Geodesic Methods in Biomedical Image Analysis

Laurent D. COHEN
Directeur de Recherche CNRS
CEREMADE, UMR CNRS 7534
Université Paris-9 Dauphine
Place du Maréchal de Lattre de Tassigny
75016 Paris, France
Cohen@ceremade.dauphine.fr
www.ceremade.dauphine.fr/~cohen

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Geodesic Minimal Paths

- Minimal paths, Eikonal Equation, Fast Marching and Front Propagation
- 3D Fast Marching, some examples
- Anisotropic Fast Marching
- Finsler Metrics for Various Active Contours Energy terms
- Closed Contour as a set of minimal paths. Key points method
- Geodesic Voting and tree structure segmentation
- Application to Virtual Endoscopy and Vessel Visualization

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Active Contours

- Energy Minimization:

$$\int_\Omega w_1\|\mathcal{C}'(s)\|^2 + w_2\|\mathcal{C}''(s)\|^2 + P(\mathcal{C}(s))ds$$

- *C(s)=(x(s),y(s))* curve drawn on the image
- Smoothing terms : length and curvature penalization
- Trapped in local minima
- Geodesic Approach removed the second term

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Active Contours limitation

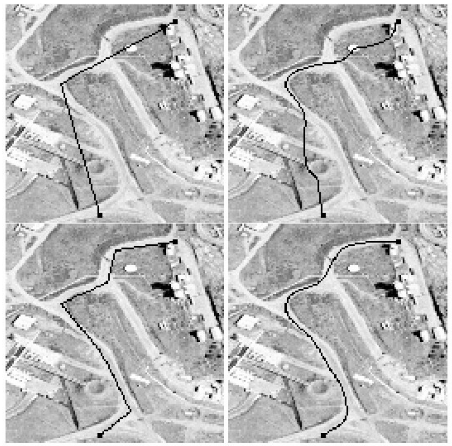

FIG. 10: **Snake Initialization** classical snakes need a very close initialization to avoid a Local Minimum.

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Minimal Paths: Eikonal Equation

simplified formulation for active contour model energy

$$E(C)=\int_0^L \{w+P(C(s))\}=\int_0^L \tilde{P}(C(s))ds$$

Potential P>0 takes lower values near interesting features :
on contours, dark structures, ...
w is a regularization parameter

Start point $C(0)=p1;$

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Paths of minimal energy

Looking for a path along which a feature Potential P(x,y) is minimal

$$E(C)=\int_0^L P(C(s))ds$$

example: a vessel
dark structure
P =gray level

Input : Start point *p1=(x1,y1)*

End point *p2 =(x2,y2)*

Image

Output: Minimal Path

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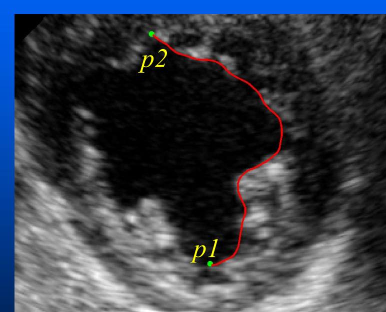
Paths of minimal energy



Looking for a path along which a feature Potential P(x,y) is minimal

$$E(C)=\int_0^L P(C(s))ds$$

example: cardiac ventricle contour
P =gradient based

Input : Start point *p1=(x1,y1)*

End point *p2 =(x2,y2)*

Image

Output: Minimal Path

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Minimal Paths: Eikonal Equation

$$E(C)=\int_0^L P(C(s))ds$$

Potential P>0 takes lower values near interesting features : on contours, dark structures, ...

STEP 1 : search for the surface of minimal action U of $p1$ as the minimal energy integrated along a path between start point $p1$ and any point p in the image

Start point $C(0)=p1;$

$$U_{p1}(p)=\inf_{C(0)=p1;C(L)=p} E(C)=\inf_{C(0)=p1;C(L)=p}\int_0^L P(C(s))ds$$

STEP 2: Back-propagation from the end point $p2$ to the start point $p1$:

Simple Gradient Descent along U_{p1}

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Minimal Paths: Eikonal Equation

STEP 1 : minimal action U of $p1$ as the minimal energy integrated along a path between start point $p1$ and any point p in the image

Start point $C(0)=p1;$

$$U_{p1}(p)=\inf_{C(0)=p1;C(L)=p} E(C)=\inf_{C(0)=p1;C(L)=p}\int_0^L P(C(s))ds$$

$$\|\nabla U_{p1}(x)\| = P(x) \text{ and } U_{p1}(p1)=0$$

Example P=1, U Euclidean distance to p1
in general, U weighted geodesic distance to p1

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Minimal Paths: back propagation

$$E(C)=\int_0^L P(C(s))ds$$

STEP 2: Back-propagation from the end point $p2$ to the start point $p1$:

Simple Gradient Descent along U_{p1}

$$\frac{dC}{ds}(s) = -\nabla U_{p_1}(C(s)) \text{ with } C(0)=p_2.$$

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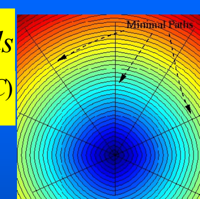
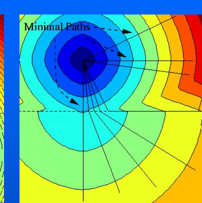
Minimal paths - 2D synthetic examples

$$E(C)=\int_0^L \tilde{P}(C(s))ds$$

$$U_{p1}(p)=\inf_{C(0)=p1;C(L)=p} E(C)$$

P=c





P1

slower

P2 < P1

faster

Examples of shortest paths on univalued or bivalued potential

Fermat Principle in Geometric Optics :
Path followed by light minimizes time

$$T = \frac{1}{c}\int_{p_0}^{p_1} n ds$$

where n>1 is refraction index v=c/n

Snell-Descartes 'law

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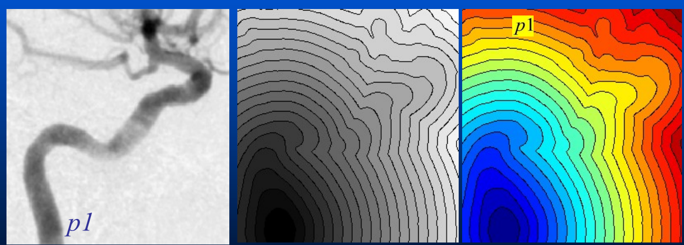
Minimal Paths and Front Propagation

Minimal Action $U_{p1}(p) = \inf_{C(0)=p1;C(L)=p} E(C) = \inf_{C(0)=p1;C(L)=p} \int_0^L P(C(s))ds$

Front Propagation $\mathcal{L}(t) = \{ p \in R^2 / U_{p0}(p) = t \}$



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Minimal Paths and Front Propagation

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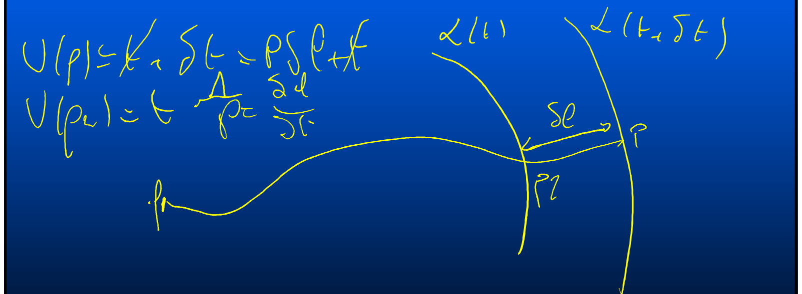

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Minimal Paths and Front Propagation

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Front Propagation $\mathcal{L}(t) = \{ p \in R^2 / U_{p1}(p) = t \}$

Evolution of t level set of U from p0

$$\frac{\partial \mathcal{L}(\sigma,t)}{\partial t} = \frac{1}{P(\mathcal{L}(\sigma,t))} \vec{n}(\sigma,t)$$

n normal vector to a level set of U is in the direction of the Gradient of U, implies Eikonal Equation :

$$\|\nabla U_{p1}(x)\| = P(x) \text{ and } U_{p1}(p1) = 0$$

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Minimal Paths and Front Propagation

Minimal Action $U_{p1}(p) = \inf_{C(0)=p1;C(L)=p} E(C) = \inf_{C(0)=p1;C(L)=p} \int_0^L P(C(s))ds$

Front Propagation $\mathcal{L}(t) = \{ p \in R^2 / U_{p1}(p) = t \}$

Evolution of t level set of U from p0

$$\frac{\partial \mathcal{L}(\sigma,t)}{\partial t} = \frac{1}{P(\mathcal{L}(\sigma,t))} \vec{n}(\sigma,t)$$

$U(\mathcal{L}(\sigma,t)) = t$ derivative w.r.t. σ and t

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Minimal Paths: Eikonal Equation

STEP 1 : minimal action U of $p1$ as the minimal energy integrated along a path between start point $p1$ and any point p in the image

Start point $C(0)=p1$;

$$U_{p1}(p)=\inf_{C(0)=p1;C(L)=p} E(C)=\inf_{C(0)=p1;C(L)=p}\int_0^L P(C(s))ds$$

$$\left\|\nabla U_{p1}(x)\right\|=P(x) \text{ and } U_{p1}(p1)=0$$

Example P=1, U Euclidean distance to p1
in general, U weighted geodesic distance to p1

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Minimal Paths: back propagation

$$E(C)=\int_0^L P(C(s))ds$$

STEP 2: Back-propagation from the end point $p2$ to the start point $p1$:
Simple Gradient Descent along U_{p1}

$$\frac{dC}{ds}(s)=-\nabla U_{p_1}(C(s)) \text{ with } C(0)=p_2.$$

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Minimal Paths: back propagation

Define C_1 by back propagation and C(s) by reverse:

$$C_1(0)=p1, C_1(L)=p0, C_1'(s)=-\frac{\nabla U_{p0}}{\left\|\nabla U_{p0}\right\|}, C(s)=C_1(L-s)$$

$$U_{p0}(p1)=\min_{\gamma(0)=p0;\gamma(L)=p1}\int_0^L P(\gamma(s))ds$$

$$U_{p0}(p1)=U_{p0}(p1)-U_{p0}(p0)=U_{p0}(C(L))-U_{p0}(C(0))$$

$$U_{p0}(p1)=\int_0^L [U_{p0}(C(s))]'\,ds=\int_0^L \nabla U_{p0}(C(s)).C'(s)ds$$

$$U_{p0}(p1)=\int_0^L \nabla U_{p0}(C(s)).\frac{\nabla U_{p0}}{\left\|\nabla U_{p0}\right\|}ds$$

$$U_{p0}(p1)=\int_0^L \left\|\nabla U_{p0}(C(s))\right\|ds=\int_0^L P(C(s))ds$$

Thus C reaches the minimum of the energy, it is a minimal path.

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Eikonal Equation- Sequential Approach

$\mathcal{U}(p_0)=0$

U steady state of $\frac{\partial \mathcal{U}}{\partial \tau}=\tilde{P}-\|\nabla \mathcal{U}\|$,

$U=\mathcal{U}_\infty$ satisfies $\|\nabla U\|=\tilde{P}$,

Iterative Sequential Scheme : $U_{i,j}$ given by

$$(\max\{u-U_{i-1,j},u-U_{i+1,j},0\})^2+(\max\{u-U_{i,j-1},u-U_{i,j+1},0\})^2=P_{i,j}^2,$$

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FAST MARCHING in 2D:
very efficient algorithm O(NlogN) for Eikonal Equation

Introduced by Sethian / Tsistsiklis

Numerical approximation of U(xij) as the solution to the discretized problem with upwind finite difference scheme

$$\|\nabla U\| = \tilde{P}$$

$$\max\left(u - U(x_{i-1,j}), u - U(x_{i+1,j}), 0\right)^2 + \max\left(u - U(x_{i,j-1}), u - U(x_{i,j+1}), 0\right)^2 = h^2 \tilde{P}(x_{i,j})^2$$

This 2nd order equation induces that :
action U at {i,j} depends only of the neighbors that have lower actions.

Fast marching introduces order in the selection of the grid points for solving this numerical scheme.

Starting from the initial point p1 with U = 0,
the action computed at each point visited can only grow.

Level sets of U can be seen as a Front propagation outwards.

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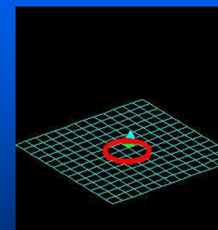
Fast Marching



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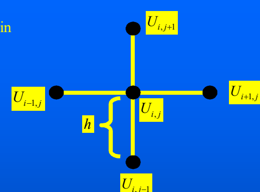
Implementation: 2 Steps for building γ_{min}

➢ Step 1: Solve the eikonal equation

$$\|\nabla U_{p1}(x)\| = P(x) \text{ and } U_{p1}(p1) = 0$$

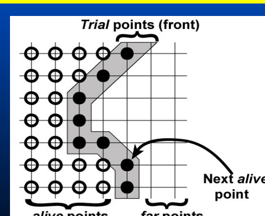


Upwind scheme

$$\max\left(\frac{U_{i,j}-U_{i-1,j}}{h}, \frac{U_{i,j}-U_{i+1,j}}{h}, 0\right)^2 + \max\left(\frac{U_{i,j}-U_{i,j-1}}{h}, \frac{U_{i,j}-U_{i,j+1}}{h}, 0\right)^2 = P_{ij}^2$$

Fast algorithm to compute the **action map** on the discretization grid :

- *Sethian* Fast Marching: N.log(N) complexity, first order.
- *Kim* Group Marching: N complexity, first order .
- Sweeping (iterative) methods



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Algorithm for 2D Fast Marching

– Definitions :
  – *Alive* set : all grid points at which the action value $\mathcal{U}$ has been reached and will not be changed ;
  – *Trial* set : next grid points (4-connexity neighbors) to be examined. An estimate U of $\mathcal{U}$ has been computed using Equation (4) from alive points only (i.e. from $\mathcal{U}$) :

$$(\max\{u - \mathcal{U}_{i-1,j}, u - \mathcal{U}_{i+1,j}, 0\})^2 + (\max\{u - \mathcal{U}_{i,j-1}, u - \mathcal{U}_{i,j+1}, 0\})^2 = \tilde{P}_{i,j}^2 \qquad (4)$$

  – *Far* set : all other grid points, there is not yet an estimate for U ;
– Initialization :
  – *Alive* set is confined to the starting point p_0, with $\mathcal{U}(p_0) = 0$;
  – *Trial* is confined to the four neighbors p of p_0 with initial value $U(p) = \tilde{P}(p)$ ($\mathcal{U}(p) = \infty$) ;
  – *Far* is the set of all other grid points with $\mathcal{U} = U = \infty$;
– Loop :
  – Let $p = (i_{min}, j_{min})$ be the *Trial* point with the smallest action U ;
  – Move it from the *Trial* to the *Alive* set (i.e. $\mathcal{U}(p) = U_{i_{min}, j_{min}}$ is frozen) ;
  – For each neighbor (i, j) (4-connexity in 2D) of (i_{min}, j_{min}) :
    – If (i, j) is *Far*, add it to the *Trial* set and compute $U_{i,j}$ using Table 2 ;
    – If (i, j) is *Trial*, update the action $U_{i,j}$ using Eqn. (4) and Table 2.

TAB. 1: *Fast Marching algorithm*

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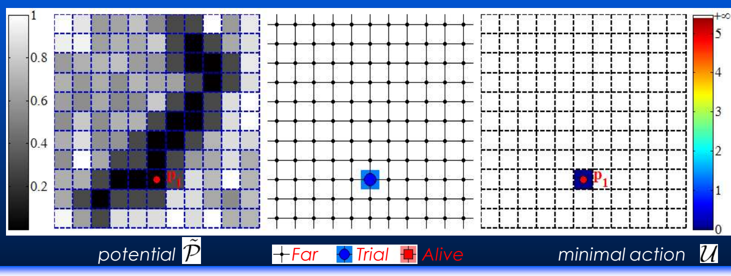
Fast Marching Algorithm

Initialization



J. A. Sethian
A fast marching level set method for monotonically advancing fronts.
P.N.A.S., **93**:1591-1595, 1996.

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Fast Marching Algorithm

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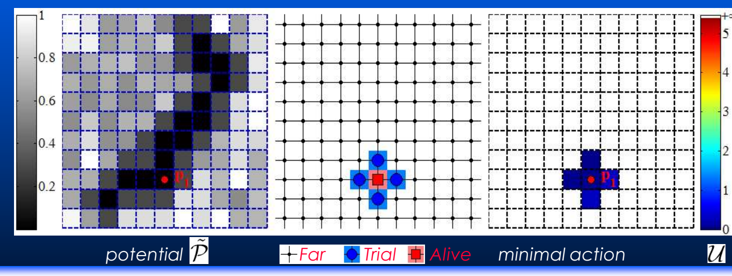
Itération #1

- Find point $\mathbf{x}_{min}$ (*Trial* point with smallest value of $\mathcal{U}$).
- $\mathbf{x}_{min}$ becomes *Alive*.
- For each of 4 neighbors $\mathbf{x}$ of point $\mathbf{x}_{min}$:
  If $\mathbf{x}$ is not *Alive*,
  Estimate $\mathcal{U}(\mathbf{x})$ with upwind scheme.
  $\mathbf{x}$ becomes *Trial*.



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P.N.A.S., **93**:1591-1595, 1996.

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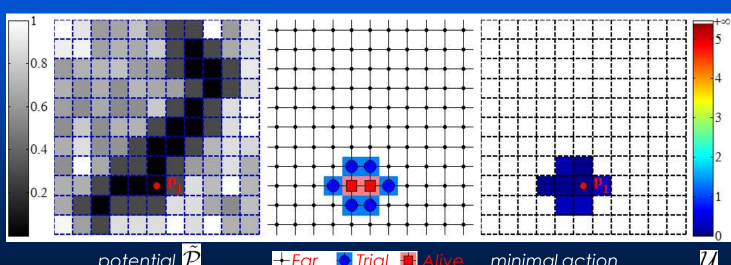
Fast Marching Algorithm

Itération #2

- Find point $\mathbf{x}_{min}$ (*Trial* point with smallest value of $\mathcal{U}$).
- $\mathbf{x}_{min}$ becomes *Alive*.
- For each of 4 neighbors $\mathbf{x}$ of point $\mathbf{x}_{min}$:
  If $\mathbf{x}$ is not *Alive*,
  Estimate $\mathcal{U}(\mathbf{x})$ with upwind scheme.
  $\mathbf{x}$ becomes *Trial*.



J. A. Sethian
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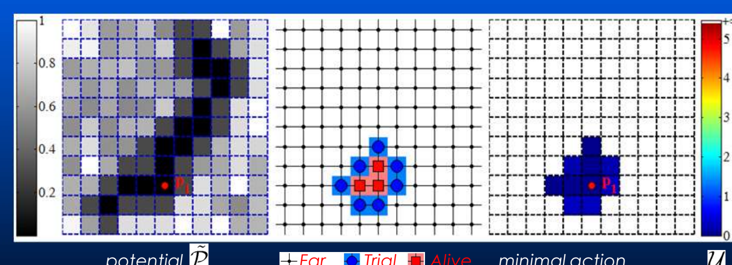
Fast Marching Algorithm

Itération #k

- Find point $\mathbf{x}_{min}$ (*Trial* point with smallest value of $\mathcal{U}$).
- $\mathbf{x}_{min}$ becomes *Alive*.
- For each of 4 neighbors $\mathbf{x}$ of point $\mathbf{x}_{min}$:
  If $\mathbf{x}$ is not *Alive*,
  Estimate $\mathcal{U}(\mathbf{x})$ with upwind scheme.
  $\mathbf{x}$ becomes *Trial*.



J. A. Sethian
A fast marching level set method for monotonically advancing fronts.
P.N.A.S., **93**:1591-1595, 1996.

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Algorithm for 2D Up-Wind Scheme

Notice that for solving Equation (4), only alive points ($\mathcal{U}$) are considered. Considering the neighbors of grid point (i,j) in 4-connexity. We note $\{A_1, A_2\}$ and $\{B_1, B_2\}$ the two couples of opposite neighbors such that we get the ordering $\mathcal{U}(A_1) \leq \mathcal{U}(A_2)$, $\mathcal{U}(B_1) \leq \mathcal{U}(B_2)$, and $\mathcal{U}(A_1) \leq \mathcal{U}(B_1)$. Considering that we have $u \geq \mathcal{U}(B_1) \geq \mathcal{U}(A_1)$, the equation derived is

$$(u - \mathcal{U}(A_1))^2 + (u - \mathcal{U}(B_1))^2 = \tilde{P}_{i,j}^2 \quad (5)$$

1. Computing the discriminant Δ of Equation (5) we have two possibilities
   - If $\Delta \geq 0$, u should be the largest solution of Equation (5);
     - If the hypothesis $u > \mathcal{U}(B_1)$ is wrong, go to 2;
     - If this value is larger than $\mathcal{U}(B_1)$, this is the solution;
   - If $\Delta < 0$, B_1 has an action too large to influence the solution. It means that $u > \mathcal{U}(B_1)$ is false. Go to 2;

   Simple calculus can replace case 1 by the test :

1bis. If $\tilde{P}_{i,j} > \mathcal{U}(B_1) - \mathcal{U}(A_1)$,
$u = \frac{\mathcal{U}(B_1)+\mathcal{U}(A_1)+\sqrt{2\tilde{P}_{i,j}^2-(\mathcal{U}(B_1)-\mathcal{U}(A_1))^2}}{2}$ is the largest solution of Equation (5),
else go to 2;

2. Considering that we have $u < \mathcal{U}(B_1)$ and $u \geq \mathcal{U}(A_1)$, we finally have $u = \mathcal{U}(A_1) + \tilde{P}_{i,j}$.

TAB. 2: Solving locally the upwind scheme

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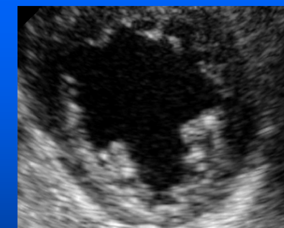
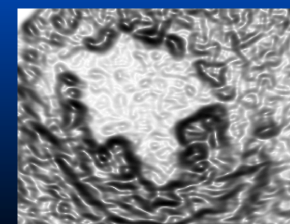
Minimal paths for 2D segmentation

Energy to minimize

$$E(\gamma) = \int_0^L P(\gamma(t))dt$$

$$P : X \in \Omega \rightarrow \frac{1}{1+\alpha.|\nabla I_\sigma(X)|^2}$$





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Minimal paths for 2D segmentation

Construction of the minimal path

- Action map

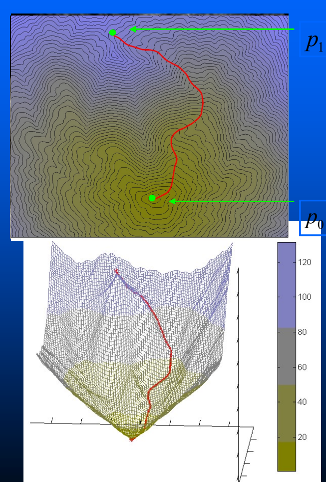
$$U_{p_0}(p) = \min_{\gamma : p_0 \rightarrow p} \left\{ \int_0^L P(\gamma(s))ds \right\}$$

- Eikonal equation

$$\|\nabla U_{p_0}\| = P, \quad U_{p_0}(p_0) = 0$$

- Minimal path obtained by back propagation from point p to p_0 solving:

$$\begin{cases} \frac{d\gamma_{min}}{dt}(t) = -\nabla U_{p_0}(\gamma_{min}(t)) \\ \gamma_{min}(0) = p \end{cases}$$



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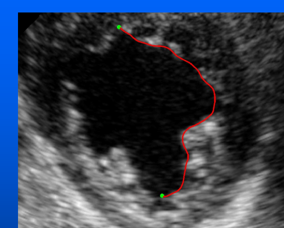
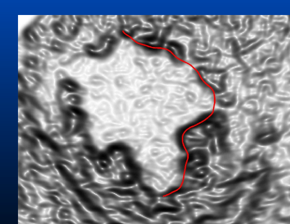
Minimal paths for 2D segmentation





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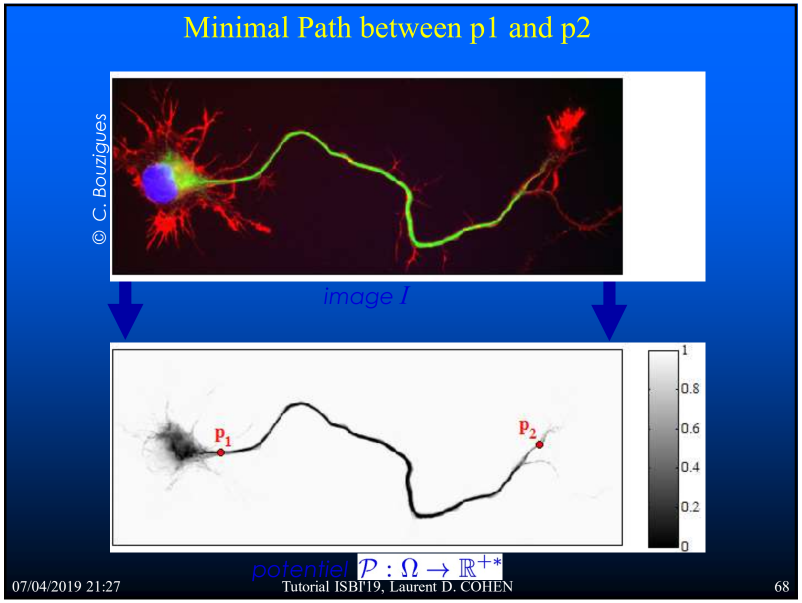
Minimal Path between p1 and p2
© C. Bouzigues
image I
p1
p2
potentiel $\mathcal{P} : \Omega \to \mathbb{R}^{+*}$
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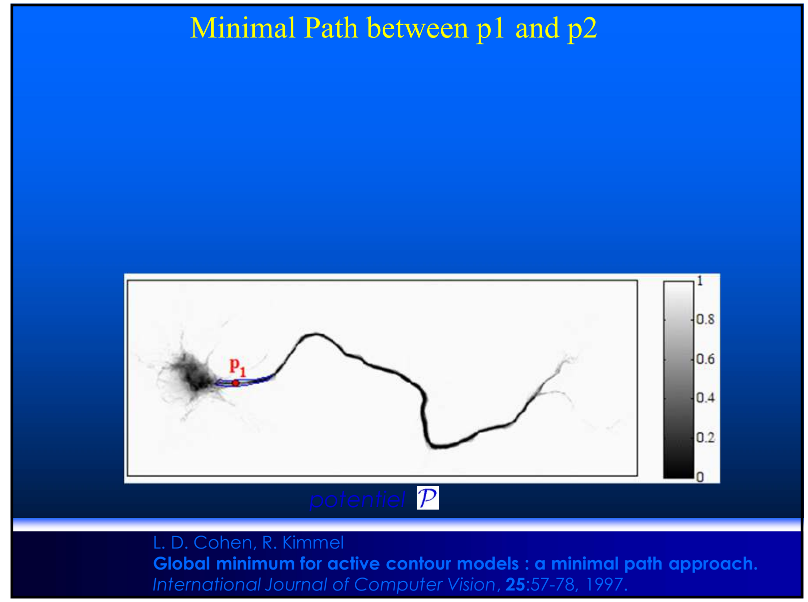
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p1
potentiel $\mathcal{P}$
L. D. Cohen, R. Kimmel
Global minimum for active contour models : a minimal path approach.
International Journal of Computer Vision, 25:57-78, 1997.

