microscopic ones. The main advantage of microscopic models (in which we describe the dynamics of each vehicle in an individual way) is that one can easily distinguish each vehicle and then associate different attributes (like maximal velocity, maximal acceleration...) to each vehicle. It is also possible to describe microscopic phenomena like red lights, slowdown or change of the maximal velocity. The main drawback is for numerical simulations where we have to treat a large number of data, which can be very expensive for example if we want to simulate the traffic at the scale of a town. On the contrary, macroscopic models consist in describing the collective behaviour of the vehicles for example by giving an evolution law on the density of vehicles. The oldest macroscopic model is the LWR model (Lighthill, Whitham [LW55], Richards [R56]), which dates back to 1955 and is inspired by the laws of fluid dynamics. More recently, some macroscopic models propose to describe the flow of vehicles in terms of the averaged spacing between the vehicles (in some sense, the inverse of the density, see the works of Leclercq, Laval and Chevallier [LLC07]). The main advantage of these macroscopic models is that it is possible to make numerical simulations on large portion of road. On the other side, it is more complicated to describe microscopic phenomena or attributes. Generally speaking, microscopic models are considered more justifiable because the behaviour of every vehicle can be described with high precision and it is immediately clear which kind of interactions are considered. On the contrary, macroscopic models are based on assumptions that are hardly correct or at least verifiable. As a consequence, it is often desirable establishing a connection between microscopic and macroscopic models so to justify and validate the latter on the basis of the verifiable modelling assumptions of the former. The goal of this talk is to show how to pass from microscopic models to macroscopic ones. As we will explain, this problem can be seen as an homogenization result on a non-local Hamilton-Jacobi equation. More precisely, at the microscopic scale, we will consider a first order model of the type « follow the leader », i.e., the velocity of a vehicle depends only on the distance with the one in front of it and we will consider a local perturbation located at the origin which make slow down the vehicles. At the macroscopic scale, we attend to recover an Hamilton-Jacobi equation on the right and on the left of the origin and a condition of junction at the origin (as studied in the work of Imbert and Monneau [IM15]). This junction condition allows us to see the influence of the microscopic perturbation at the macroscopic scale. We will also consider the case of a simple junction, i.e., one road that separates in several roads.

This is joint works with W. Salazar and M. Zaydan

Jessica Guérand — ENS

Effective boundary conditions for Hamilton-Jacobi equations

In this presentation, we introduce a result about dynamic boundary conditions for Hamilton-Jacobi equations of first order. It is well-known that stability and existence of viscosity solutions are assured when the boundary condition is relaxed (i.e. at the boundary, either the boundary condition or the equation in the interior of the domain is satisfied). Here, we will exhibit all the boundary conditions for which stability and existence of solutions are valid in the strong sense (i.e. at the boundary, only the boundary condition is satisfied). Moreover, considering the equivalence relation « to have the same solutions », we will see that many boundary conditions are in the same equivalence class. Also in each class there exists a unique boundary condition satisfied in the strong sense. We will explain the relation between a general boundary condition and the one in its class which is satisfied in the strong sense.

Francisco Silva — Limoges

Finite mean field games: fictitious play and convergence analysis

In this talk, based on an ongoing work with S. Hadikhanloo (U. Paris-Dauphine), we consider a class of finite state and discrete time Mean Field Games (MFGs) introduced by Gomes, Mohr and Rigao Souza in 2009. In this framework we first study an adaptation of the fictitious play procedure for continuous MFGs, introduced recently by Cardaliaguet and Hadikhanloo, and we prove the convergence to the solution of the finite MFG. In the second part of the talk, we consider a first order continuous MFG and an associated family of finite MFGs, parameterized by a finite time and space grid. We prove that, as the time and space steps tend to 0, the solutions of the finite MFGs converge to a solution of the continuous one.

François Delarue — Nice

CLT and LDP for mean field games

We establish central limit theorems (CLT) and large deviation principles (LDP) for mean field games, when the so-called master equation associated with the mean field game admits a sufficiently smooth solution. The key idea is to use the solution to the master equation to construct an associated McKean-Vlasov interacting n-particle system that is sufficiently close to the Nash equilibrium dynamics of the n-player game for large n, and then deduce scaling limit theorems for the former from scaling limit theorems for the latter.

Joint work with D. Lacker (Columbia) and K. Ramanan (Brown)

LIST OF PARTICIPANTS

Yves Achdou (U. Paris Diderot)
Guy Barles (U. Tours)
Charles Bertucci (U. Paris-Dauphine)
Adrien Blanchet (U. Toulouse 1)
Pierre Cardaliaguet (U. Paris-Dauphine)
Annalisa Cesaroni (U. Padova)
Jean-François Chassagneux (U. Paris Diderot)
Emmanuel Chasseigne (U. Tours)
Adina Ciomaga (U. Paris Diderot)
Marco Cirant (U. Padova)
Manh-Khang Dao (U. Rennes)
François Delarue (U. Nice)
Olav Ersland (NTNU Trondheim)
Adriano Festa (INSA Rouen)
Markus Fischer (U. Padova)
Nicolas Forcadel (INSA Rouen)
Christine Georgelin (U. Tours)
Alessandro Goffi (U. Padova)
Christine Grün (U. Toulouse 1)
Jessica Guerand (ENS)
Saeed Hadikhanloo (Polytechnique)
Ying Hu (U. Rennes 1)
Cyril Imbert (ENS)
Ziad Kobeissi (U. Paris Diderot)
Olivier Ley (INSA Rennes)
Sten Madec (U. Tours)
Florent Malrieu (U. Tours)
Marco Masoero (U. Paris-Dauphine)
Guilherme Mazanti (U. Paris Sud)
Ziad Qriouet (U. Paris 12 Marne la Vallée)
Vincent Perrollaz (U. Tours)
Alessio Porretta (U. Roma Tor Vergata)
Catherine Rainer (U. Brest)
Michele Ricciardi (U. Roma Tor Vergata)
Francesco Salvarani (U. Pavia and U. Paris-Dauphine)
Etienne Sandier (U. Paris 12 Marne-la-Vallée)
Filippo Santambrogio (U. Paris Sud)
Francisco Silva (U. Limoges)
Agathe Soret (U. Colombia)
Daniela Tonon (U. Paris-Dauphine)