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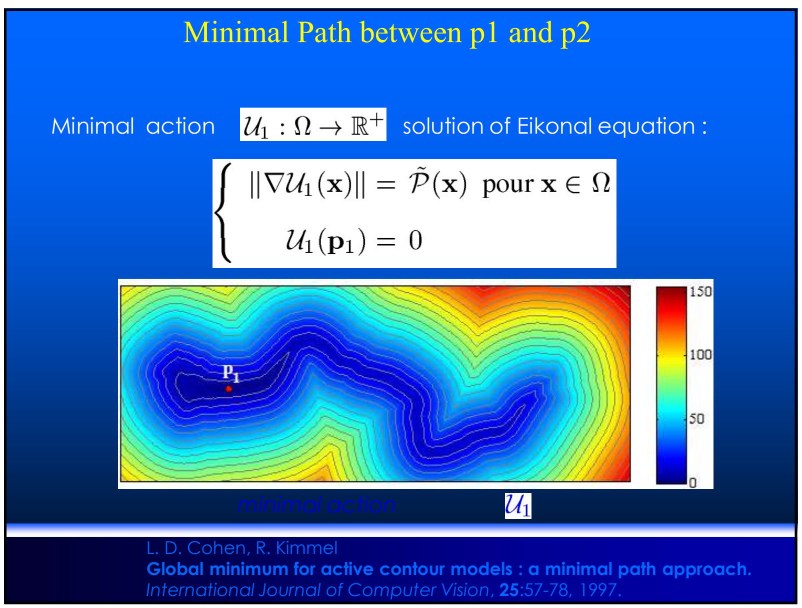
Minimal Path between p1 and p2
Minimal action $\mathcal{U}_1 : \Omega \to \mathbb{R}^+$ solution of Eikonal equation :
$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \text{ pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$
p1
minimal action $\mathcal{U}_1$
L. D. Cohen, R. Kimmel
Global minimum for active contour models : a minimal path approach.
International Journal of Computer Vision, 25:57-78, 1997.

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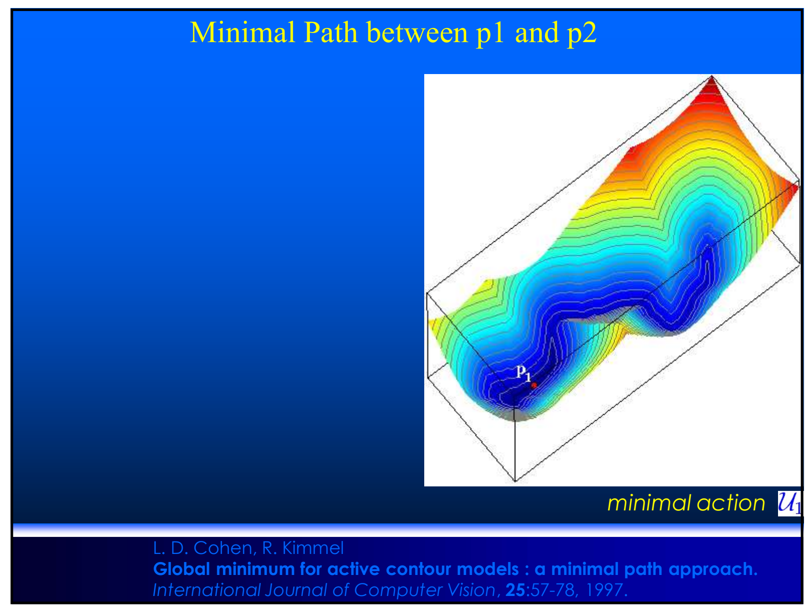
Minimal Path between p1 and p2
p1
minimal action $\mathcal{U}_1$
L. D. Cohen, R. Kimmel
Global minimum for active contour models : a minimal path approach.
International Journal of Computer Vision, 25:57-78, 1997.

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Minimal Path between p1 and p2

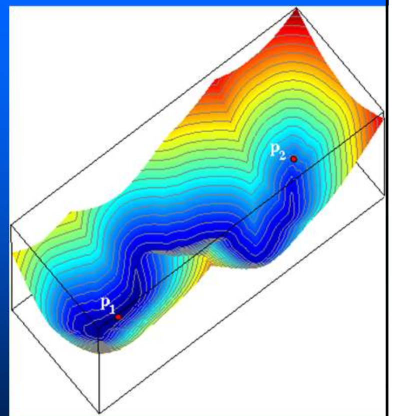
minimal path

$$\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2} = \min_{\gamma \in \mathcal{A}_{\mathbf{p}_1,\mathbf{p}_2}} \int_\gamma \tilde{\mathcal{P}}(\gamma(s))\mathrm{d}s$$

Is obtained by solving ODE:

$$\begin{cases} \dfrac{\partial \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)}{\partial s} = -\nabla \mathcal{U}_1(\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)) \\ \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(0) = \mathbf{p}_2 \end{cases}$$

⇨ simple gradient descent on $\mathcal{U}_1$ from $\mathbf{p_2}$ to $\mathbf{p_1}$



minimal action $\mathcal{U}_1$

L. D. Cohen, R. Kimmel
Global minimum for active contour models : a minimal path approach.
International Journal of Computer Vision, **25**:57-78, 1997.

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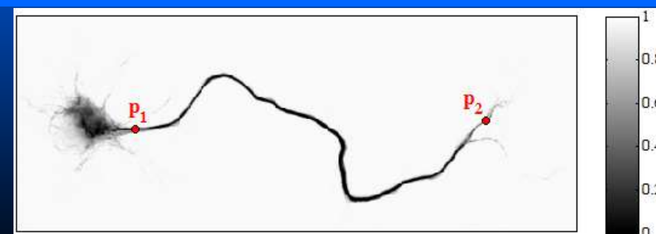
Minimal Path between p1 and p2

Step #1

$$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \text{ pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$$



potential $\mathcal{P}$

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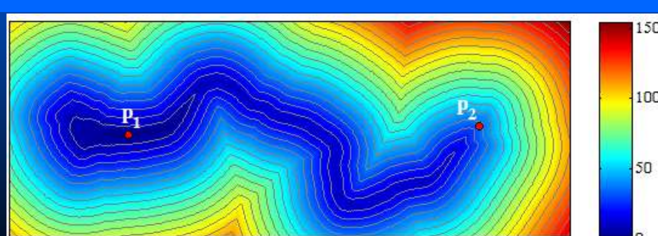
Minimal Path between p1 and p2

Step #1

$$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \text{ pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$$



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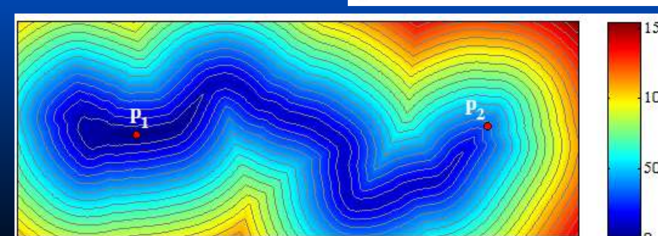
Minimal Path between p1 and p2

Step #1

$$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \text{ pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$$

Step #2
gradient descent on $\mathcal{U}_1$ for extraction of minimal path $\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}$

$$\begin{cases} \dfrac{\partial \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)}{\partial s} = -\nabla \mathcal{U}_1(\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)) \\ \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(0) = \mathbf{p}_2 \end{cases}$$



07/04/2019 21:27 *minimal action* $\mathcal{U}_1$ al ISBI'19, Laurent D. COHEN 75

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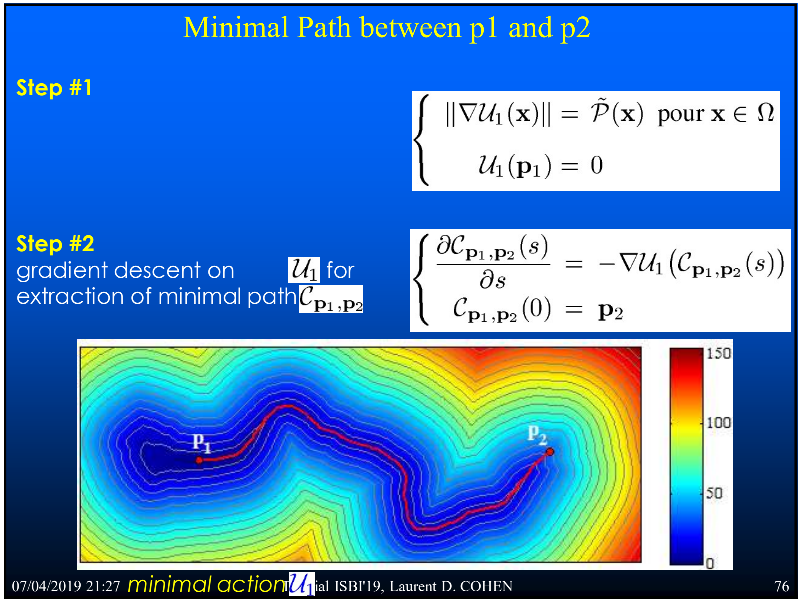
Minimal Path between p1 and p2
Step #1
$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \text{ pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$
Step #2
gradient descent on $\mathcal{U}_1$ for
extraction of minimal path $\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}$
$\begin{cases} \frac{\partial \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)}{\partial s} = -\nabla \mathcal{U}_1(\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)) \\ \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(0) = \mathbf{p}_2 \end{cases}$
p1
p2
150
100
50
0
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minimal action $\mathcal{U}_1$
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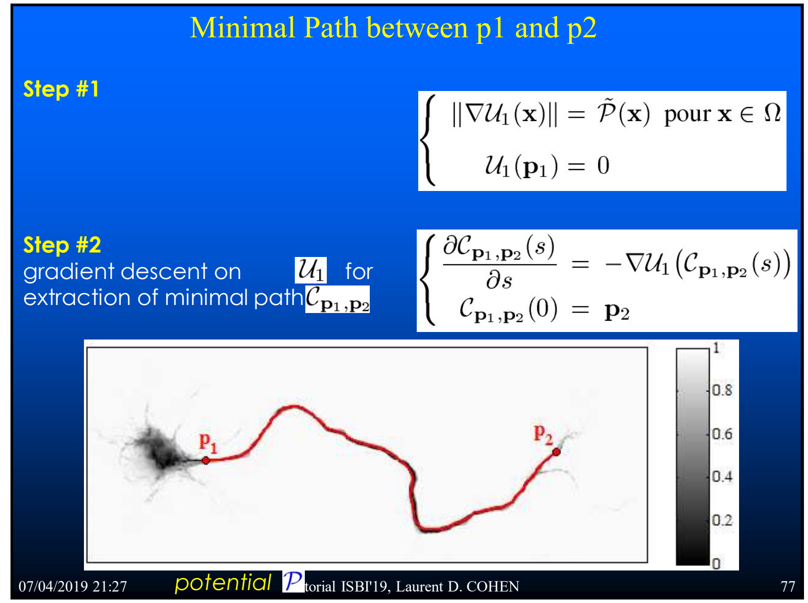
Minimal Path between p1 and p2
Step #1
$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \text{ pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(\mathbf{p}_1) = 0 \end{cases}$
Step #2
gradient descent on $\mathcal{U}_1$ for
extraction of minimal path $\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}$
$\begin{cases} \frac{\partial \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)}{\partial s} = -\nabla \mathcal{U}_1(\mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(s)) \\ \mathcal{C}_{\mathbf{p}_1,\mathbf{p}_2}(0) = \mathbf{p}_2 \end{cases}$
p1
p2
1
0.8
0.6
0.4
0.2
0
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potential $\mathcal{P}$
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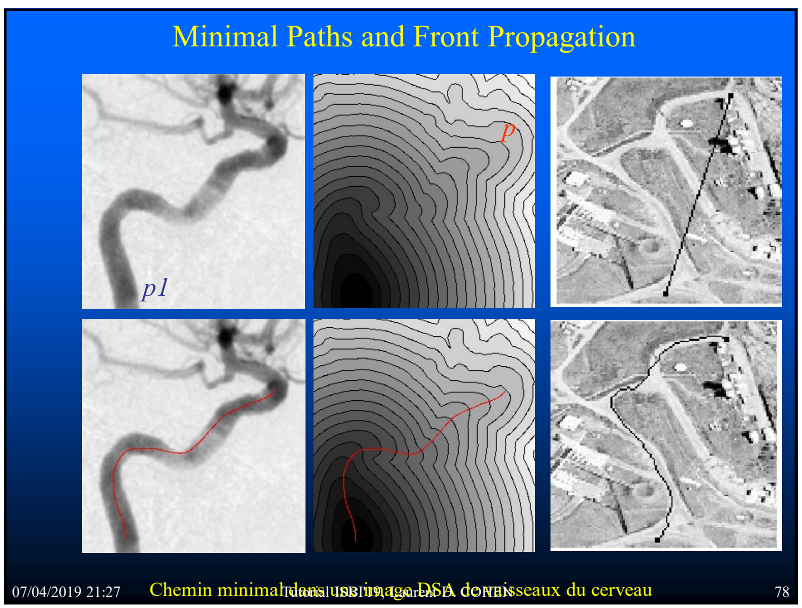
Minimal Paths and Front Propagation
p1
p
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Chemin minimal dans une image DSA de vaisseaux du cerveau
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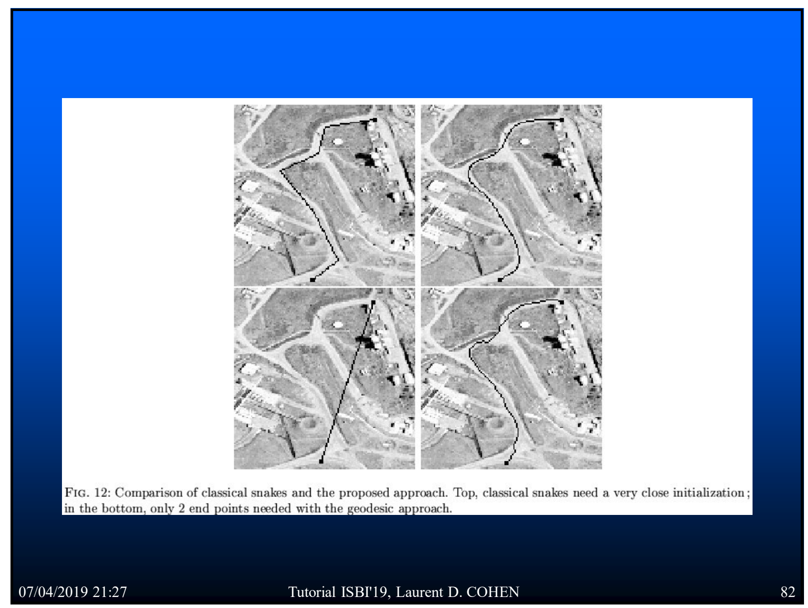
FIG. 12: Comparison of classical snakes and the proposed approach. Top, classical snakes need a very close initialization; in the bottom, only 2 end points needed with the geodesic approach.
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Paths with same origin

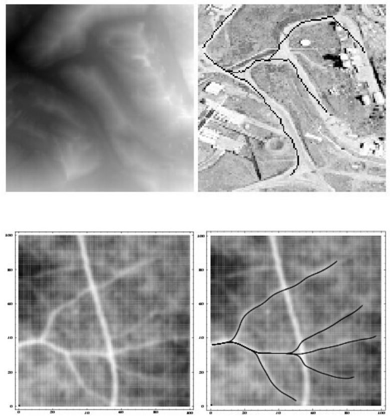

FIG. 14: Many paths are obtained simultaneously connecting the start point on the upper left to 4 other points. Second example for finding vessels in medical angiographic

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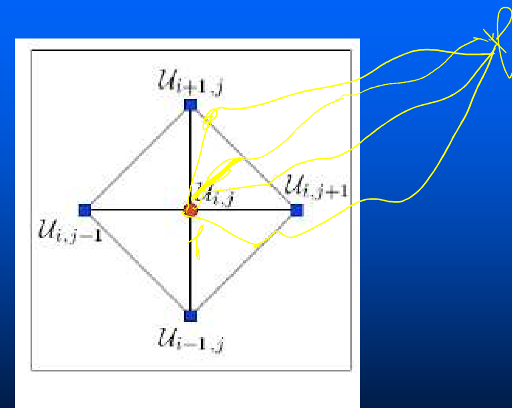
Tsitsiklis Algorithm



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Graph Search algorithms
Dynamic Programming

$$\text{Minimization of } \int_\Omega \tilde{P}(\mathcal{C})ds \qquad (19)$$

- A^* algorithm : Dijkstra 1959
  - distance image initialized with value ∞,
  - expands to a neighbor pixel a previously obtained minimal path ending at the vertex with smallest current cost value.
  - 1 iteration per pixel and a search for the best pixel to update : $O(NlogN)$.
  - Similar to Fast marching but **not consistent**.
- F^* algorithm : Fischler, Tenenbaum, Wolf 1981.
  - same but sequential ;
  - image scanned iteratively top to bottom, row by row, left to right followed by right to left, and then bottom to top.
  - similar in spirit to shape from shading but **not consistent**.

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- Metrication error -

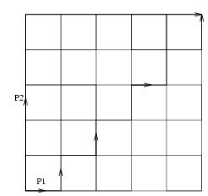

FIG. 22: An L^1 norm cause the shortest path to suffer from errors of up to 41%. In this case both P_1 and P_2 are optimal, and will stay optimal no matter how much we refine the (4-neighboring) grid.

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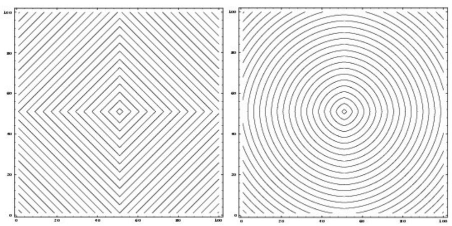

FIG. 23: Illustration of metrication error for computation of the distance map to a single point, showing level sets of the distance. On the left : a graph search-like discrete distance computation gives squares ; On the right : the distance is obtained by our approach, giving circles.

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Partial front propagation

A minimal path in a DSA image of brain vessels — Complete front propagation — Partial front propagation

During front propagation, the action computed at each point visited can only grow due to the resolution of Eikonal equation.

There is no need to propagate further when the end point is reached.

The number of points visited during the partial propagation is reduced, thus saving computing time.

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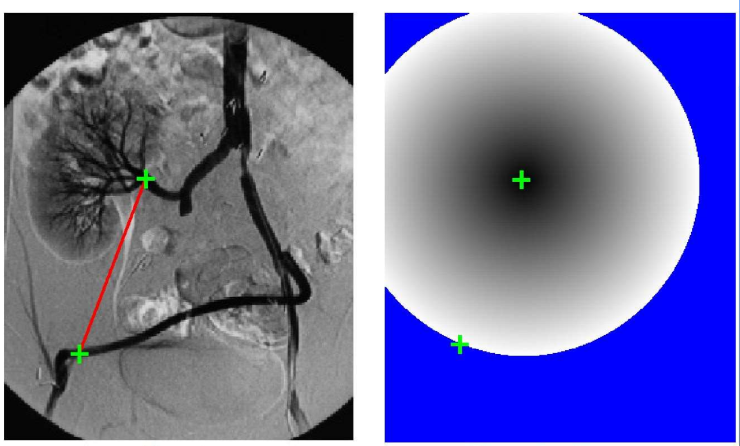
Minimal paths for 2D segmentation

- $P(\mathbf{x}) = 1$ ⟹ droite (plus court chemin euclidien)



Chemin — Carte de distance

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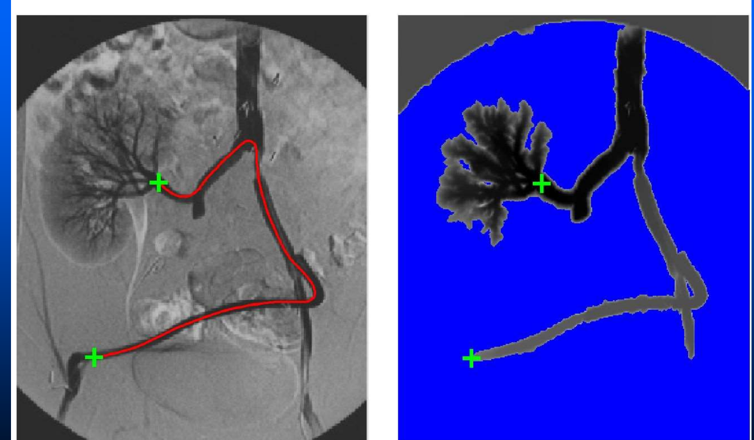
Minimal paths for 2D segmentation

- $P(\mathbf{x}) = w + (I(\mathbf{x}) - I(\mathbf{x}_0))^2$ ⟹ chemin d'intensité homogène



Chemin — Carte de distance

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Simultaneous front propagation

Simultaneous propagation from the two ends

Meeting point: saddle point of U on the path at the middle with respect to energy.

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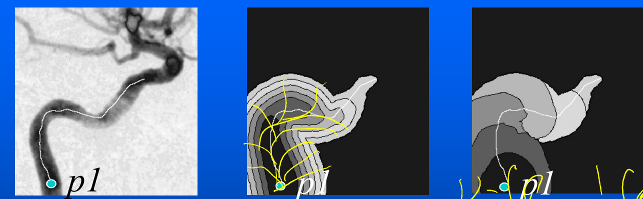
One end point front propagation



- Sometimes difficult to give a second end point (3D, complex shape).
- Simultaneously compute the Euclidean length of the minimal path at each point visited. Low cost: included in the fast marching with potential P = 1.
- Stopping point for propagation by maximum length condition.
- It reduces user interaction to the definition of the starting point.

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Geodesic Minimal Paths

- Minimal paths, Eikonal Equation, Fast Marching and Front Propagation
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- Closed Contour as a set of minimal paths. Key points method
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3D FAST MARCHING:

Extension to 3D by the same numerical resolution of Eikonal Equation

Ui,j,k solution of the discrete problèm

$$\|\nabla U\| = \tilde{P}$$

$$\begin{array}{ll}(\max\{u-U_{i-1,j,k}, u-U_{i+1,j,k}, 0\})^2 & + \\ (\max\{u-U_{i,j-1,k}, u-U_{i,j+1,k}, 0\})^2 & + \\ (\max\{u-U_{i,j,k-1}, u-U_{i,j,k+1}, 0\})^2 & = \tilde{P}^2_{i,j,k}\end{array}$$

2nd degree Equation,
action U at {i,j,k} depends only on smaller action neighbors

Fast marching : order in the selection of points to solve the local scheme.

Starting point p0 with U = 0.

Level sets of U can be seen as a front propagation outwards.

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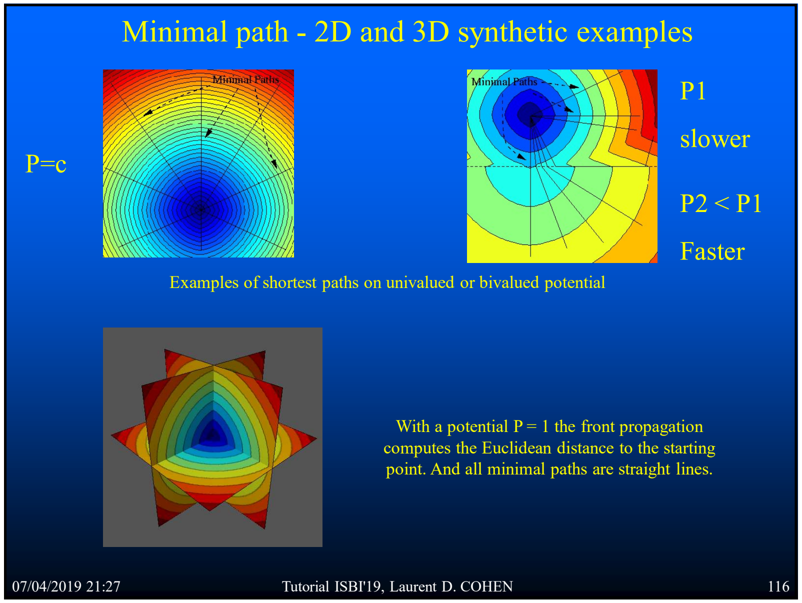

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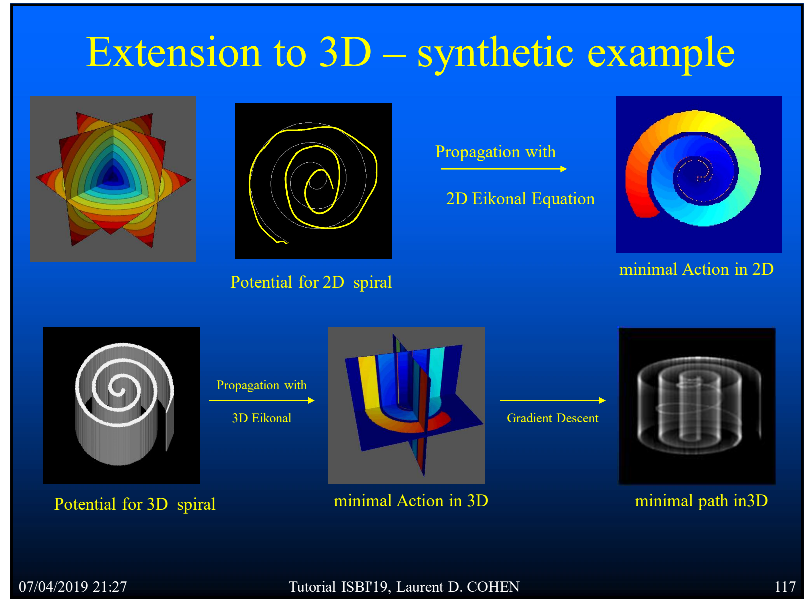

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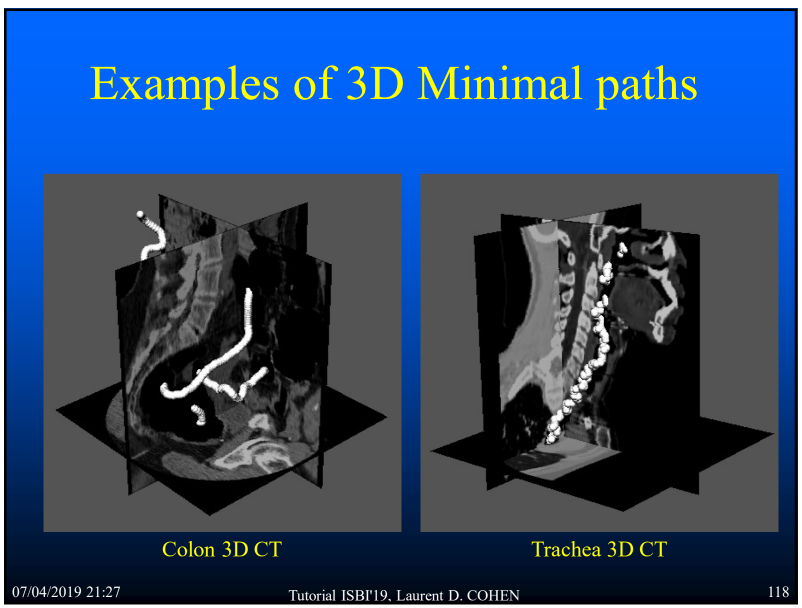

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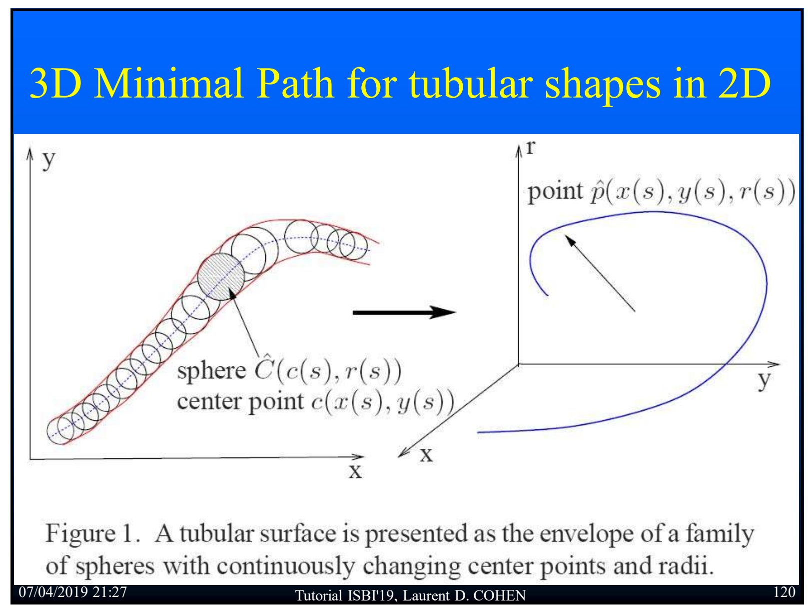

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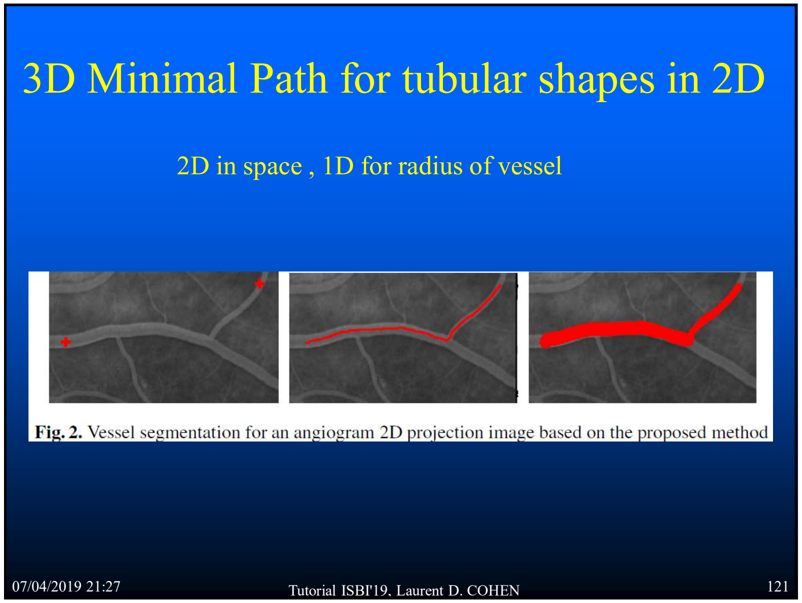

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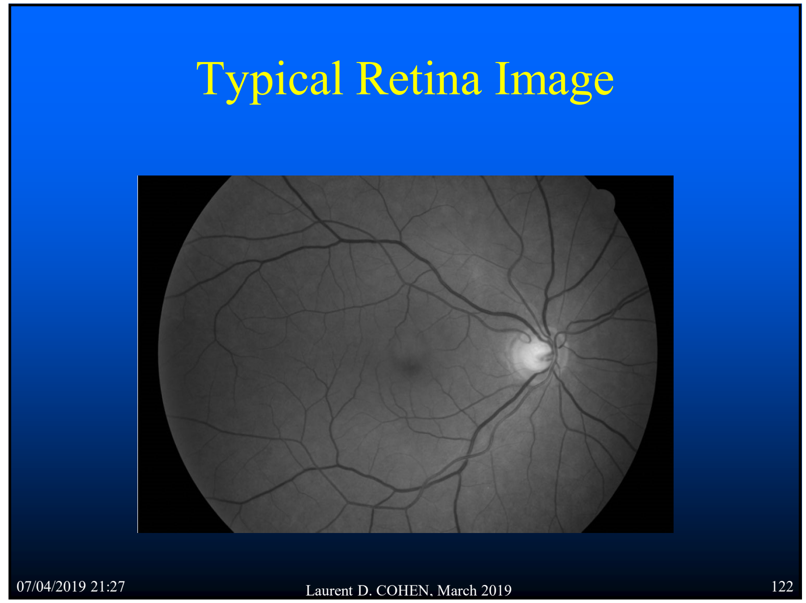

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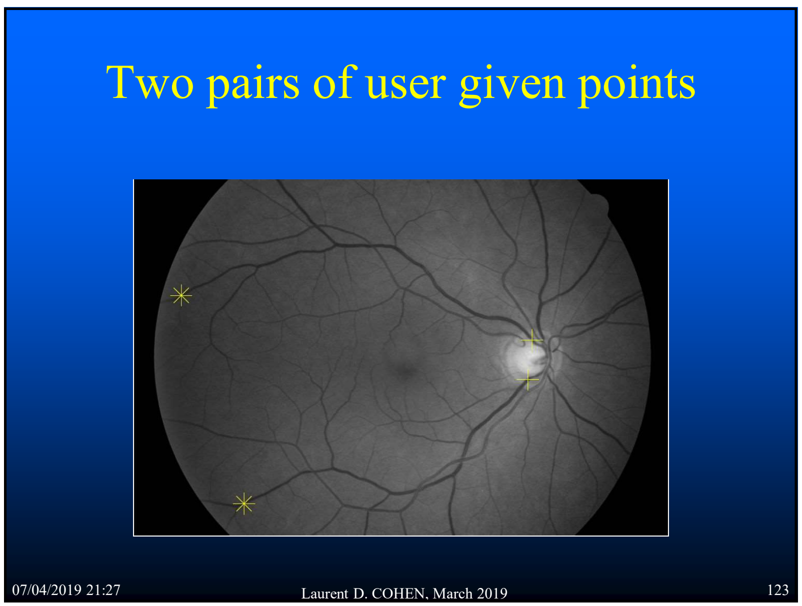

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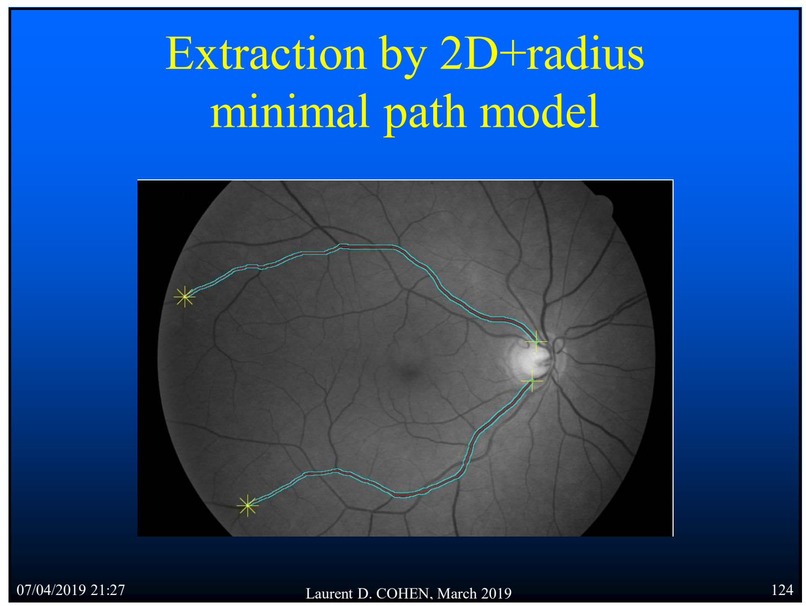

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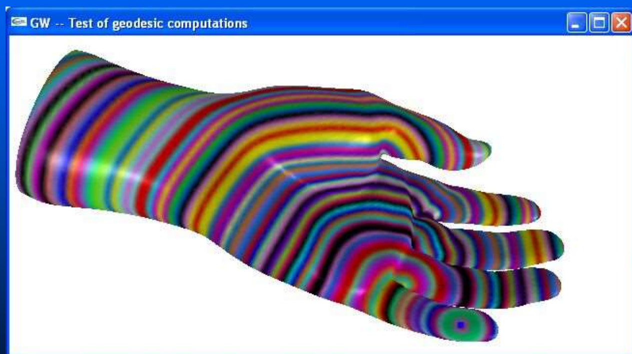
Fast Marching on a surface

Front Propagation on a surface from one point.



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Fast Marching on a surface

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Geodesic Minimal Paths

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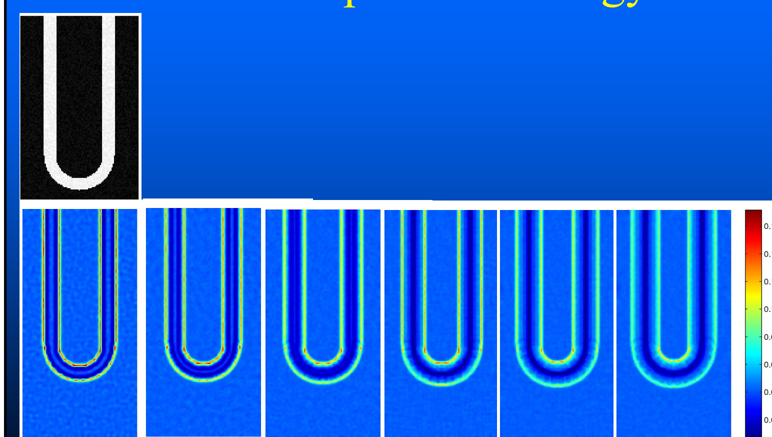
Radius dependent Energy



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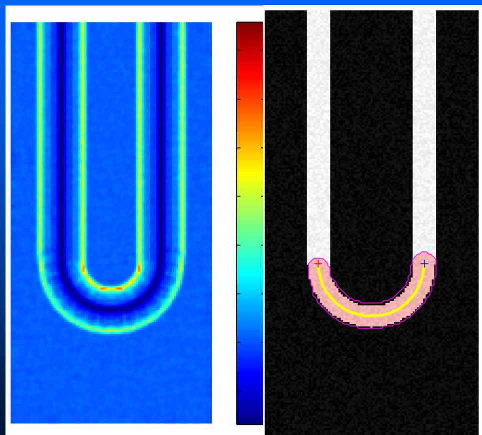
Radius dependent Energy



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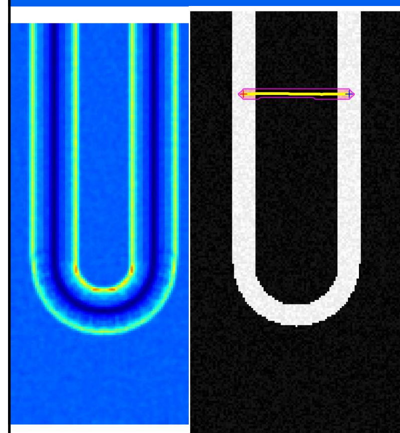
Orientation dependent Energy



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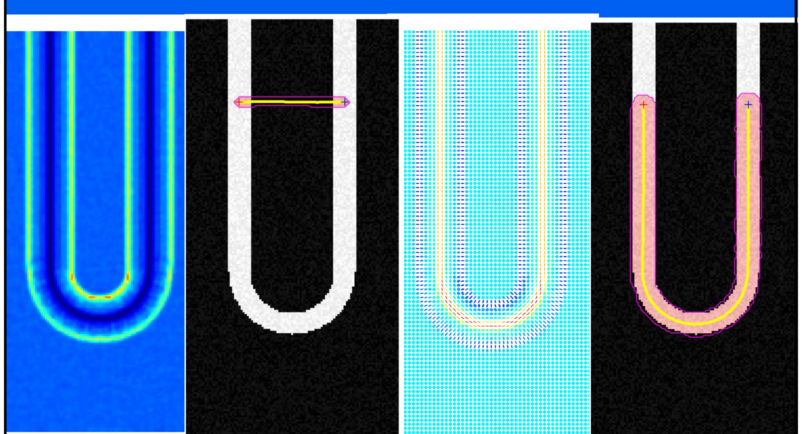
Orientation dependent Energy



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Anisotropic Energy

$$E(C)=\int_0^L P(C(s),C'(s))ds$$

Considers the local orientations of the structures

$$P(C(s),C'(s))=\sqrt{C'(s)^T H(C(s))C'(s)}$$

Describes an infinitesimal distance along an oriented pathway *C*, relative to a metric H

Geodesic Methods for Shape and Surface Processing, Gabriel Peyre and Laurent D. Cohen in Advances in Computational Vision and Medical Image Processing: Methods and Applications, Springer, 2009.

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Anisotropic Energy: Eikonal Equation

$$E(C)=\int_0^L \sqrt{C'(s)^T H(C(s))C'(s)}ds$$

Start point $C(0)=p1$; $U_{p1}(p)=\inf_{C(0)=p1;C(L)=p} E(C)$

$$\left\|\nabla U_{p1}(p)\right\|_{H(p)^{-1}} = \sqrt{\nabla U_{p1}{}^T H^{-1}\nabla U_{p1}} = 1$$

and $U_{p1}(p1)=0$

Geodesic Methods for Shape and Surface Processing, Gabriel Peyre and Laurent D. Cohen in Advances in Computational Vision and Medical Image Processing: Methods and Applications, Springer, 2009.

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Anisotropic Energy: Gradient descent

$$E(C)=\int_0^L \sqrt{C'(s)^T H(C(s))C'(s)}ds$$

Start point $C(0)=p1$; $U_{p1}(p)=\inf_{C(0)=p1;C(L)=p} E(C)$

$$C'(s) = -H^{-1}(C(s))\nabla U_{p1}(C(s))$$

and $U_{p1}(p1)=0$

Geodesic Methods for Shape and Surface Processing, Gabriel Peyre and Laurent D. Cohen in Advances in Computational Vision and Medical Image Processing: Methods and Applications, Springer, 2009.

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Anisotropic Energy: includes Isotropic case

$$E(C)=\int_0^L \sqrt{C'(s)^T H(C(s))C'(s)}ds$$

Start point $C(0)=p1$; $H(p)=P^2(p)Id$

$$\left\|\nabla U_{p1}(p)\right\| = P \qquad C'(t) = -\nabla U_{p1}(C(t))$$

Geodesic Methods for Shape and Surface Processing, Gabriel Peyre and Laurent D. Cohen in Advances in Computational Vision and Medical Image Processing: Methods and Applications, Springer, 2009.

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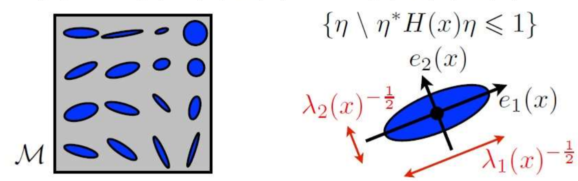
Anisotropy and Geodesics

Tensor eigen-decomposition:

$$H(x) = \lambda_1(x)e_1(x)e_1(x)^{\mathrm{T}} + \lambda_2(x)e_2(x)e_2(x)^{\mathrm{T}} \quad \text{with} \quad 0 < \lambda_1 \leqslant \lambda_2.$$



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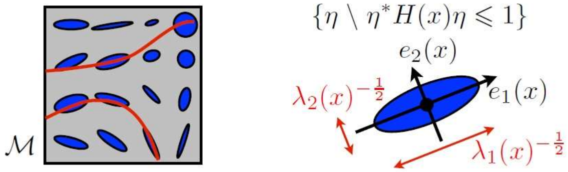
Anisotropy and Geodesics

Tensor eigen-decomposition:

$$H(x) = \lambda_1(x)e_1(x)e_1(x)^{\mathrm{T}} + \lambda_2(x)e_2(x)e_2(x)^{\mathrm{T}} \quad \text{with} \quad 0 < \lambda_1 \leqslant \lambda_2,$$

$$\{\eta \setminus \eta^* H(x)\eta \leqslant 1\}$$



Geodesics tend to follow $e_1(x)$.

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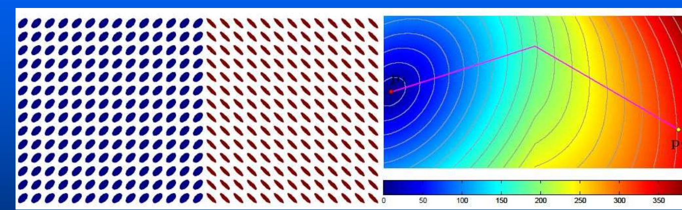
Anisotropy and Geodesics



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Anisotropy and Geodesics

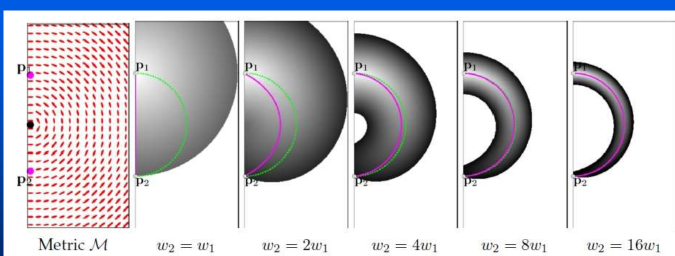

FIG. 2.14: Given an elliptic metric $\mathcal{M} = w_1^2 e_r e_r^T + w_2^2 e_\theta e_\theta^T$ with standard polar notations, influence of anisotropy ratio $\frac{w_2}{w_1}$ is shown.

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Anisotropy and Geodesics Mirebeau 2014

- Recent Numerical advance to solve Anisotropic Geodesics: JM Mirebeau
- Similar as Fast Marching but adapting the discrete neighborhood to anisotropy
- Stable when metric has a large anisotropic ratio
- Fast and accurate enough

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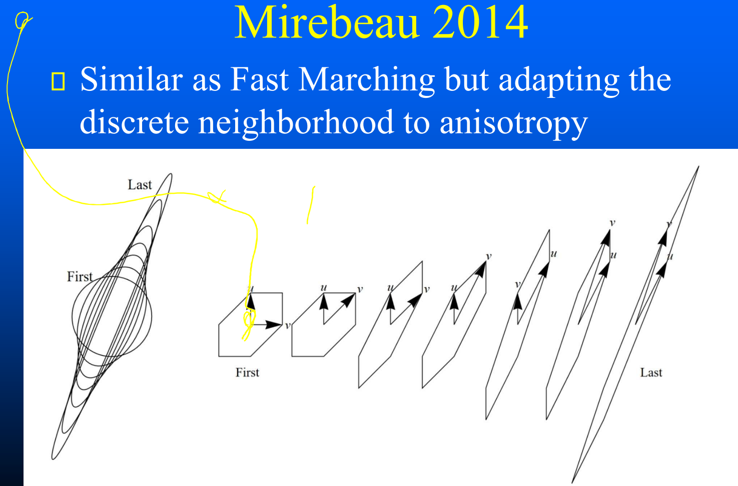
Anisotropy and Geodesics Mirebeau 2014

- Similar as Fast Marching but adapting the discrete neighborhood to anisotropy



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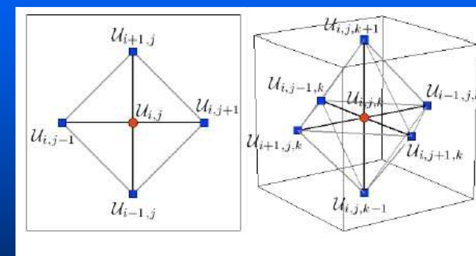
Anisotropic Fast Marching



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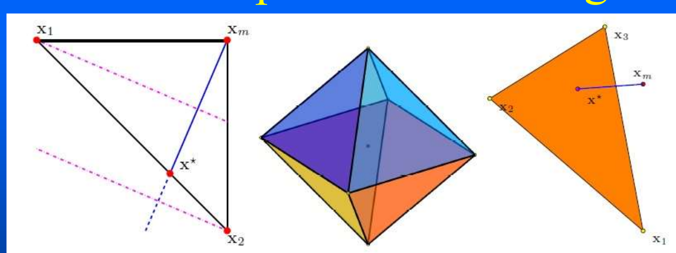
Anisotropic Fast Marching



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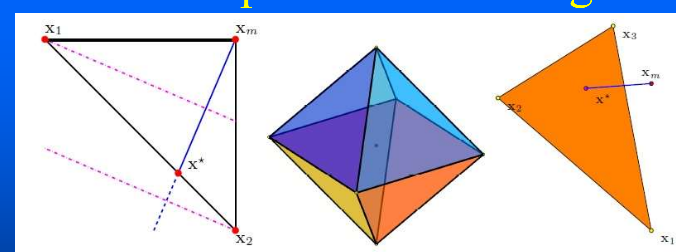
Isotropic Fast Marching



$$U(\mathbf{x}_m) = \min_{\mathbf{x} \in \mathbf{x}_1\mathbf{x}_2\mathbf{x}_3} \left\{ U(\mathbf{x}) + \int_{\mathbf{x}}^{\mathbf{x}_m} \tilde{P}(C) \right\}$$

$$f(\alpha) = \sum_{i=1}^{3} \alpha_i U(\mathbf{x}_i) + \tilde{P}(\mathbf{x}_m) \left\| \mathbf{x}_m - \sum_{i=1}^{3} \alpha_i \mathbf{x}_i \right\|_2$$

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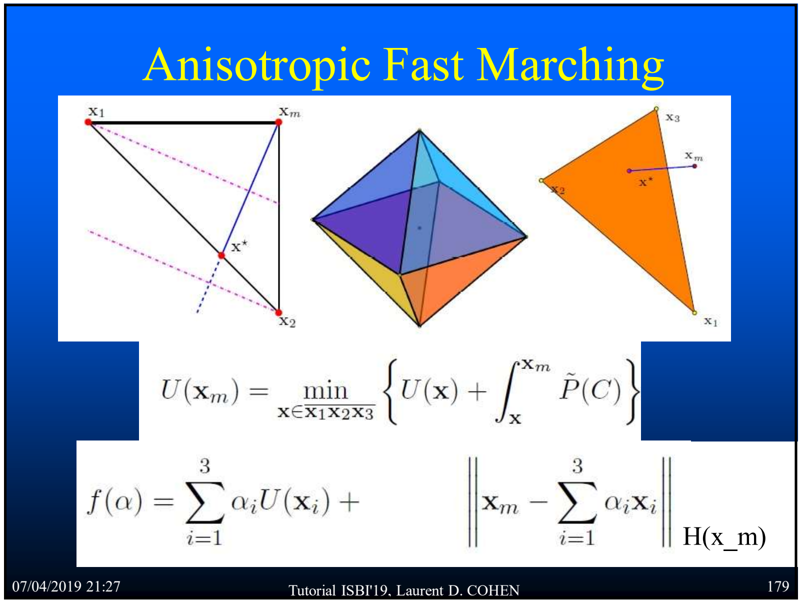

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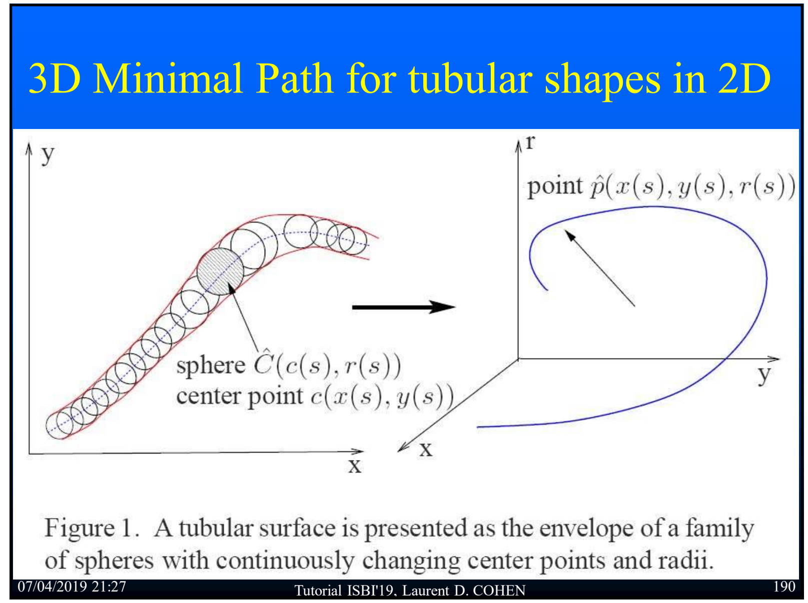

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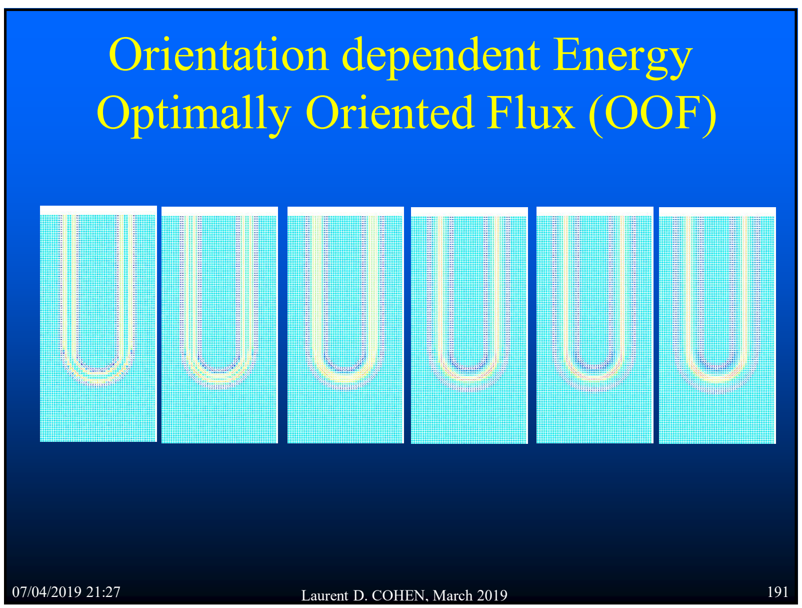

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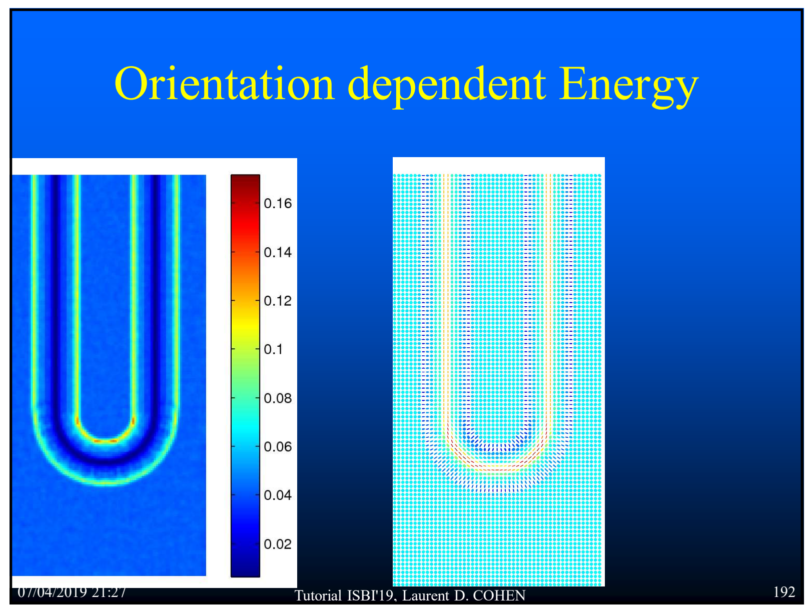

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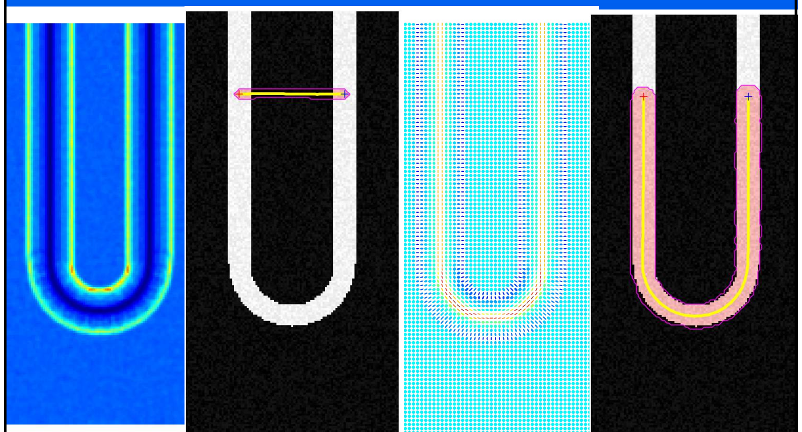
Orientation dependent Energy



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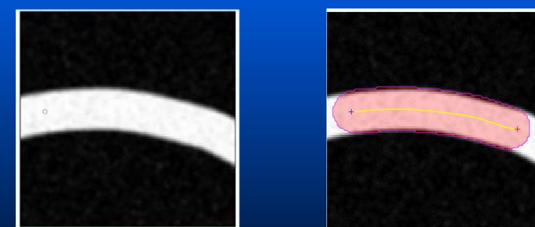
Examples of 3D Minimal Paths for tubular shapes in 2D

Anisotropic Fast Marching algorithm to solve

$$\|\nabla\mathcal{U}(x)\|_{\mathcal{M}^{-1}} = \sqrt{\nabla\mathcal{U}(x)^T\mathcal{M}^{-1}(x)\nabla\mathcal{U}(x)} = 1 \text{ and } \mathcal{U}_{p_0}(p_0) = 0$$

and back-propagation $\mathcal{C}' \propto \mathcal{M}^{-1}\nabla\mathcal{U}$



Tubular anisotropy for 3D vessels segmentation. Fethallah Benmansour and Laurent D. Cohen. Preprint, 2009.

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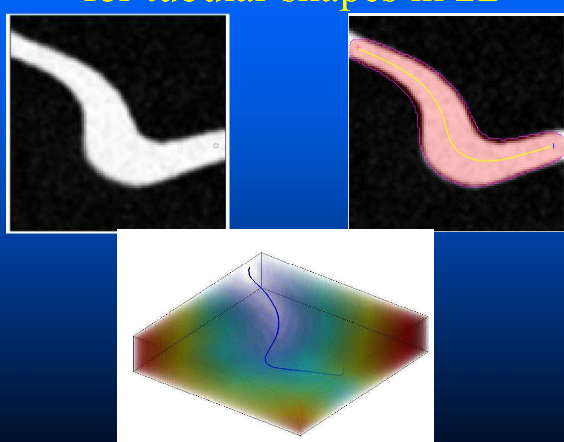
Examples of 3D Minimal Paths for tubular shapes in 2D



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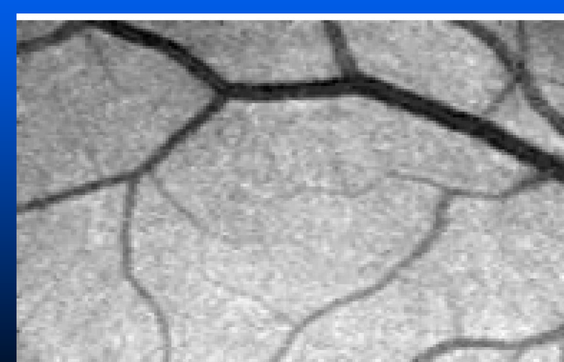
3D Minimal Paths for tubular shapes in 2D

2D in space , 1D for radius of vessel



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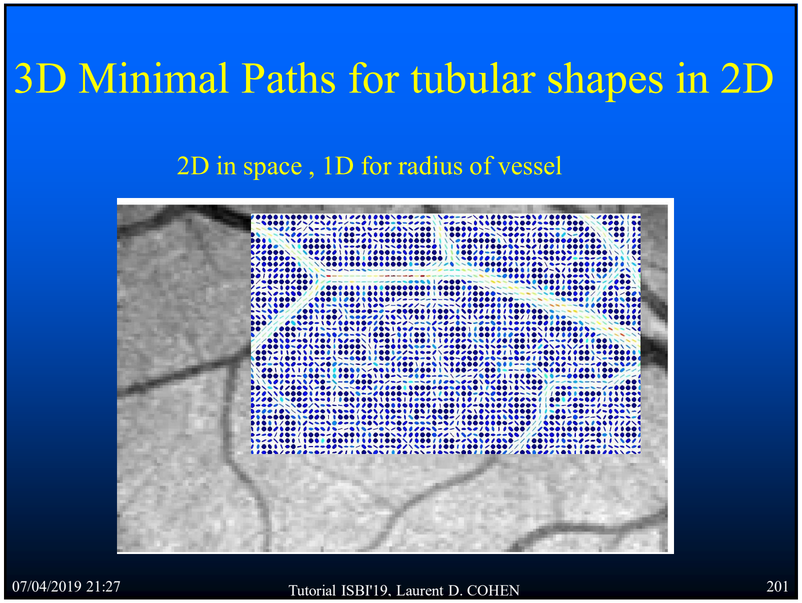

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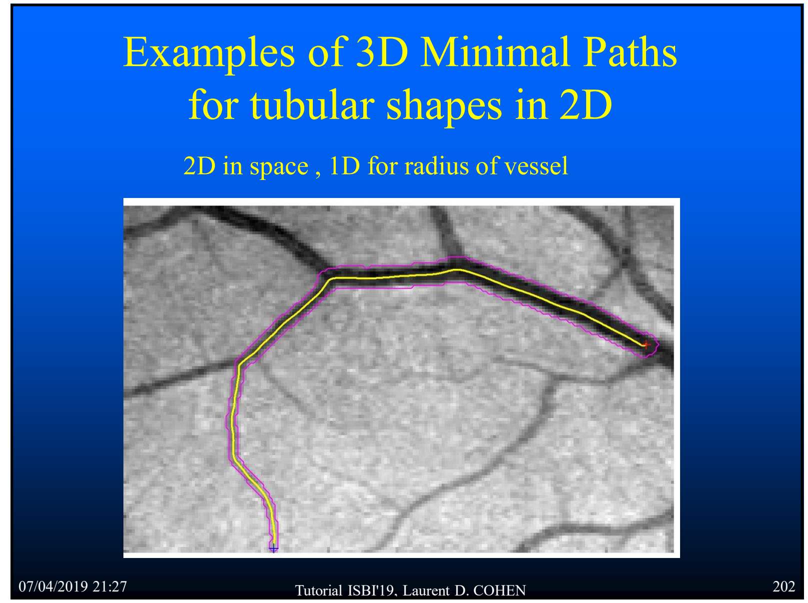

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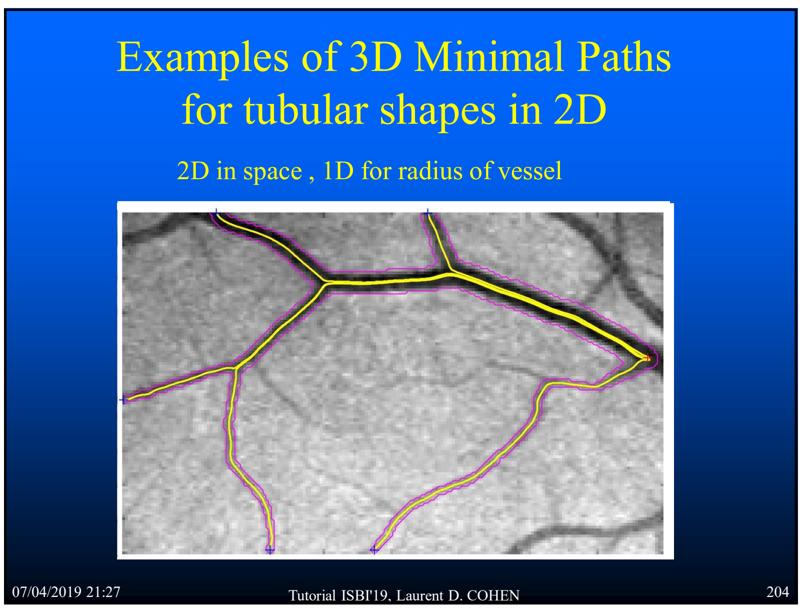

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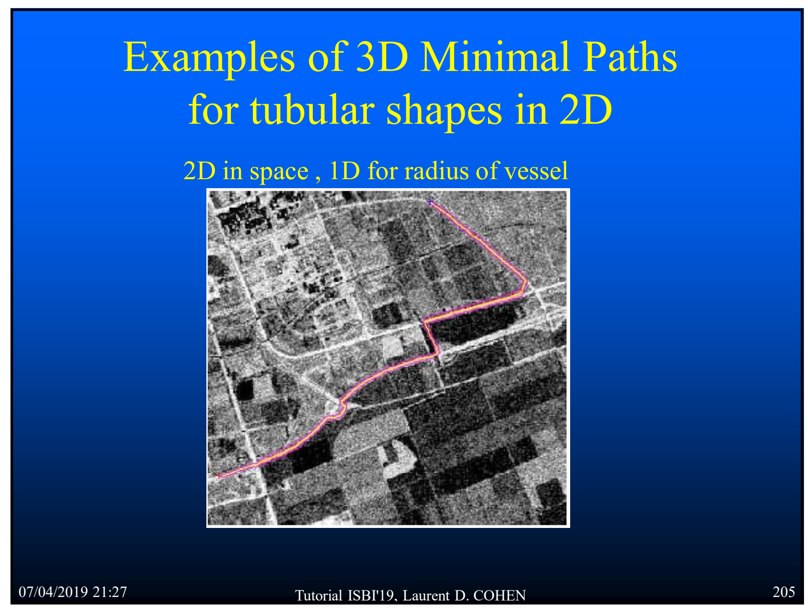

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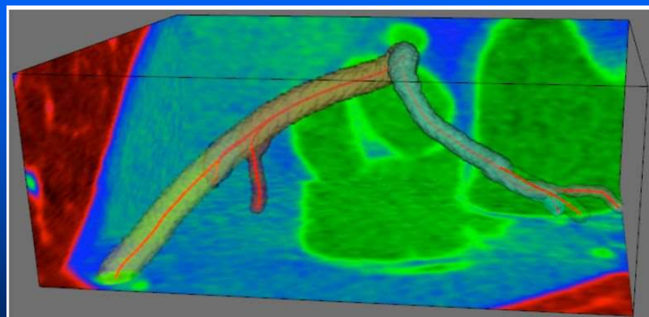
Examples of 4D Minimal Paths for tubular shapes in 3D



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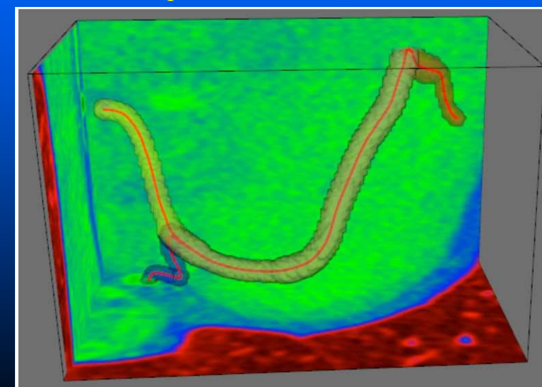
Examples of 4D Minimal Paths for tubular shapes in 3D

3D in space , 1D for radius of vessel



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Geodesic Minimal Paths

- Minimal paths, Eikonal Equation, Fast Marching and Front Propagation
- 3D Fast Marching, some examples
- Anisotropic Fast Marching
- Finsler Metrics for Various Active Contours Energy terms
- Closed Contour as a set of minimal paths. Key points method
- Geodesic Voting and tree structure segmentation
- Application to Virtual Endoscopy and Vessel Visualization

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Finsler Metrics

- Finsler Metric: A more general setting
- Finsler Metric: Segmentation with Curvature Penalization
- Finsler Metric: Region-Based Segmentation
- Finsler Metric: Active Contour with Alignment term

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General Finsler Metric

$$\ell(\gamma) = \int_0^1 \mathcal{F}_{\gamma(t)}\big(\gamma'(t)\big)\,dt, \quad (1)$$

where $\gamma'(t) = \frac{d}{dt}\gamma(t)$. The minimal action map $\mathcal{U}(\mathbf{x})$, or geodesic distance from the source point $\mathbf{p}$, is the minimal length (1) among all path joining starting point $\mathbf{p}$ to $\mathbf{x} \in \Omega$:

$$\mathcal{U}(\mathbf{x}) := \min\{\ell(\gamma);\, \gamma \in \mathcal{A}_{\mathbf{p},\mathbf{x}}\}. \quad (2)$$

The minimal action map $\mathcal{U}$ is the unique viscosity solution to an Eikonal PDE:

$$\begin{cases} \mathcal{F}^*_{\mathbf{x}}(\nabla\mathcal{U}(\mathbf{x})) = 1, & \text{for all } \mathbf{x} \in \Omega, \\ \mathcal{U}(\mathbf{p}) = 0, \end{cases} \quad (3)$$

where $\mathcal{F}^*_{\mathbf{x}}$ is defined as

$$\mathcal{F}^*_{\mathbf{x}}(\mathbf{u}) = \sup_{\mathbf{v}\neq 0} \frac{\langle \mathbf{u},\mathbf{v}\rangle}{\mathcal{F}_{\mathbf{x}}(\mathbf{v})}. \quad (4)$$

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General Finsler Metric

$$\mathcal{F}_{\mathbf{x}}(\mathbf{u}) = \sqrt{\langle \mathbf{u}, \mathcal{M}\mathbf{u}\rangle} - \langle \boldsymbol{\omega}, \mathbf{u}\rangle. \quad (5)$$

The asymmetric part should obey the following *smallness* condition to ensure that the Finsler metric $\mathcal{F}_{\mathbf{x}}$ is positive:

$$\forall \mathbf{x} \in \Omega, \quad \langle \boldsymbol{\omega}(\mathbf{x}), \mathcal{M}^{-1}(\mathbf{x})\boldsymbol{\omega}(\mathbf{x})\rangle < 1. \quad (6)$$

Equation (5) defines an anisotropic Finsler metric in general. This is an anisotropic Riemannian metric if the vector field $\boldsymbol{\omega}$ is identically zero, and an isotropic metric if in addition the tensor field $\mathcal{M}$ is proportional to the identity matrix.

The geodesic $\mathcal{C}$, joining $\mathbf{x}$ from the initial source point $\mathbf{p}$, can be recovered by solving the following ODE involving $\mathcal{U}$ and the dual metric $\mathcal{F}^*$:

$$\mathcal{C}'(t) = -\nabla\mathcal{F}^*_{\mathcal{C}(t)}\Big(\nabla\mathcal{U}(\mathcal{C}(t))\Big). \quad (7)$$

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Finsler Metrics

- Finsler Metric: A more general setting
- Finsler Metric: Segmentation with Curvature Penalization
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- Finsler Metric: Active Contour with Alignment term

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Active Contours

- Energy Minimization:

$$\int_\Omega w_1\|\mathcal{C}'(s)\|^2 + w_2\|\mathcal{C}''(s)\|^2 + P(\mathcal{C}(s))ds$$

- *C(s)=(x(s),y(s))* curve drawn on the image
- Smoothing terms : length and curvature penalization
- Trapped in local minima
- Geodesic Approach removed the second term

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Curvature Penalized Minimal Path Method with A Finsler Metric

with Da Chen and JM Mirebeau, 2015-2016

- The metric may depend on the orientation
- Orientation-lifted metric: the curve length of Euler elastica can be exactly computed by this metric

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Curvature Penalized Minimal Path Method with A Finsler Metric

$$\mathcal{L}_E(\Gamma) = \int_0^L (1+\kappa^2(s))\,ds$$

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Curvature Penalized Minimal Path Method with A Finsler Metric

$$\mathcal{L}_E(\Gamma) = \int_0^L (1+\kappa^2(s))\,ds$$

- Let $\Gamma : [0,1] \to \Omega$ and $\theta : [0,1] \to [0, 2\pi)$.

$$\dot\Gamma(t)/\|\dot\Gamma(t)\| := (\cos\theta(t), \sin\theta(t)) \quad \Rightarrow \quad \kappa(t) = \dot\theta(t)/\|\dot\Gamma(t)\|, \quad \forall\, t \in [0,1]$$

- κ is the curvature of Γ and $ds = \|\Gamma'(t)\|dt$. An elastica is a path minimizing

$$\mathcal{L}(\Gamma) := \int_0^L (1+\alpha\,\kappa^2(s))ds = \int_0^1 (1+\alpha\kappa^2(t))\|\dot\Gamma(t)\|dt = \int_0^1 \left(\|\dot\Gamma(t)\| + \alpha\frac{|\dot\theta(t)|^2}{\|\dot\Gamma(t)\|}\right)dt$$

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Curvature Penalized Minimal Path Method with A Finsler Metric Orientation Lifting

- Let $\gamma = (\Gamma, \theta) \in C^1([0,1], \Omega \times S^1)$, where $\theta \in S^1 = [0, 2\pi[$
- Let $\vec{v}_\theta = (\cos\theta, \sin\theta)$ be the unit direction vector

Orientation-lifted Metric

$$\mathcal{F}^\infty_\gamma(\dot\gamma) := \begin{cases} \|\dot\Gamma\| + \frac{|\dot\theta|^2}{\|\dot\Gamma\|}, & \text{if } \dot\Gamma \text{ is positively proportional with } \vec{v}_\theta, \\ \infty, & \text{otherwise.} \end{cases}$$

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Curvature Penalized Minimal Path Method with A Finsler Metric Orientation Lifting

- Let $\gamma = (\Gamma, \theta) \in C^1([0,1], \Omega \times S^1)$, where $\theta \in S^1 = [0, 2\pi[$
- Let $\vec{v}_\theta = (\cos\theta, \sin\theta)$ be the unit direction vector

Orientation-lifted Metric

$$\mathcal{F}^\infty_\gamma(\dot\gamma) := \begin{cases} \|\dot\Gamma\| + \frac{|\dot\theta|^2}{\|\dot\Gamma\|}, & \text{if } \dot\Gamma \text{ is positively proportional with } \vec{v}_\theta, \\ \infty, & \text{otherwise.} \end{cases}$$

$$\int_0^L (1+\kappa^2(s))\, ds = \int_0^L \mathcal{F}^\infty_\gamma(\dot\gamma(s))\, ds$$

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Curvature Penalized Minimal Path Method with A Finsler Metric Orientation Lifting

- Let $\gamma = (\Gamma, \theta) \in C^1([0,1], \Omega \times S^1)$, where $\theta \in S^1 = [0, 2\pi[$
- Let $\vec{v}_\theta = (\cos\theta, \sin\theta)$ be the unit direction vector

Orientation-lifted Metric

$$\mathcal{F}^\infty_\gamma(\dot\gamma) := \begin{cases} \|\dot\Gamma\| + \frac{|\dot\theta|^2}{\|\dot\Gamma\|}, & \text{if } \dot\Gamma \text{ is positively proportional with } \vec{v}_\theta, \\ \infty, & \text{otherwise.} \end{cases}$$

$$\mathcal{F}^\lambda_\gamma(\gamma') := \sqrt{\lambda^2\|\Gamma'\|^2 + 2\alpha\lambda|\theta'|^2} - (\lambda-1)\langle \vec{v}_\theta, \Gamma'\rangle, \quad (14)$$

for any $\gamma = (\Gamma, \theta) \in \Omega \times \mathbb{S}^1$ and any $\gamma' = (\Gamma', \theta') \in \mathbb{R}^2 \times \mathbb{R}$, and where $\vec{v}_\theta = (\cos\theta, \sin\theta)$.

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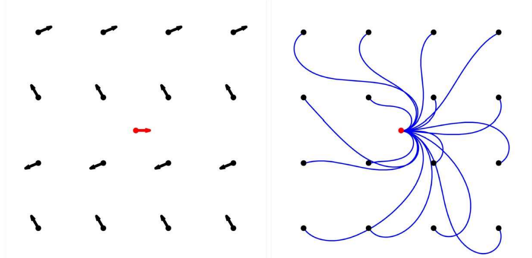
Finsler Elastica curves

$$\mathcal{L}_E(\Gamma) = \int_0^L (1+\kappa^2(s))\, ds$$



(a) (b)

Figure 10: Finsler elastica curves associated with the metric $\mathfrak{F}_\varepsilon$. **a** The source position and the end positions are denoted by the red and black dots respectively. Each of the positions is assigned a direction indicated by a arrow. **b** Minimal Paths (blue lines) associated to the initializations shown in **a**.

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Curvature Penalized Minimal Path Method with A Finsler Metric

Euler's Elastica Bending Energy

$$\mathcal{L}(\Gamma) = \int_0^L \left(\frac{1}{\alpha(s)} + \frac{1}{\beta(s)}\kappa^2(s)\right) ds, \quad (5)$$

where $\Gamma : [0, L] \to \Omega$ is a regular curve, s is arc-length parameter, κ is the curvature and L is the classical Euclidean curve length.

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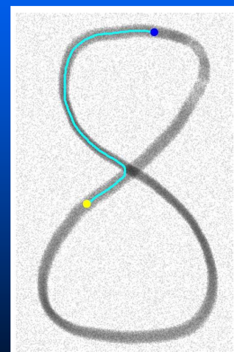
Curvature Penalized Minimal Path Method with A Finsler Metric



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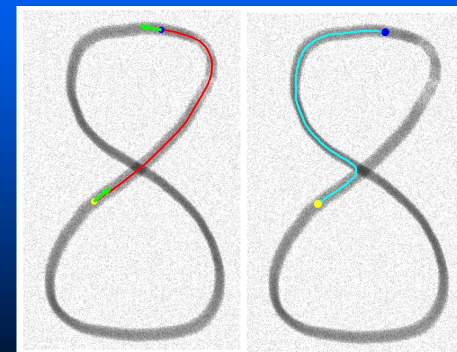
Curvature Penalized Minimal Path Method with A Finsler Metric



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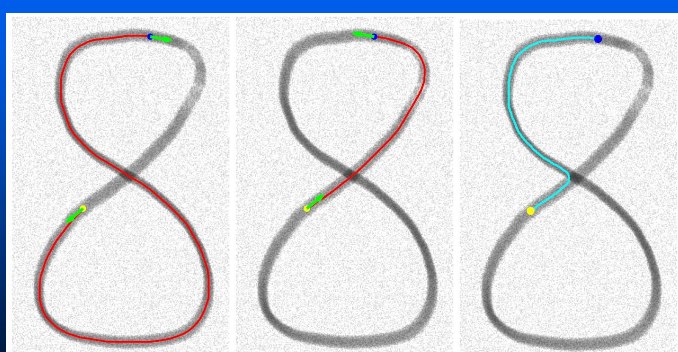
Curvature Penalized Minimal Path Method with A Finsler Metric



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Curvature Penalized Minimal Path Method with A Finsler Metric

Fig. 8 Geodesics extraction results using the proposed Finsler metric. Red and green dots are the initial and end positions respectively. Arrows indicate the corresponding tangents.

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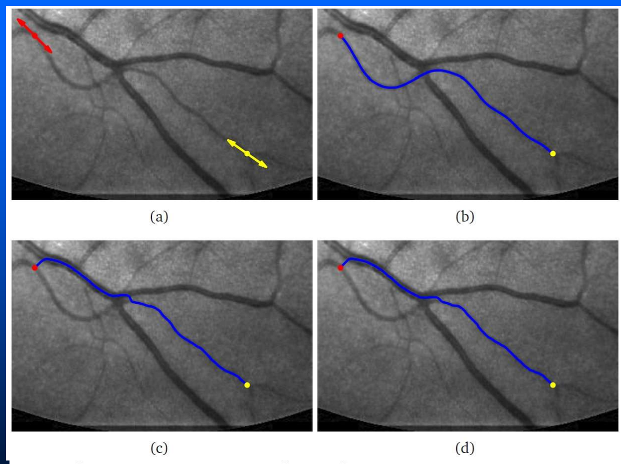

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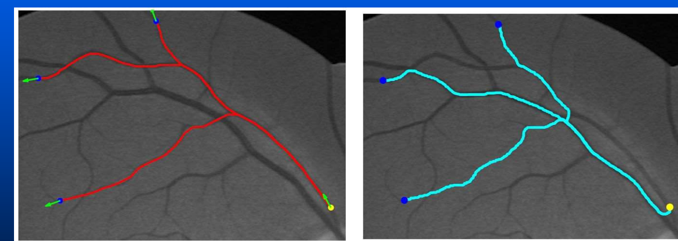

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Curvature Penalized Minimal Path Method with A Finsler Metric

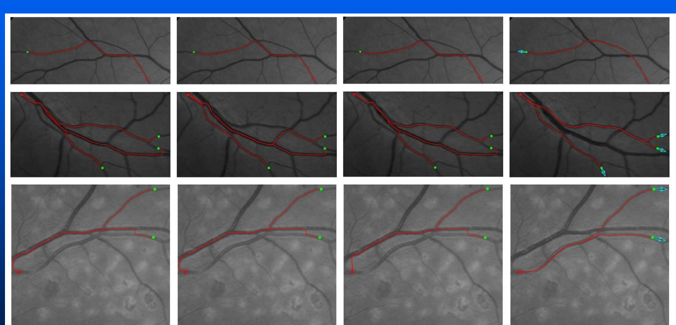

Fig. 15 Comparative blood vessels extraction results on retinal images. **Columns 1-4** present the extracted minimal paths using the IR metric, the AR metric, the IOLR metric, and the proposed Finsler metric respectively.

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Curvature Penalized Minimal Path Method with A Finsler Metric

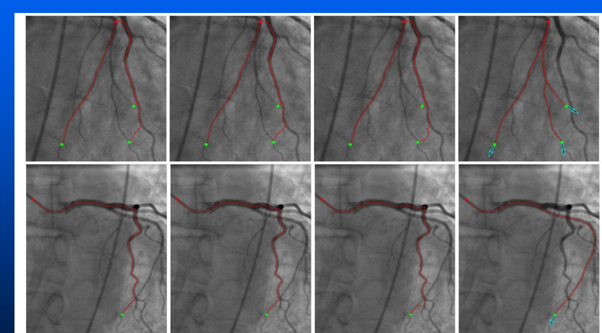

Fig. 16 Comparative blood vessel extraction results on fluoroscopy images. **Columns 1-4** present the extracted minimal paths using the IR metric, the AR metric, the IOLR metric, and the proposed Finsler metric respectively.

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Curvature Penalized Minimal Path Method with A Finsler Metric

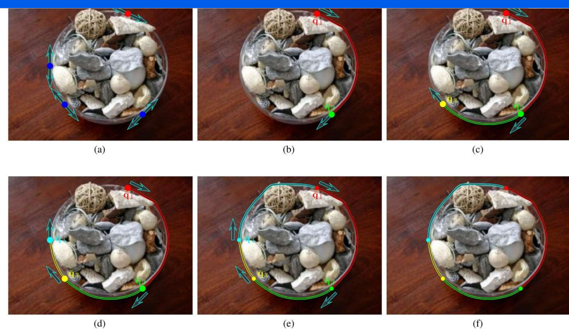

Fig. 4 Steps for the closed contour detection procedure. (**a**) original image and all vertice in $\mathcal{D}$. (**b**) The first pairs of vertice: $\{\bar{\mathbf{q}}_1, \bar{\mathbf{q}}_2\}$ are found. (**c**-**d**) the remained vertice are detected. **e** joining $\bar{\mathbf{q}}_4$ and $\bar{\mathbf{q}}_1$ to form a closed contour. (**f**) final closed contour detection results.

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Curvature Penalized Minimal Path Method with A Finsler Metric

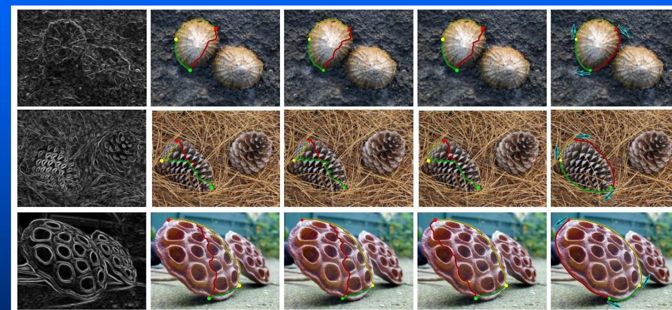

Fig. 9 Comparative closed contour detection results. **Column 1**: edge saliency map. **Columns 2-5**: results from the IR metric, the AR metric, the IOLR metric and the proposed Finsler metric. In Column 5, arrows indicate the tangents for the corresponding physical positions denoted by dots.

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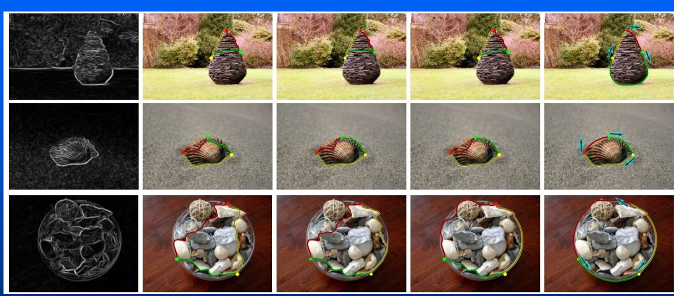
Curvature Penalized Minimal Path Method with A Finsler Metric



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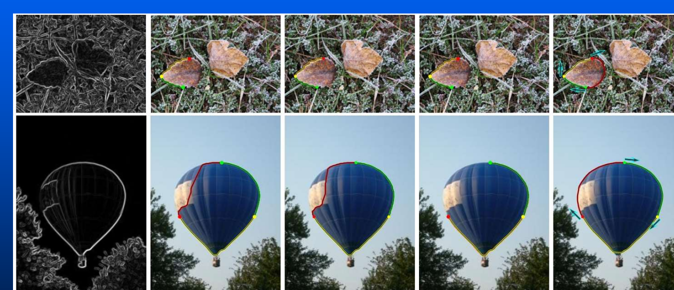
Curvature Penalized Minimal Path Method with A Finsler Metric



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Curvature Penalized Minimal Path Method with A Finsler Metric

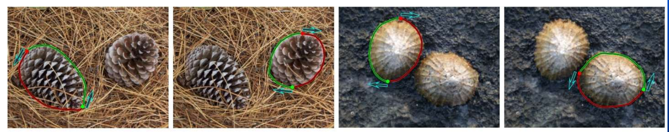

Fig. 10 Closed contour detection results using only two given physical positions.

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Finsler Metrics

- Finsler Metric: A more general setting
- Finsler Metric: Segmentation with Curvature Penalization
- Finsler Metric: Region-Based Segmentation
- Finsler Metric: Active Contour with Alignment term

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Region Based Minimal Path Method with A Finsler Metric

with Da Chen and JM Mirebeau, 2016

- Region Based Active Contour Energy

$$F(B) = c + \alpha \int_B f(\mathbf{x}) d\mathbf{x} + \ell(\gamma)$$

- Transformed to Finsler Geodesic path

$$length(\gamma) := \int_0^1 \mathcal{F}(\gamma(t), \gamma'(t))\, dt, \quad \left(\mathcal{F} : \Omega \times \mathbb{R}^n \to \mathbb{R}^+\right)$$

- Region B delimited by a set of paths

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Region Based Minimal Path Method with A Finsler Metric

Example of Region Based Active Contour

$$E(\mathcal{C}, \mu_{\text{in}}, \mu_{\text{out}}) = \alpha \int_{A_\mathcal{C}} (I - \mu_{\text{in}})^2 d\mathbf{x} + \alpha \int_{\Omega \setminus A_\mathcal{C}} (I - \mu_{\text{out}})^2 d\mathbf{x} + \mathscr{L}_{\text{Euclid}}(\mathcal{C})$$

$$\rho_{\tilde{\mathcal{C}}} = (I(\mathbf{x}) - \mu_{\text{in}}[\mathcal{C}_k])^2 - (I - \mu_{\text{out}}[\mathcal{C}_k])^2$$

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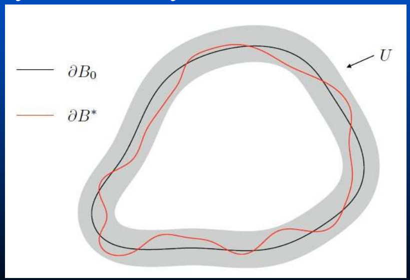
Region Based Minimal Path: Problem Formulation

- Find a shape B* minimizing the region-based functional F, with boundary included in U (tubular neighborhood of initial boundary).
- Boundary formed by a set of minimal paths.



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Region Based Minimal Path: Reformulation

- Define $\mathcal{V}_\perp$ such that $\nabla \cdot \mathcal{V}_\perp(\mathbf{x}) = \alpha f(\mathbf{x})\chi_U(\mathbf{x})$
- The region-based energy can be reformulated as:

$$\mathcal{E}_U(B) = \alpha \int_B f(\mathbf{x})\chi_U(\mathbf{x})d\mathbf{x} + \ell(\gamma) + \text{Constant}$$
$$= \int_B \nabla \cdot \mathcal{V}_\perp(\mathbf{x})d\mathbf{x} + \ell(\gamma)$$
divergence theorem →
$$= \int_0^1 \Big(\|\gamma'(t)\| + \langle M^T\mathcal{V}_\perp, M^T\mathcal{N}\rangle\|\gamma'(t)\|\Big)dt$$
$$= \int_0^1 \Big(\|\gamma'(t)\| + \langle \mathcal{V}, \gamma'(t)\rangle\Big)dt$$

M: counter-clockwise rotation matrix with angle $\pi/2$

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Region Based Minimal Path: Reformulation

$$\mathcal{E}_U(B) = \int_0^1 \Big(\|\gamma'(t)\| + \langle \mathcal{V}(\gamma(t)), \gamma'(t)\rangle\Big)dt, \quad (\gamma := \partial B)$$
$$= \int_0^1 \mathcal{F}(\gamma(t), \gamma'(t))\,dt$$

- Positivity condition

$\|\mathcal{V}\|_\infty < 1 \quad \Rightarrow \mathcal{F}$ is a Finsler (Randers) metric

- $\mathcal{E}_U$ thus can be minimized using the Eikonal framework

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Region Based Minimal Path: Field Computation

- $\mathcal{V}_\perp$ is obtained for $\forall \mathbf{x} \in U$ by solving

$$\min\left\{\int_U \|\mathcal{V}_\perp(\mathbf{x})\|^2\, d\mathbf{x}\right\}, \quad s.t. \quad \nabla \cdot \mathcal{V}_\perp(\mathbf{x}) = \alpha\, f(\mathbf{x}).$$

- $\|\mathcal{V}\|_\infty$ is bounded by the area of U
- U is the search space for the next evolutional curve
- A new vector field can be constructed:

$$\bar{\mathcal{V}}(\mathbf{x}) = T(\|\mathcal{V}_\perp(\mathbf{x})\|)\,\frac{M^{-1}\mathcal{V}_\perp(\mathbf{x})}{\|\mathcal{V}_\perp(\mathbf{x})\|}, \quad \forall \mathbf{x} \in \Omega$$

where $T(x) = 1 - \exp(-x)$, $x > 0$.

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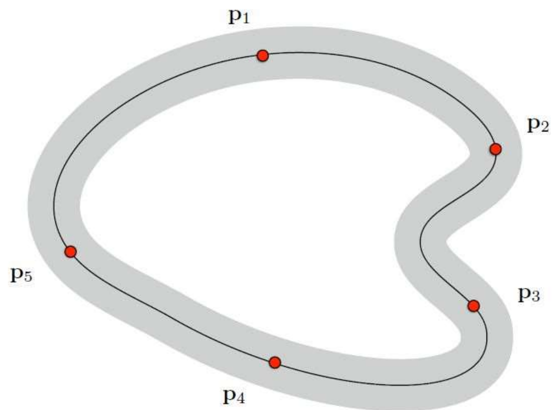
Region Based Minimal Path: Geodesic Evolution

1. Sample successive points in **clock-wise** order



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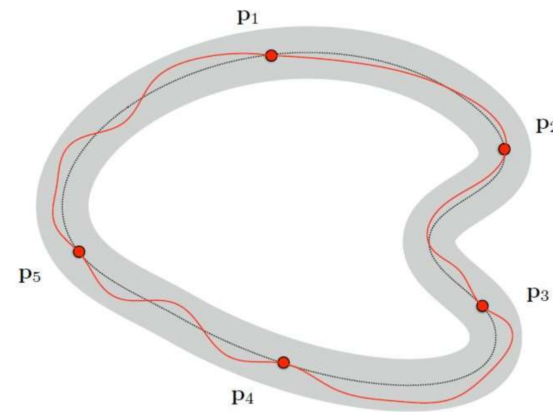
Region Based Minimal Path: Geodesic Evolution

2. Join these successive points by geodesics



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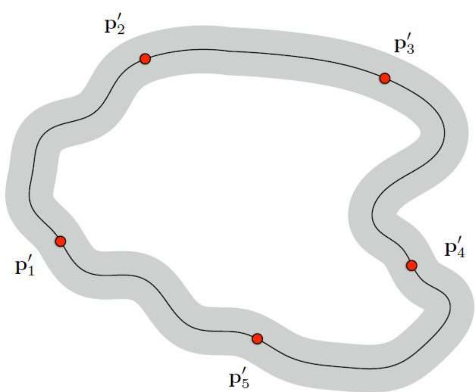
Region Based Minimal Path: Geodesic Evolution

3. Update the tubular neighbourhood and the Finsler metric



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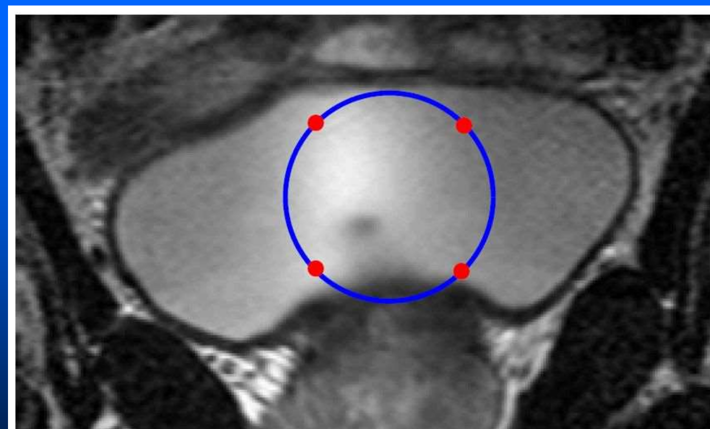
Region Based Minimal Path Method with a Finsler Metric



Initialization

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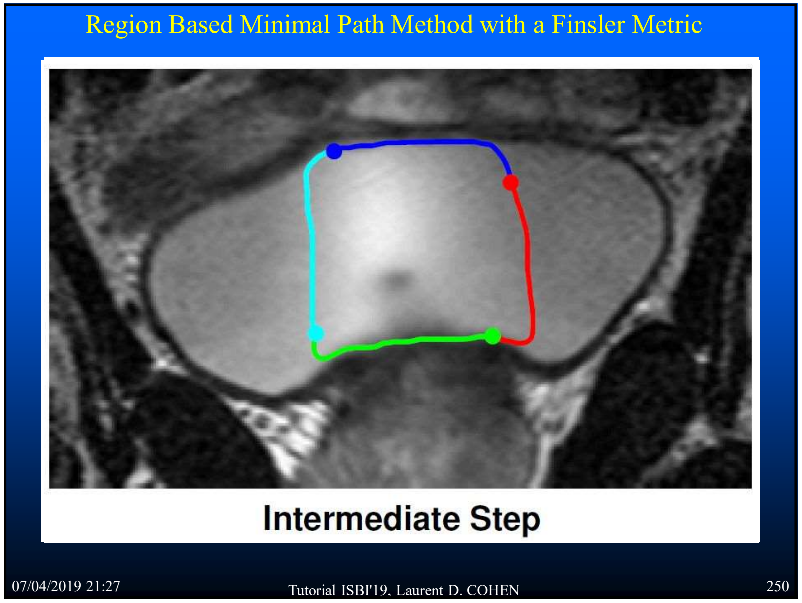

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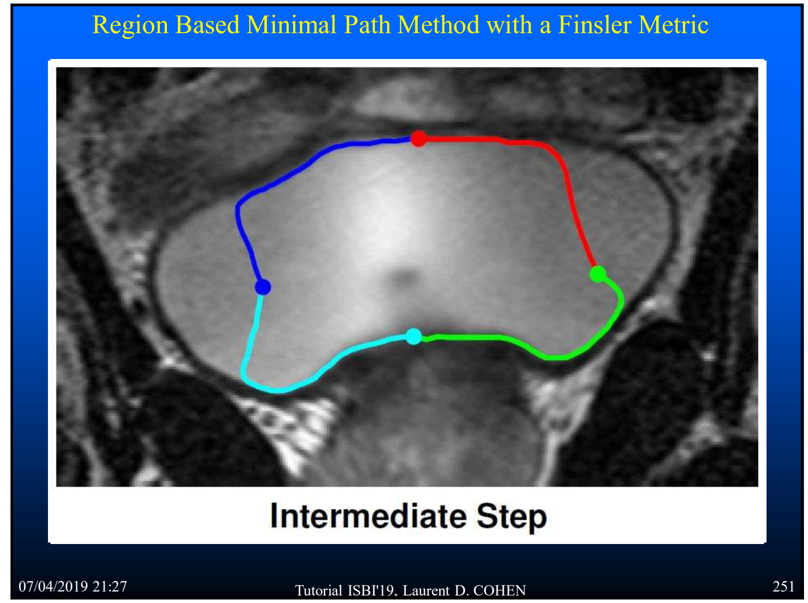

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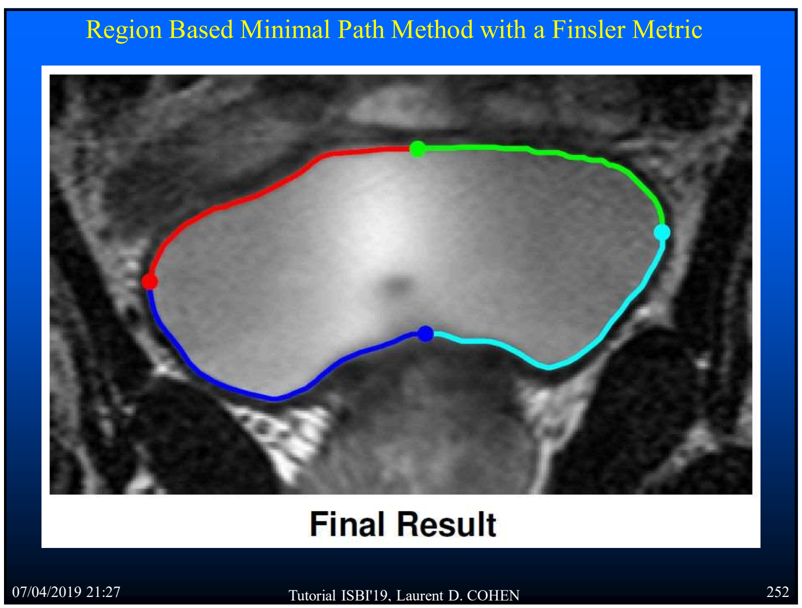

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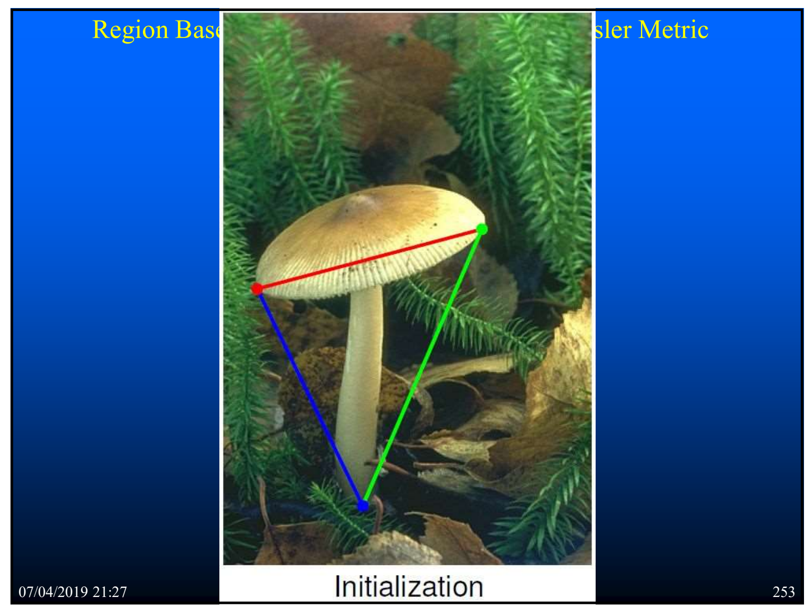

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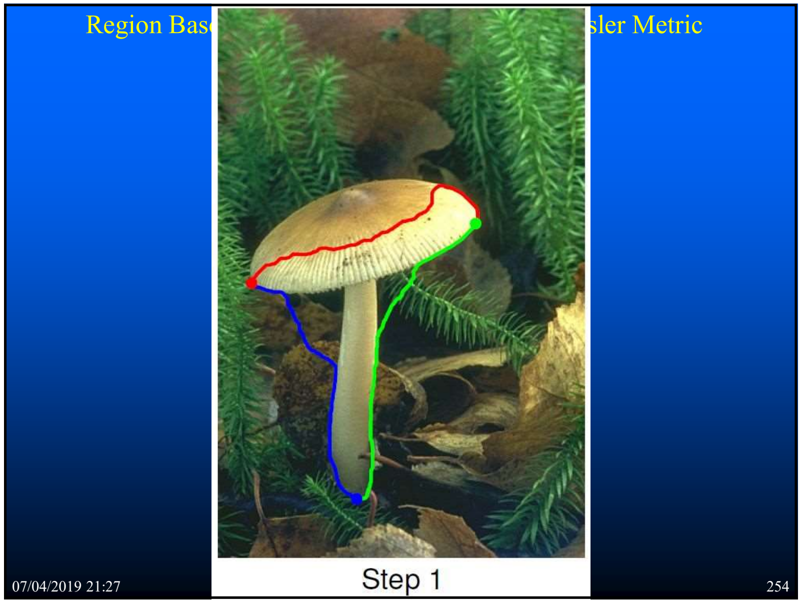

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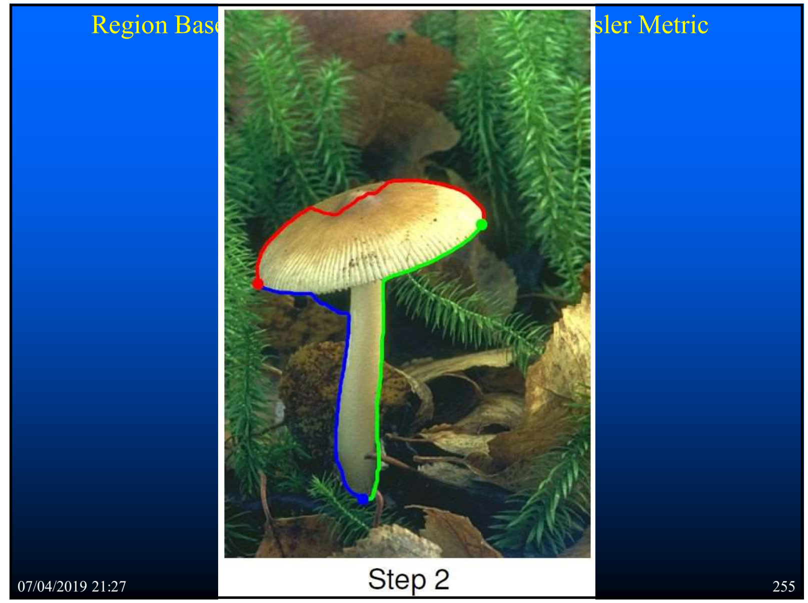

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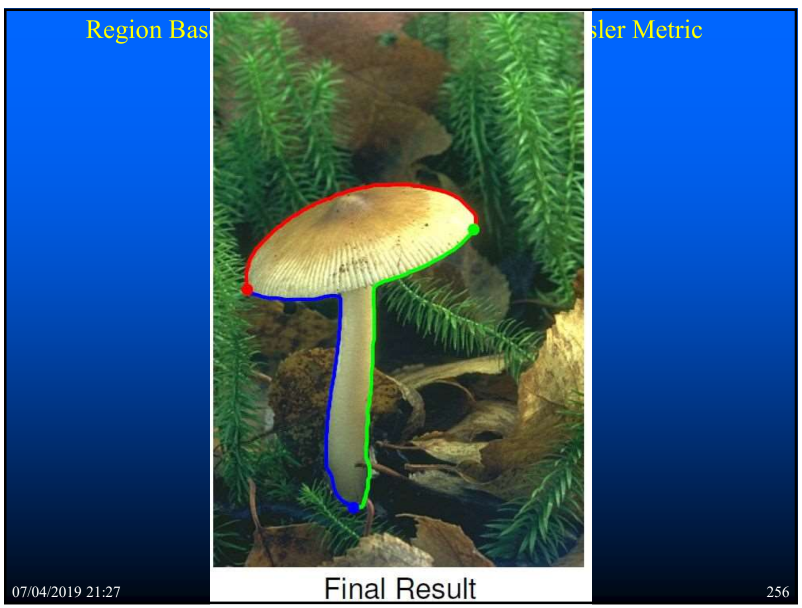

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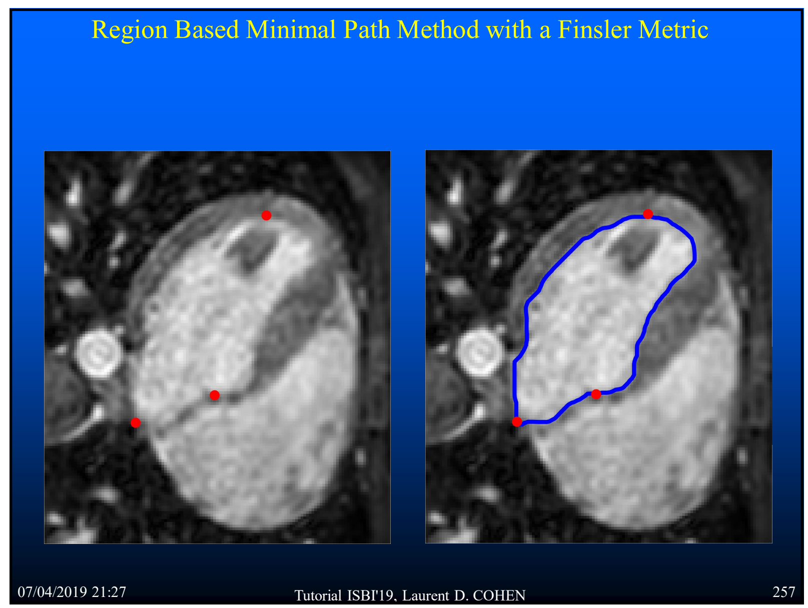

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Finsler Metrics

- Finsler Metric: A more general setting
- Finsler Metric: Segmentation with Curvature Penalization
- Finsler Metric: Region-Based Segmentation
- Finsler Metric: Active Contour with Alignment term

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Geometric active contour models with alignment terms

- Energy Minimization:

$$\mathscr{L}_\varrho(\mathcal{C}) = \int_0^1 \varrho(\langle V_{\text{align}}(\mathcal{C}), \mathcal{N}\rangle)\|\mathcal{C}'\|\,du$$

- *C(s)=(x(s),y(s))* curve drawn on the image
- The curve should align in order to be orthogonal to the Image Gradient vector

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Geometric active contour models with alignment terms

- Energy Minimization:

$$\mathscr{L}_{\text{align}}(\mathcal{C}) = \int_0^1 \langle \nabla I(\mathcal{C}), \mathcal{N}\rangle\,\|\mathcal{C}'\|\,du$$

- *C(s)=(x(s),y(s))* curve drawn on the image
- The curve should align in order to be orthogonal to the Image Gradient vector
- Curve Evolution: $\frac{\partial \mathcal{C}_\tau}{\partial \tau} = \Delta I_\sigma(\mathcal{C}_\tau)\,\mathcal{N}_\tau$

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Geometric active contour models with alignment terms

- Energy Minimization:

$$\mathscr{L}_{\text{align}}(\mathcal{C}) = \int_0^1 \langle \nabla I(\mathcal{C}), \mathcal{N}\rangle\,\|\mathcal{C}'\|\,du$$

$$\begin{aligned}\mathcal{L}_{\text{align}}(\mathcal{C}) &= \mathscr{L}_{\text{Euclid}}(\mathcal{C}) - \beta\mathscr{L}_{\text{align}}(\mathcal{C})\\ &= \int_0^1 \|\mathcal{C}'\|du - \beta\int_0^1 \langle \mathcal{N}, \nabla I_\sigma(\mathcal{C}\rangle\,\|\mathcal{C}'\|du\\ &= \int_0^1 \left(1 - \beta\langle \mathcal{N}, \nabla I_\sigma(\mathcal{C})\rangle\right)\|\mathcal{C}'\|du,\end{aligned}$$

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Geometric active contour models with alignment terms

- Energy Minimization:

$$\mathscr{L}_{\text{align}}(\mathcal{C}) = \int_0^1 \langle \nabla I(\mathcal{C}), \mathcal{N} \rangle \, \|\mathcal{C}'\| \, du$$

$$\begin{aligned}\mathcal{L}_{\text{align}}(\mathcal{C}) &= \int_0^1 \left(1 - \beta \langle \mathbf{M}\mathcal{N}, \mathbf{M}\nabla I_\sigma(\mathcal{C}) \rangle\right) \|\mathcal{C}'\| du,\\ &= \int_0^1 \left(\|\mathcal{C}'\| - \beta\langle \mathcal{C}', \xi(\mathcal{C})\rangle\right) du,\\ &= \int_0^1 \mathfrak{F}_{\text{align}}(\mathcal{C}(u), \mathcal{C}'(u)) du,\end{aligned}$$

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Geometric active contour models with alignment terms

- Energy Minimization:

$$\begin{aligned}\mathcal{L}_{\text{align}}(\mathcal{C}) &= \int_0^1 \left(1 - \beta \langle \mathbf{M}\mathcal{N}, \mathbf{M}\nabla I_\sigma(\mathcal{C}) \rangle\right) \|\mathcal{C}'\| du,\\ &= \int_0^1 \left(\|\mathcal{C}'\| - \beta\langle \mathcal{C}', \xi(\mathcal{C})\rangle\right) du,\\ &= \int_0^1 \mathfrak{F}_{\text{align}}(\mathcal{C}(u), \mathcal{C}'(u)) du,\end{aligned}$$

$$\mathfrak{F}_{\text{align}}(\mathbf{x}, \vec{\mathbf{u}}) = \|\vec{\mathbf{u}}\| - \langle \vec{\mathbf{u}}, \beta\xi(\mathbf{x})\rangle$$

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Geometric active contour models with alignment terms

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Geometric active contour models with alignment terms

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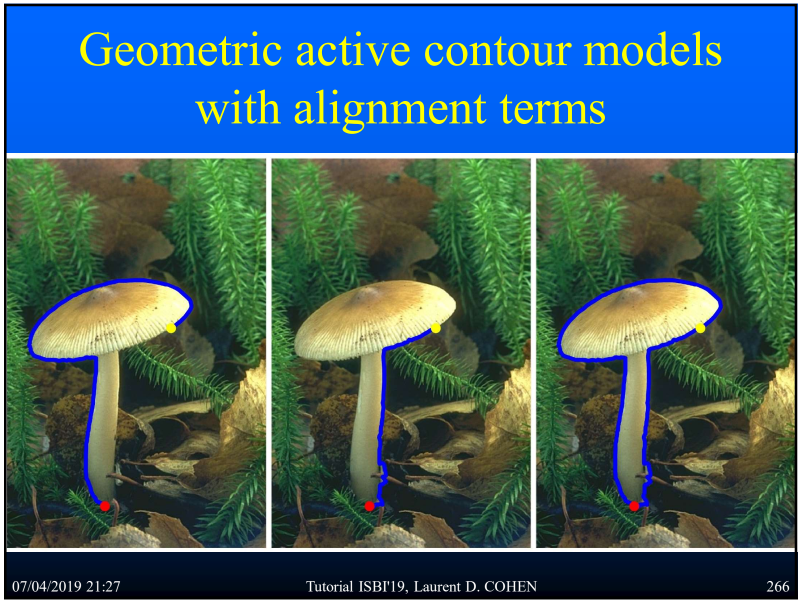

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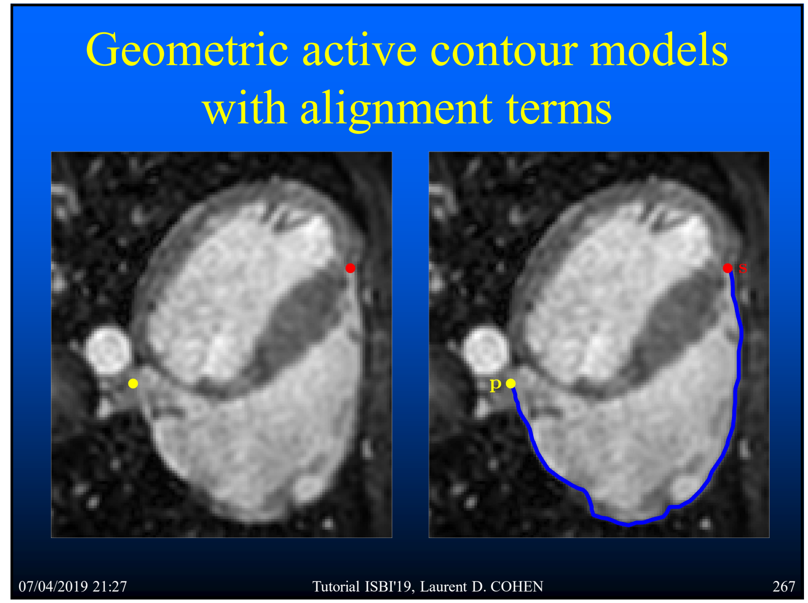

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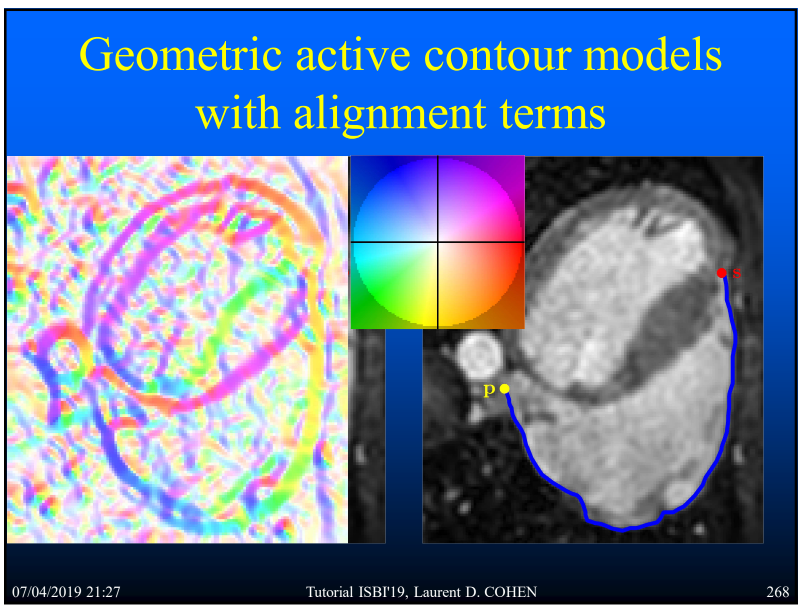

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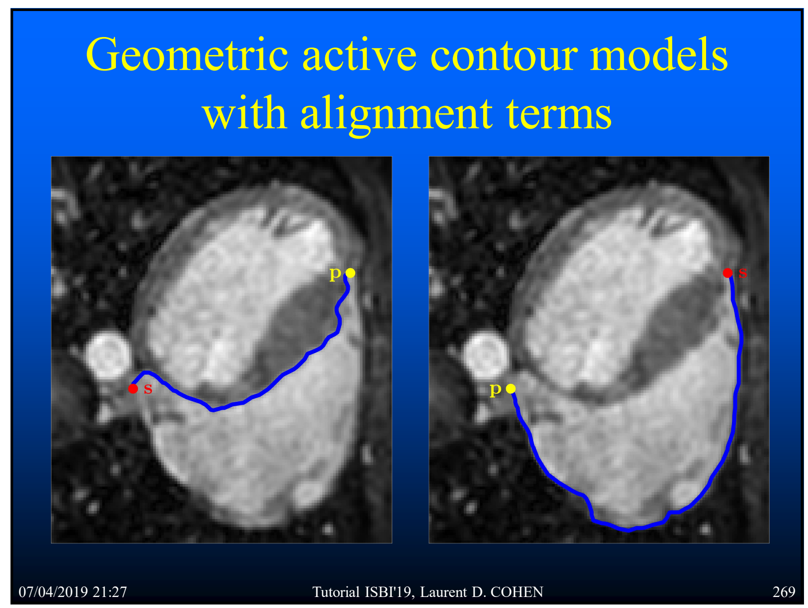

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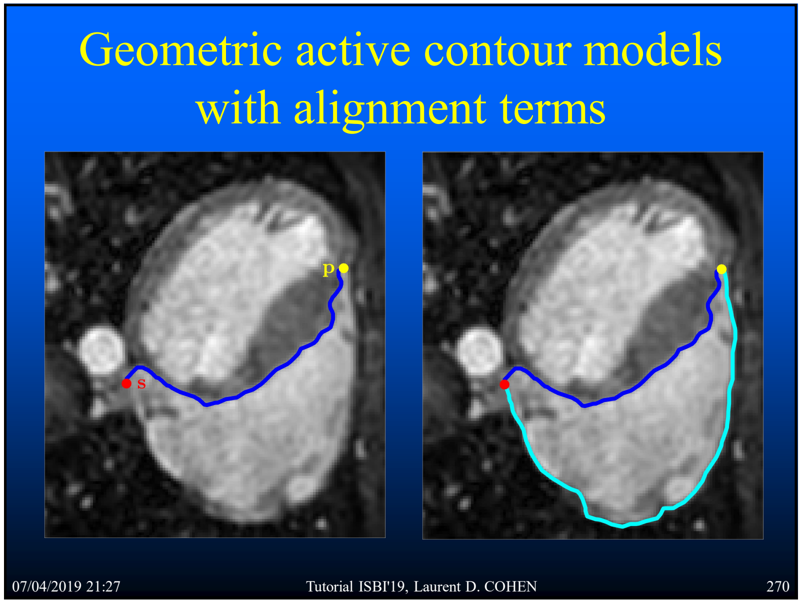

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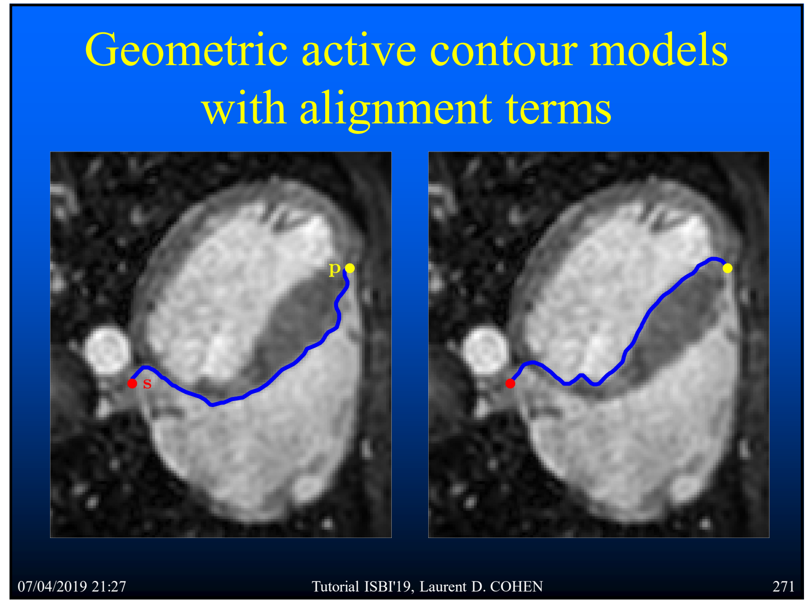

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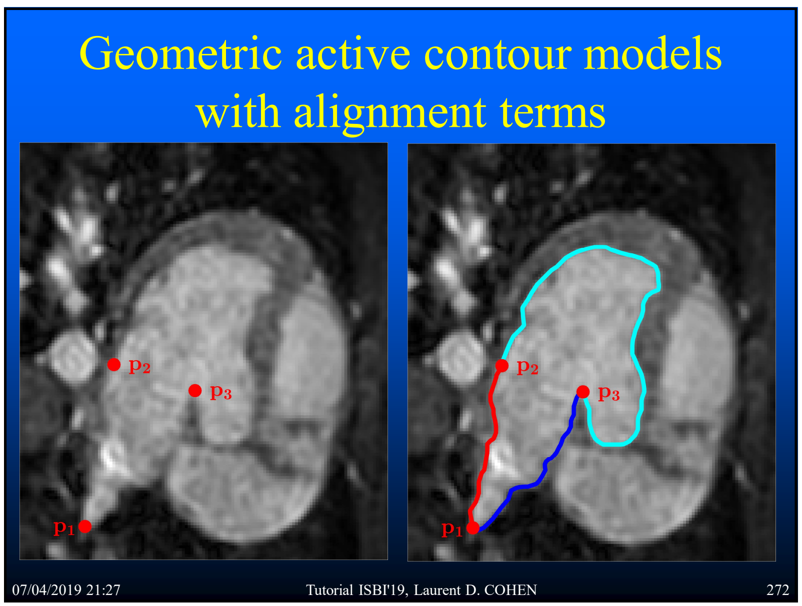

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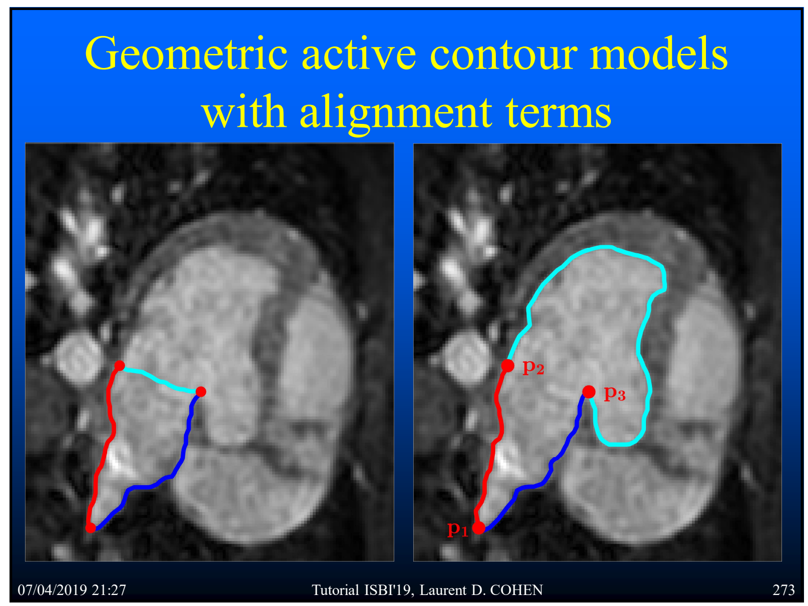

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Geodesic Minimal Paths

- Minimal paths, Eikonal Equation, Fast Marching and Front Propagation
- 3D Fast Marching, some examples
- Anisotropic Fast Marching
- Finsler Metrics for Various Active Contours Energy terms
- Closed Contour as a set of minimal paths. Key points method
- Geodesic Voting and tree structure segmentation
- Application to Virtual Endoscopy and Vessel Visualization

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Geodesic Minimal Paths

- Minimal paths, Eikonal Equation, Fast Marching and Front Propagation
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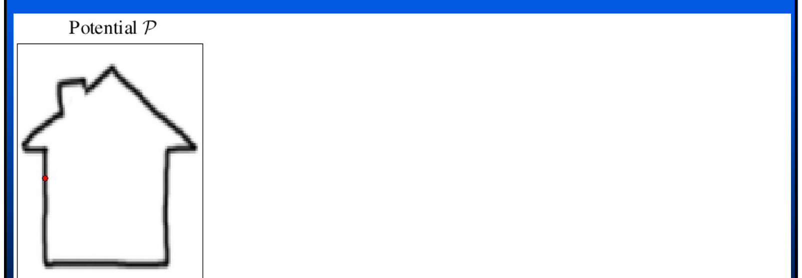
Finding a closed contour by growing minimal paths and adding keypoints



Finding a Closed Boundary by Growing Minimal Paths from a Single Point on 2D or 3D Images. Fethallah Benmansour and Laurent D. Cohen. Journal of Mathematical Imaging and Vision. To appear, 2009.

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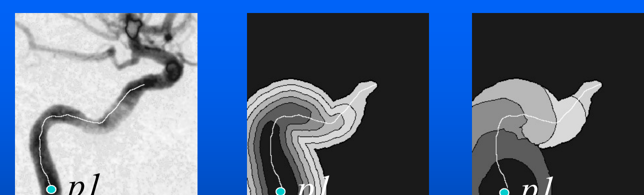
One end point front propagation



A minimal path in a DSA image of brain vessels — Action map — Euclidean path length

- Sometimes difficult to give a second end point (3D, complex shape).
- Simultaneously compute the Euclidean length of the minimal path at each point visited. Low cost: included in the fast marching with potential P = 1.
- Stopping point for propagation by maximum length condition.
- It reduces user interaction to the definition of the starting point.

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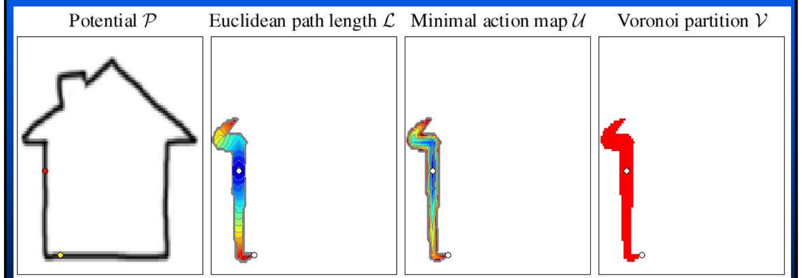
Finding a closed contour by growing minimal paths and adding keypoints



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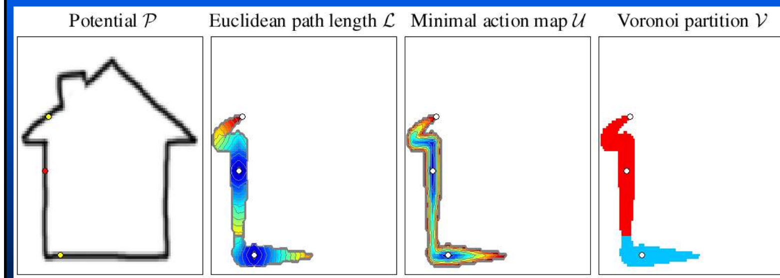
Finding a closed contour by growing minimal paths and adding keypoints



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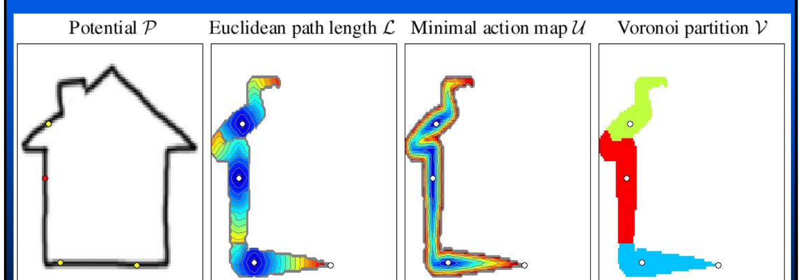
Finding a closed contour by growing minimal paths and adding keypoints



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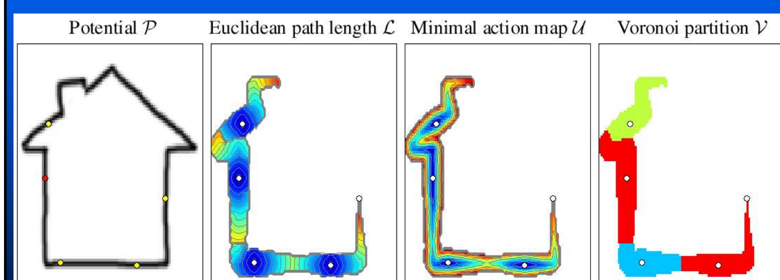
Finding a closed contour by growing minimal paths and adding keypoints



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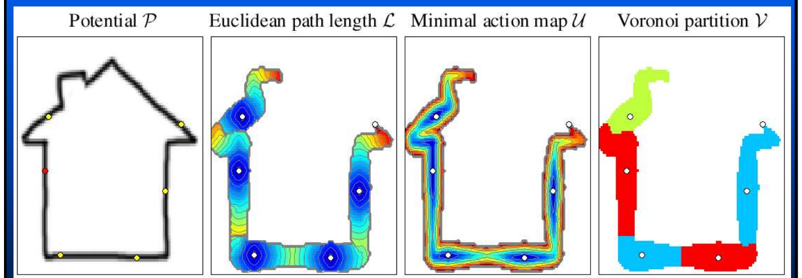
Finding a closed contour by growing minimal paths and adding keypoints



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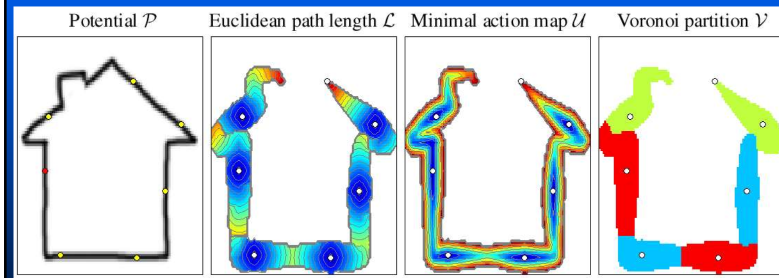
Finding a closed contour by growing minimal paths and adding keypoints



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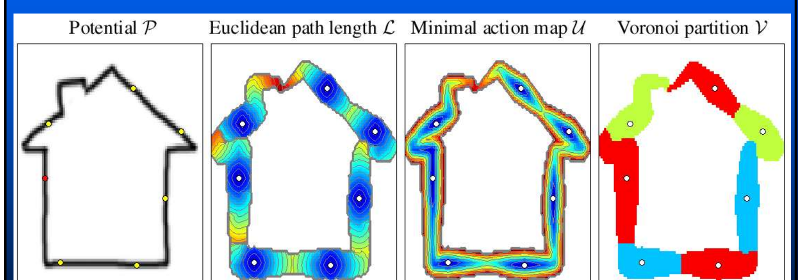
Finding a closed contour by growing minimal paths and adding keypoints



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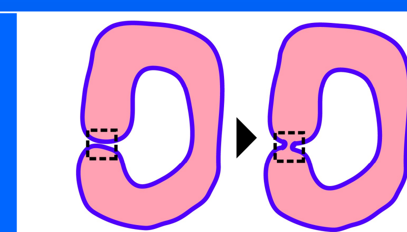
Adding keypoints: Stopping criterion

The propagation must be stopped as soon as the domain visited by the fronts has the same topology as a ring.



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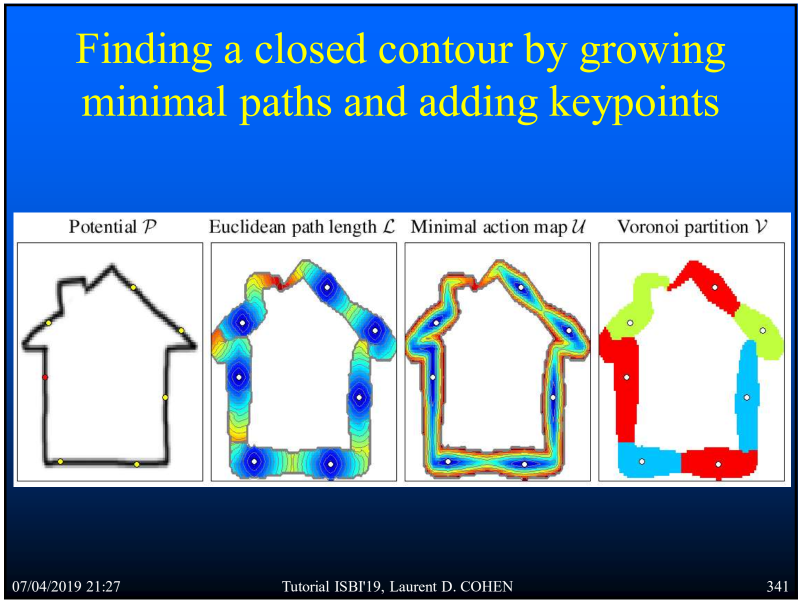

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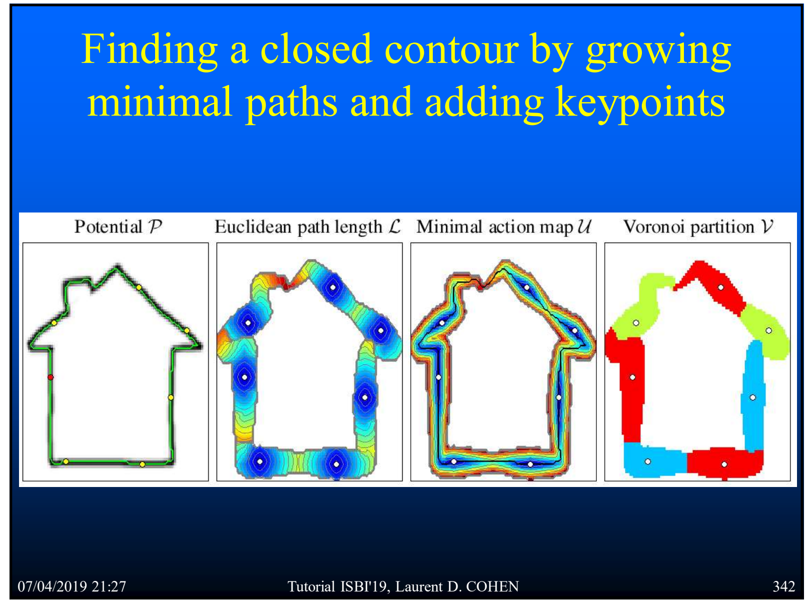

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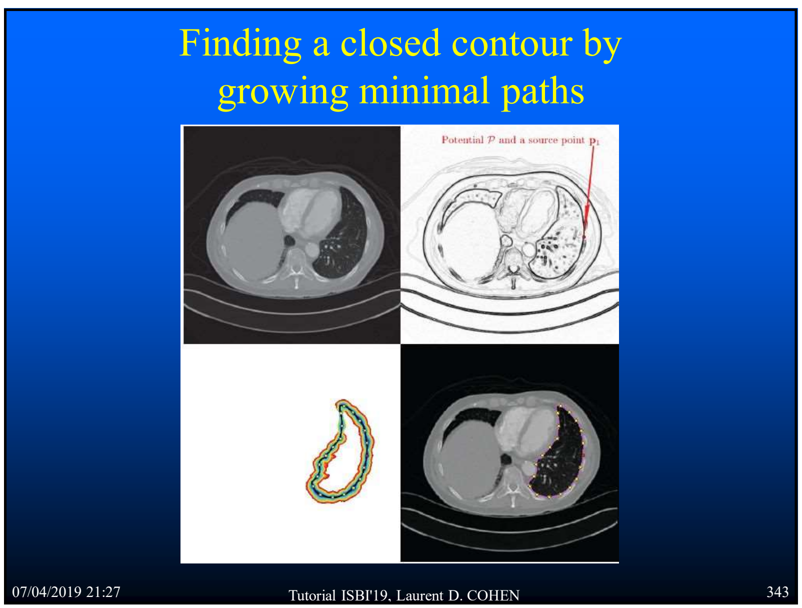

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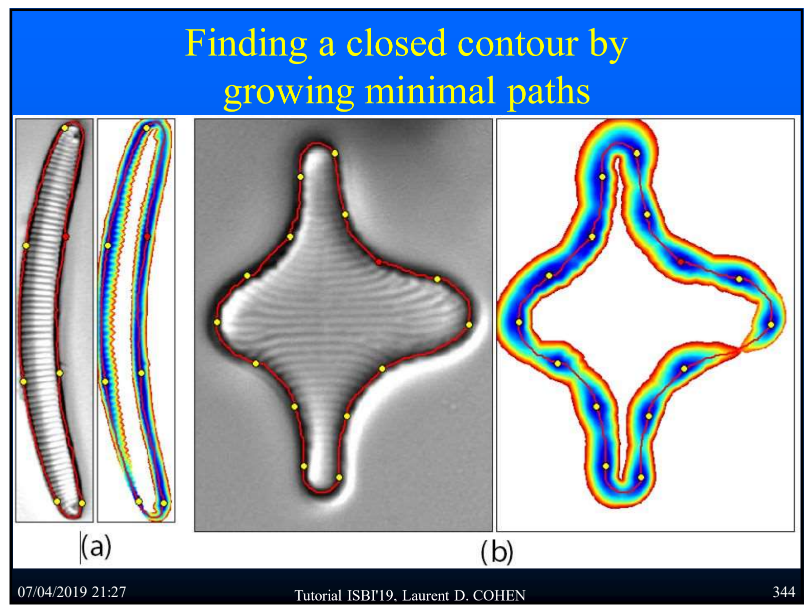

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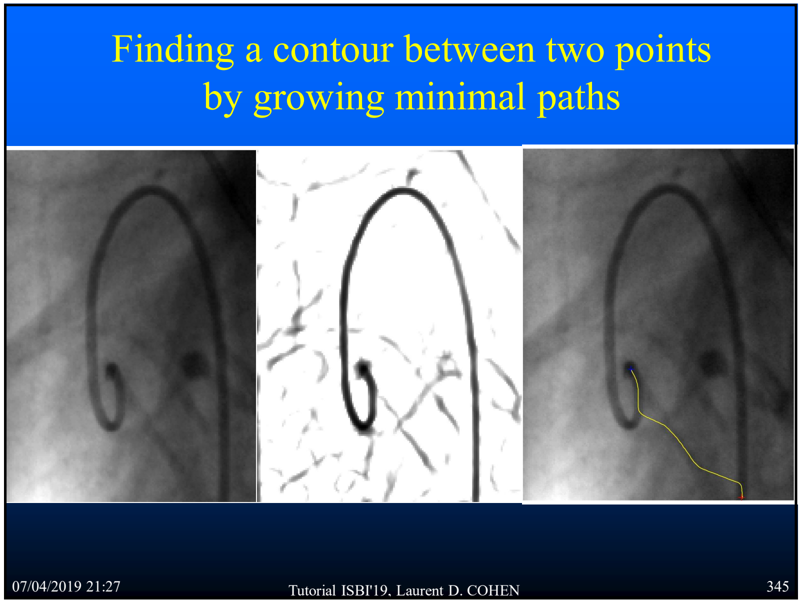

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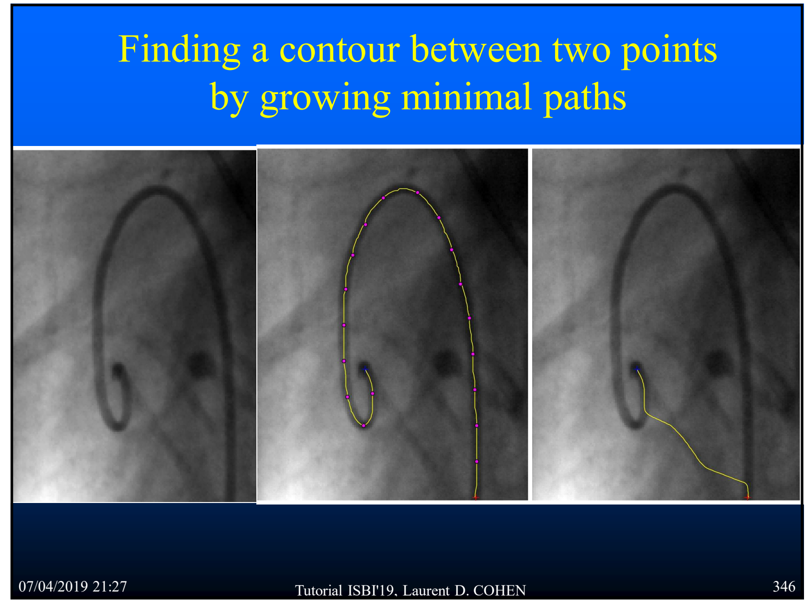

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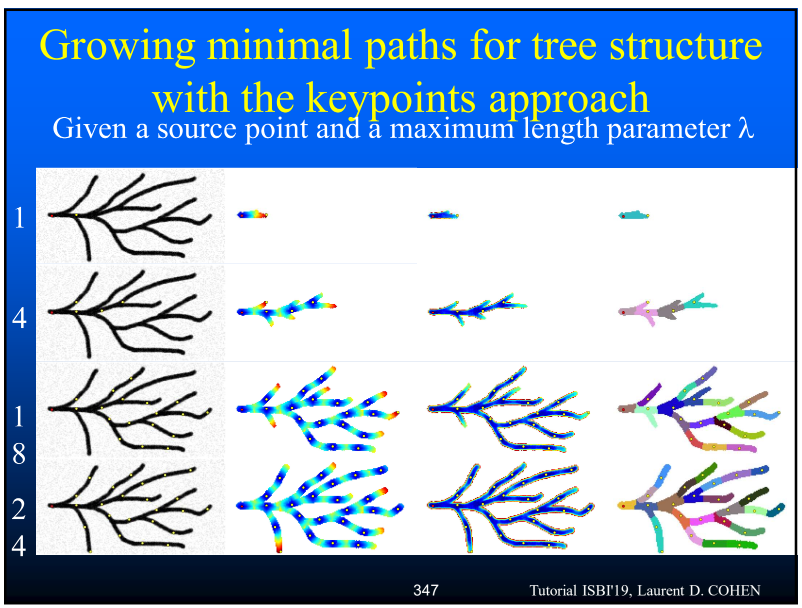

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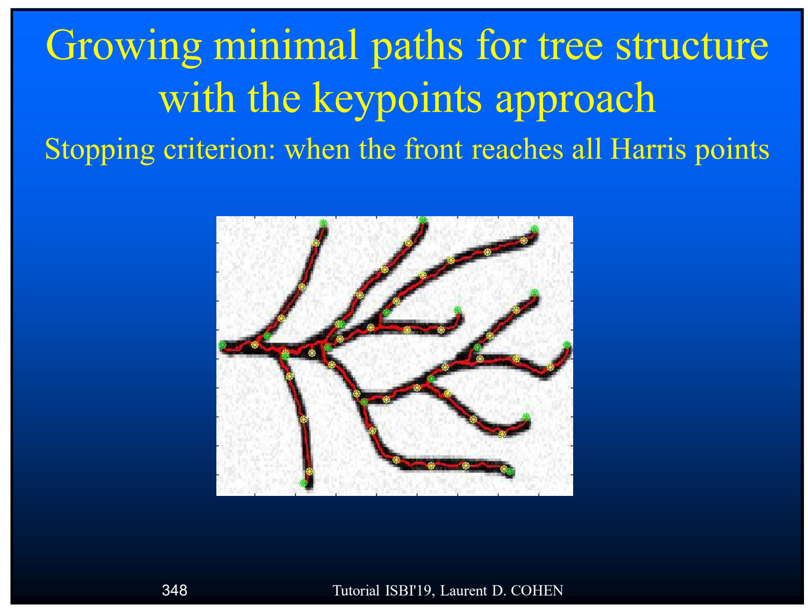

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Extension to 3D vessel segmentation

Example of results of the keypoints method in a 3D image of Pulmonary Arteries

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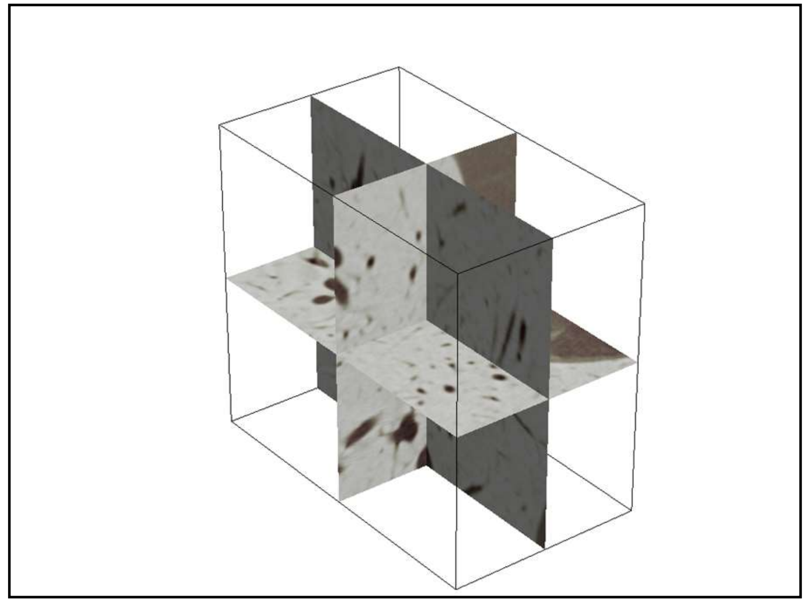

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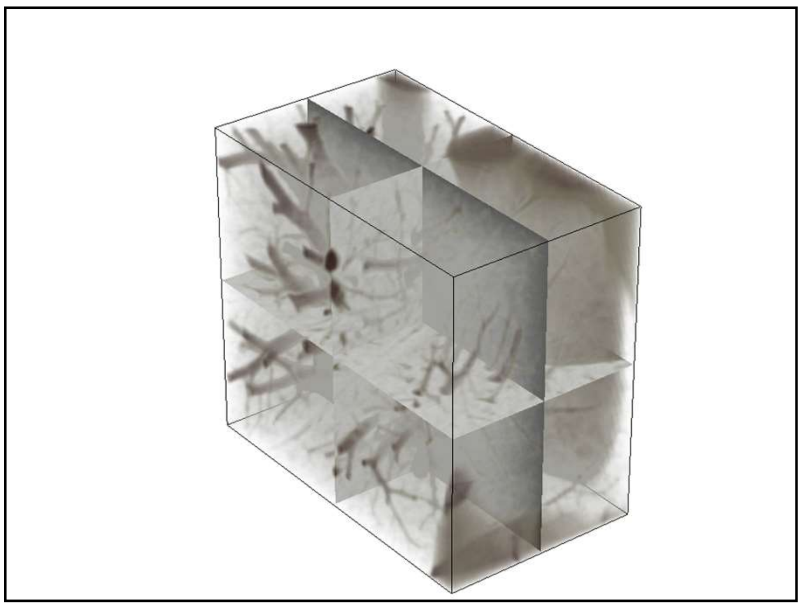

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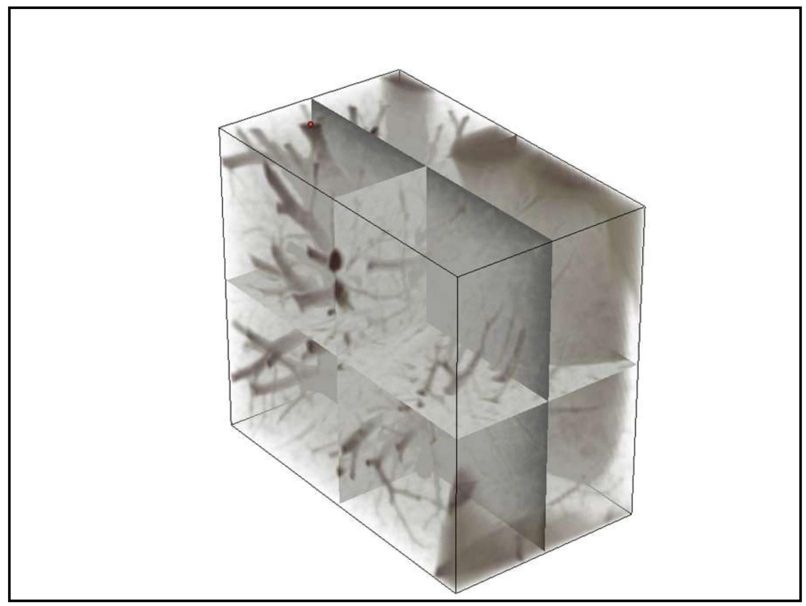

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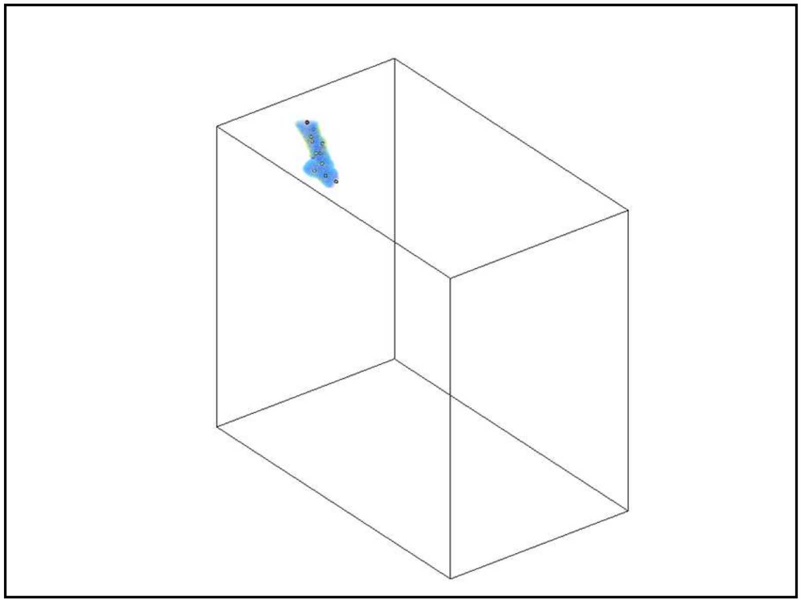
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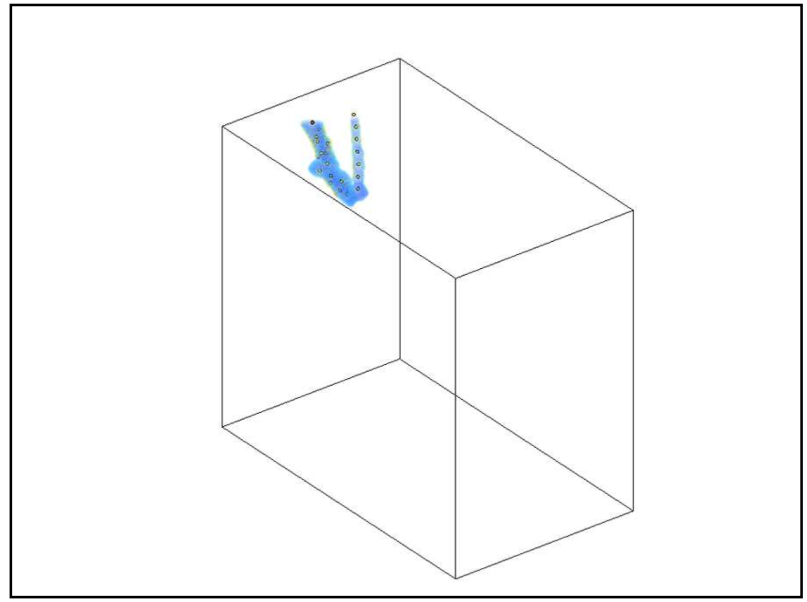
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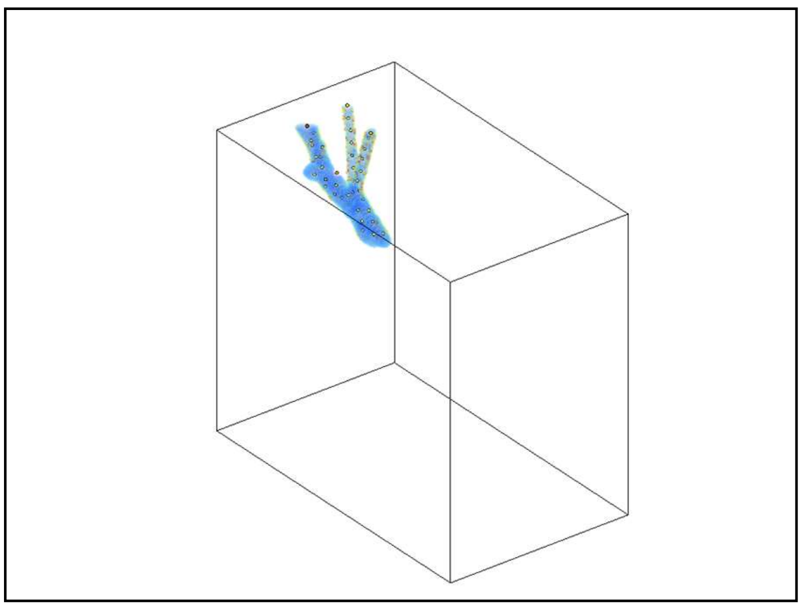
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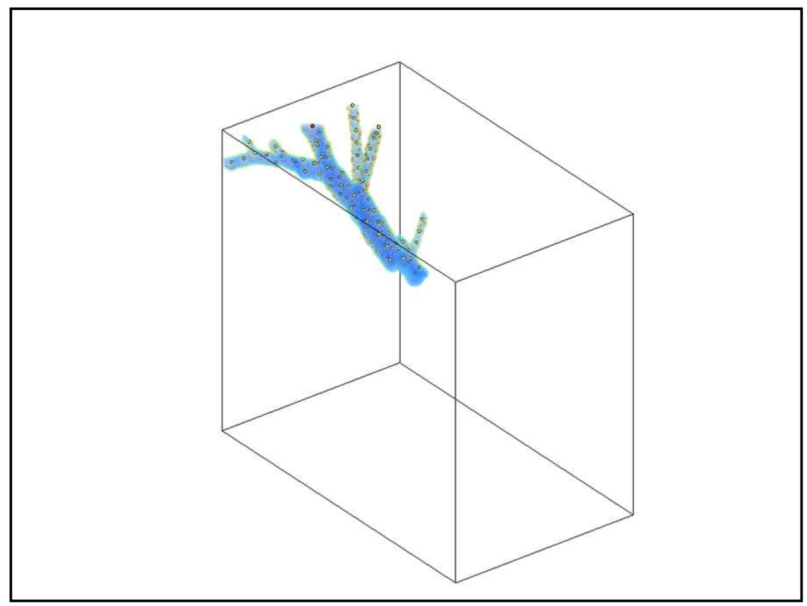
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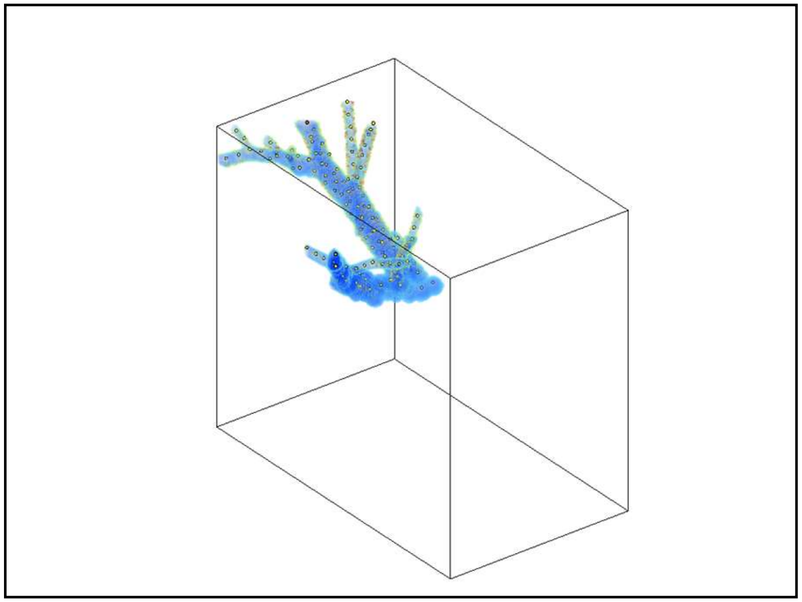

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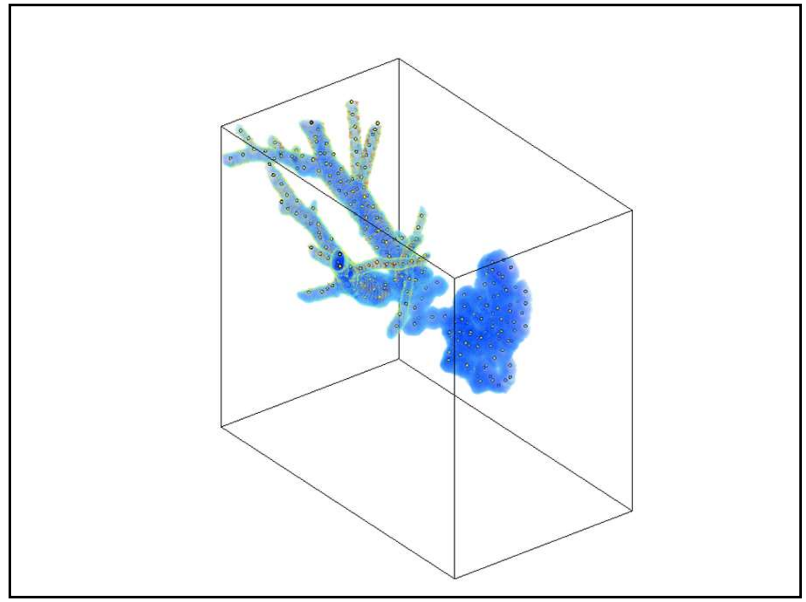

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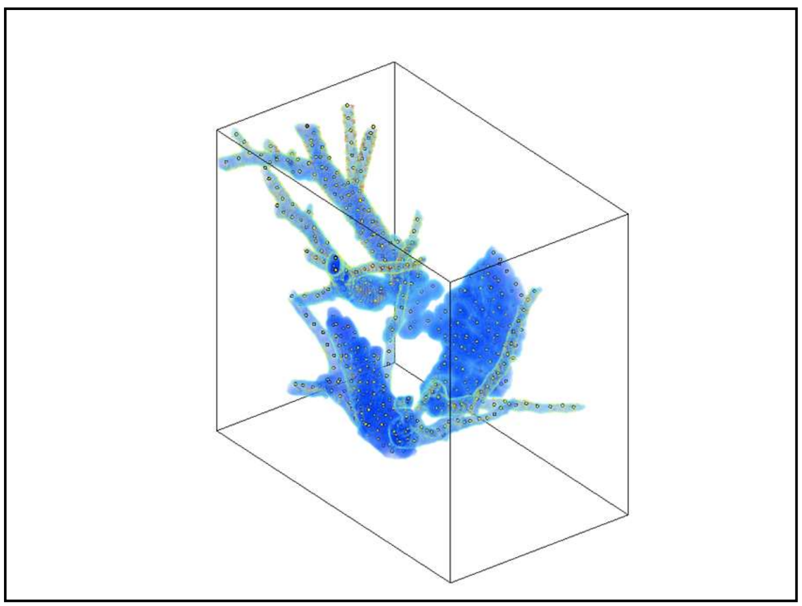

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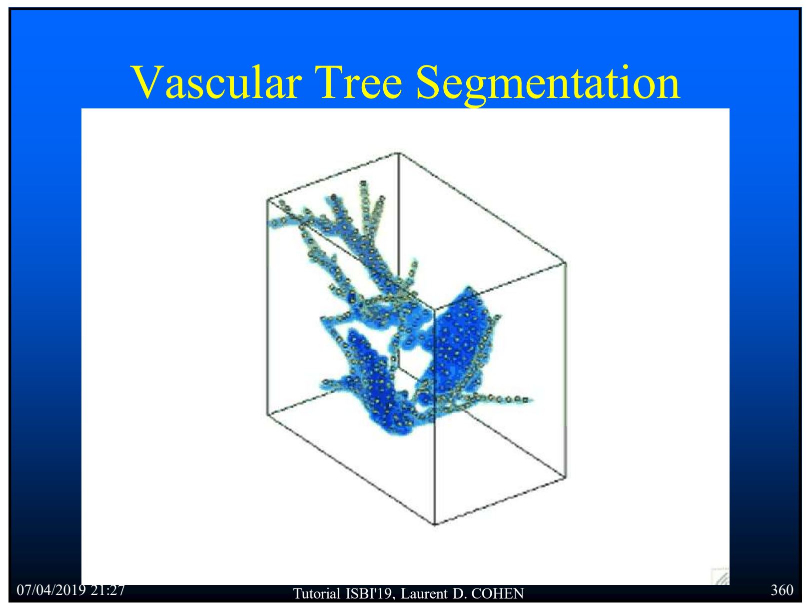

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Keypoints and 3D Minimal Paths for tubular shapes in 2D+radius

(with Li and Yezzi, MICCAI'09)

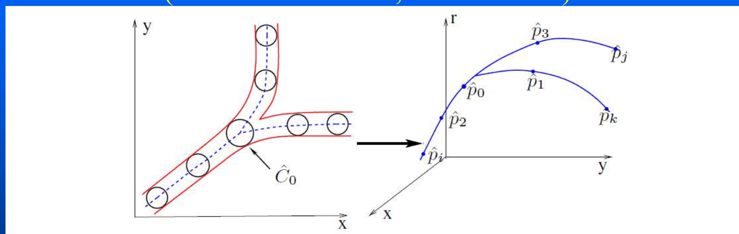

Fig. 1. The entire multi-branch structure extraction is reduced to finding structures between all adjacent key point pairs. The 4D path length D between each key point pair is equal to d_{step}. For easier visualization, the same concept is illustrated here using circles instead of spheres.

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Keypoints and 3D Minimal Paths for tubular shapes in 2D

2D in space , 1D for radius of vessel

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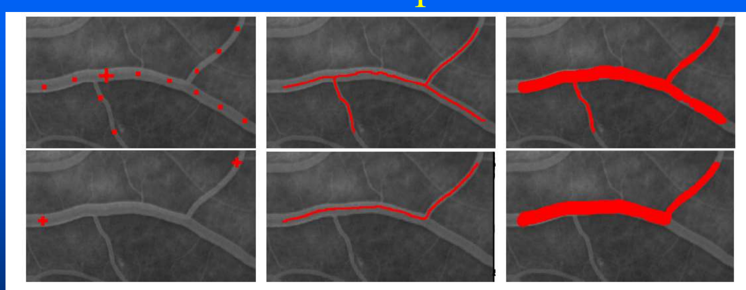
Keypoints and 3D Minimal Paths for tubular shapes in 2D



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Keypoints and 3D Minimal Paths for tubular shapes in 2D

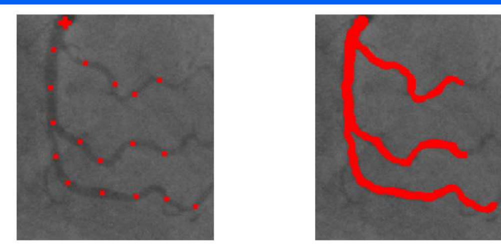

Fig. 3. Segmentation results via the proposed method on another 2D projection angiogram image. Panels from left to right show the initial point and the detected iterative key points and the detected vessel surfaces.

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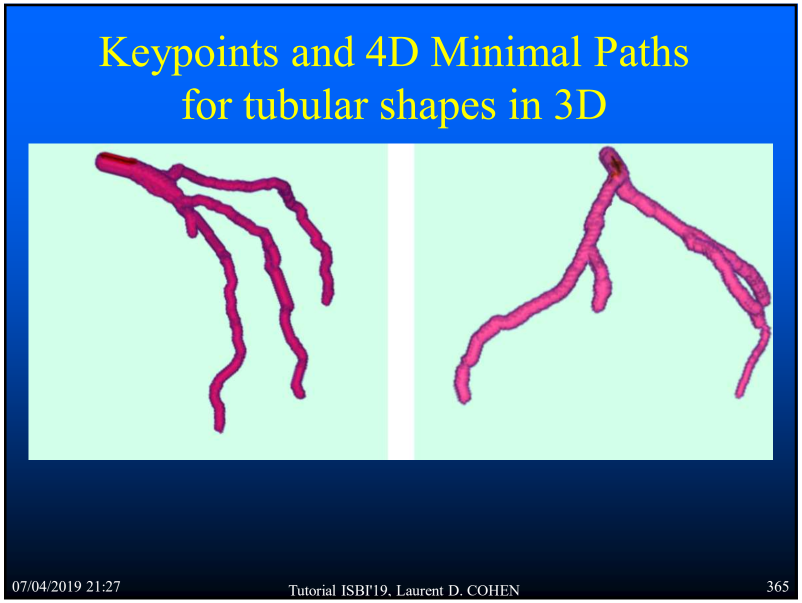

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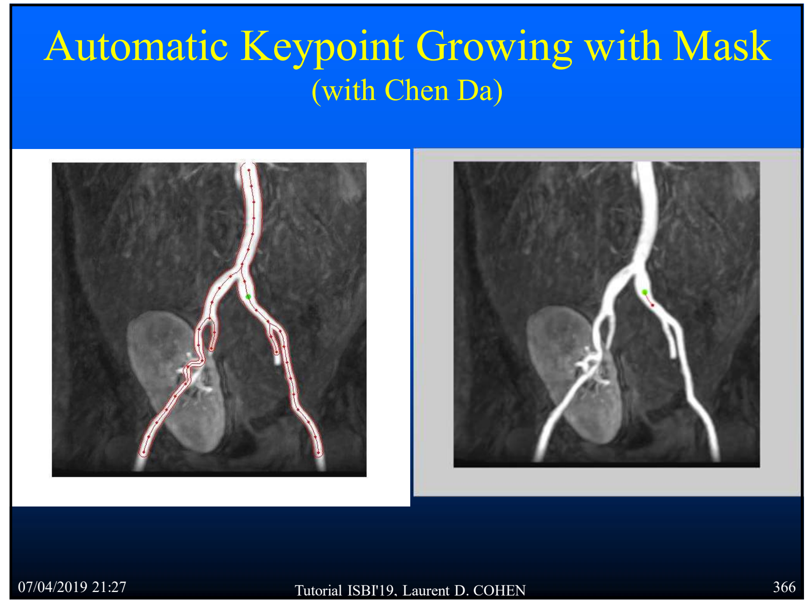

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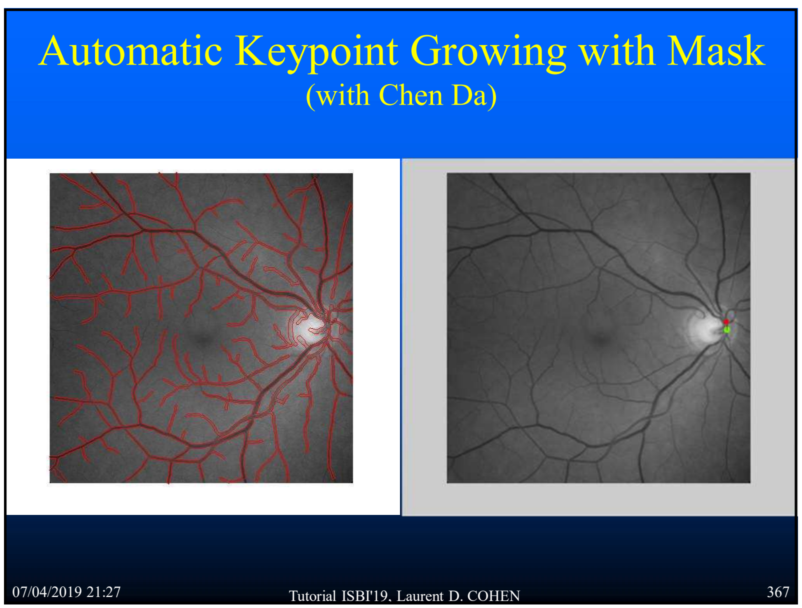

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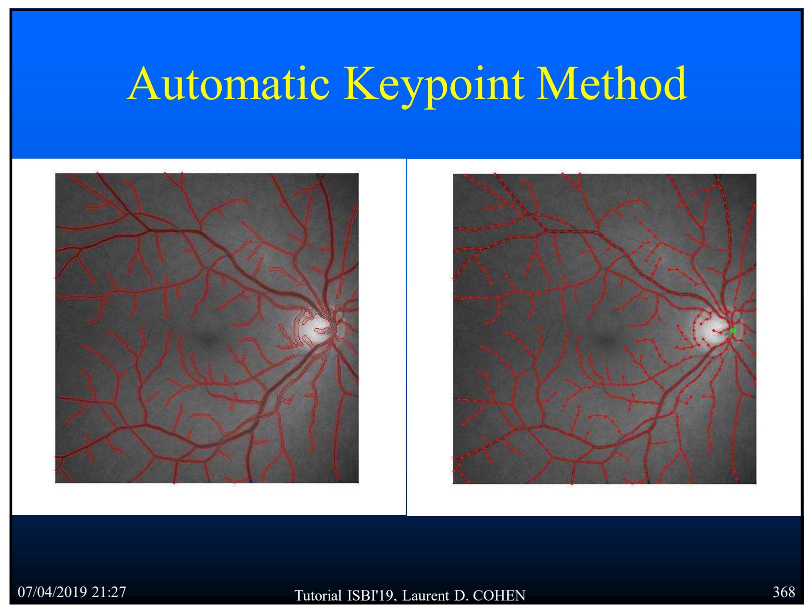

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Geodesic Minimal Paths

- Minimal paths, Eikonal Equation, Fast Marching and Front Propagation
- 3D Fast Marching, some examples
- Anisotropic Fast Marching
- Finsler Metrics for Various Active Contours Energy terms
- Closed Contour as a set of minimal paths. Key points method
- Geodesic Voting and tree structure segmentation
- Application to Virtual Endoscopy and Vessel Visualization

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Geodesic Minimal Paths

- Minimal paths, Eikonal Equation , Fast Marching and Front Propagation
- 3D Fast Marching, some examples
- Fast Marching on a surface and adaptive Remeshing
- Anisotropic Fast Marching
- Closed Contour as a set of minimal paths. Perceptual Grouping. Key points method
- Geodesic Voting and tree structure segmentation
- Adding iteratively Key points for geodesic meshing
- Surface between two curves as a network of paths
- Path Network and Transport Equation
- Application to Virtual Endoscopy
- Segmentation by Fast Marching : Freezing, Dual fronts

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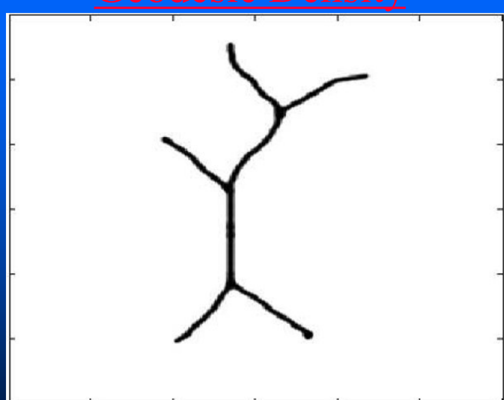
Geodesic Density



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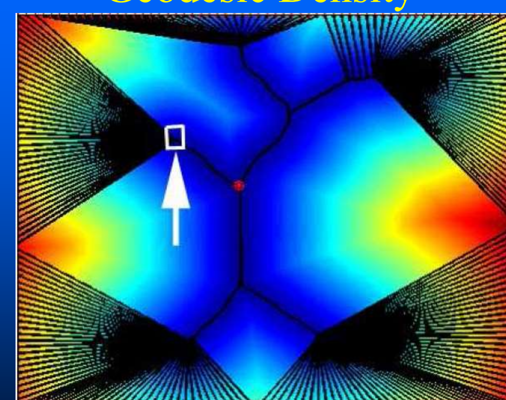
Geodesic Density



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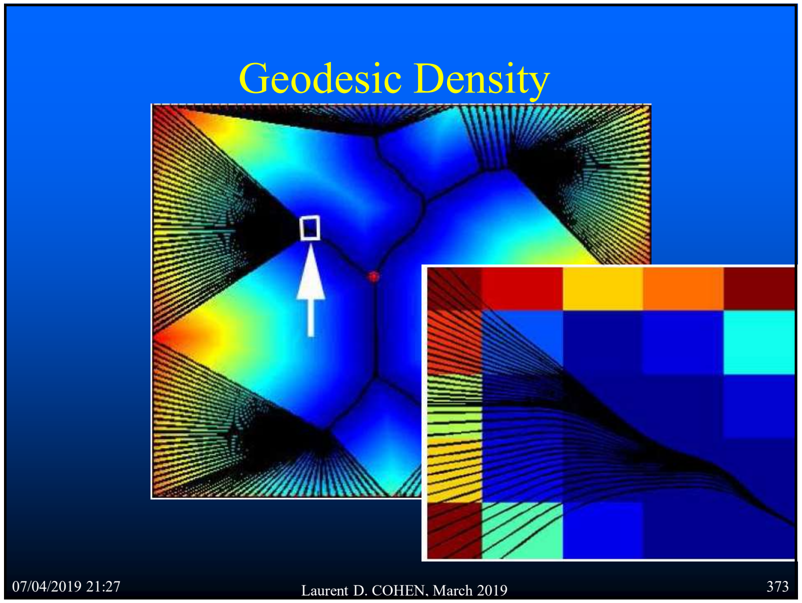

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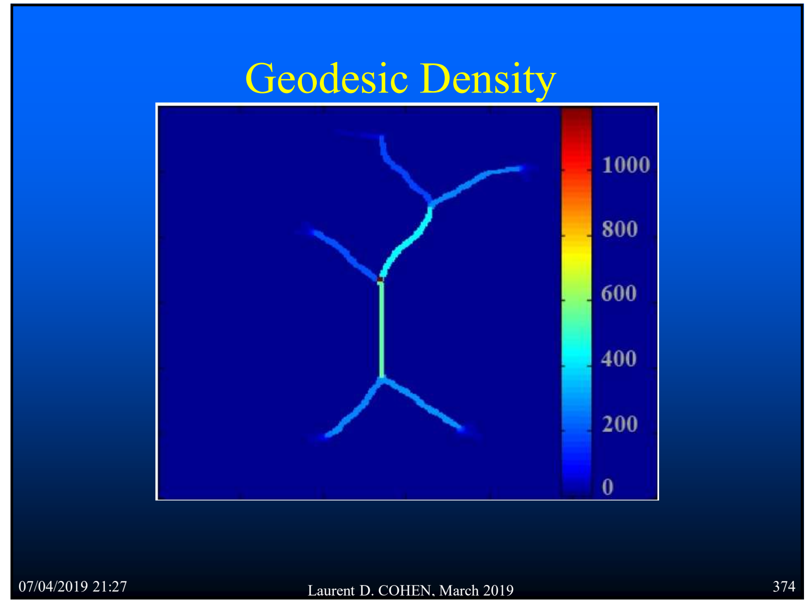

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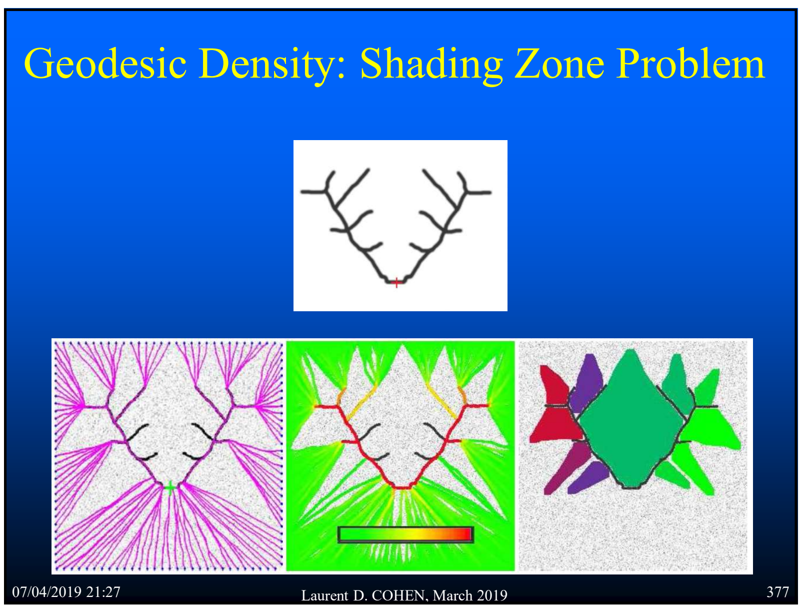

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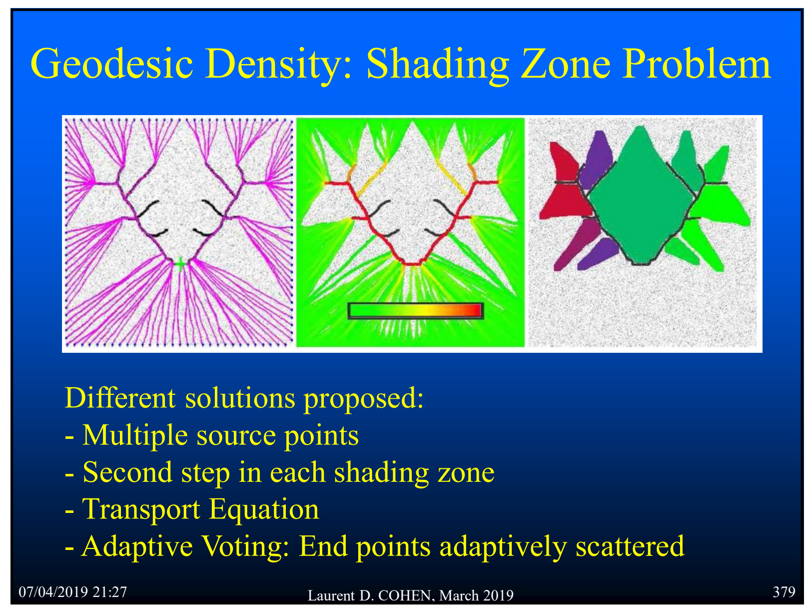

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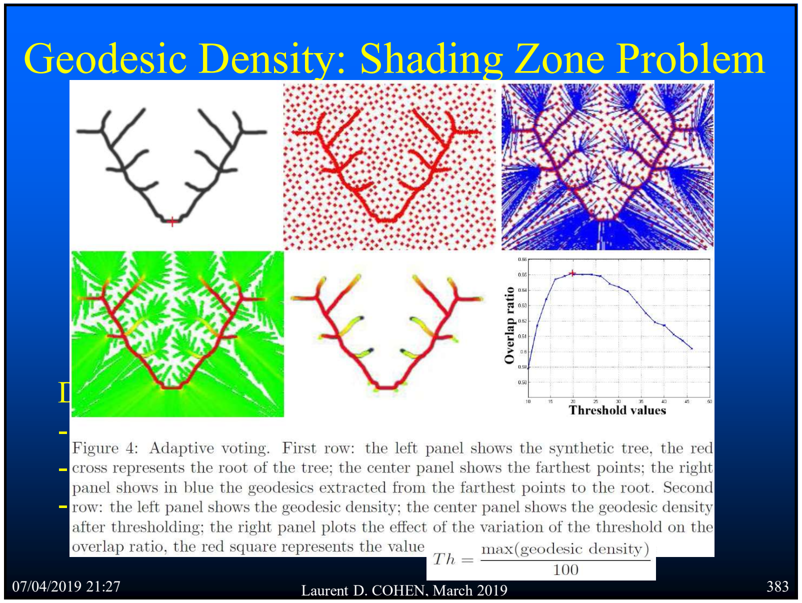

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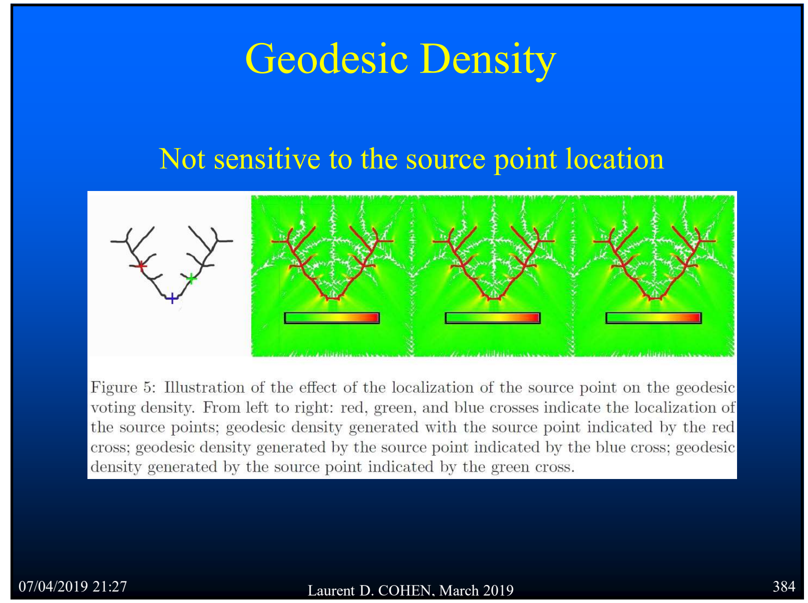

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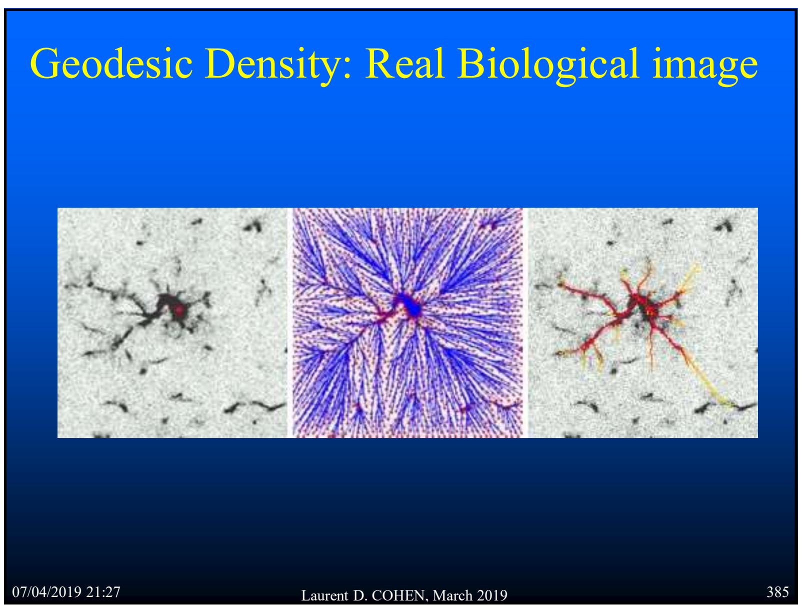

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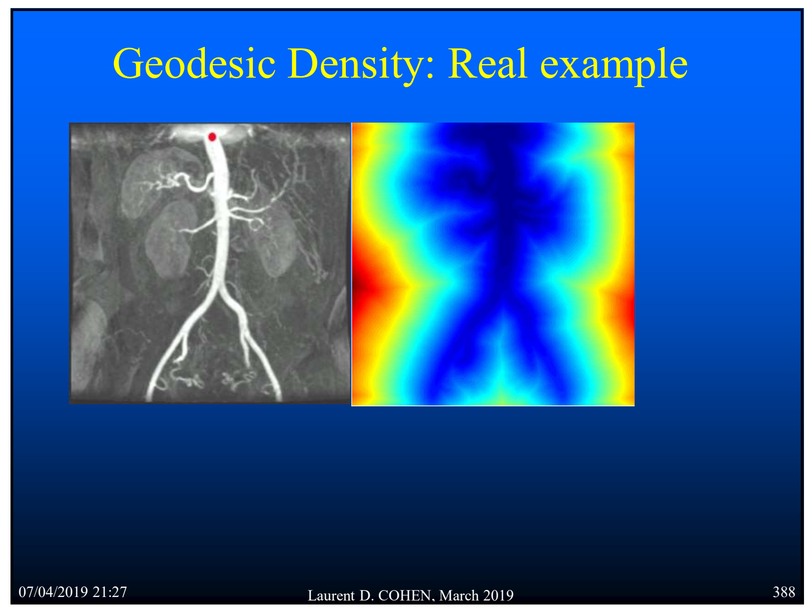

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Geodesic Density: from the image boundary

Boundary : 4*256 end points

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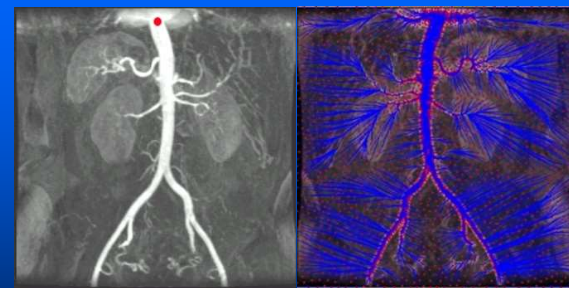
Geodesic Density: adaptive voting



Adaptive voting : 1000 end points

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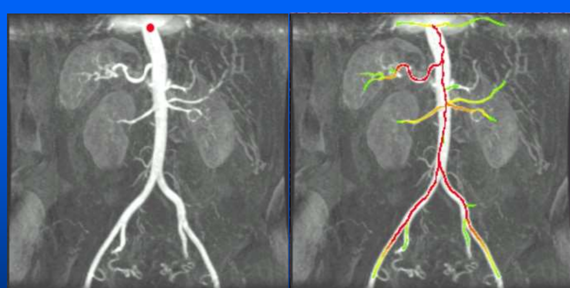
Geodesic Density: adaptive voting



Adaptive voting : 1000 end points

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Geodesic Voting and Segmentation

3D extension

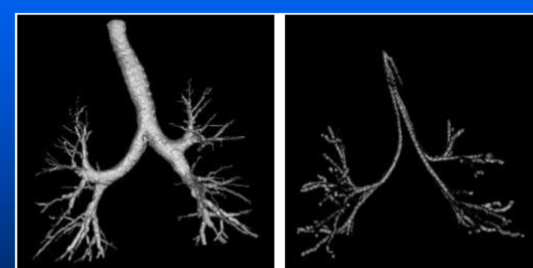

Figure 18: Segmentation of the airway tree with the geodesic voting method. The left panel shows the surface rendering of the manual segmentation. The right panel shows the rendering of the voting tree after dilation of 1 mm.

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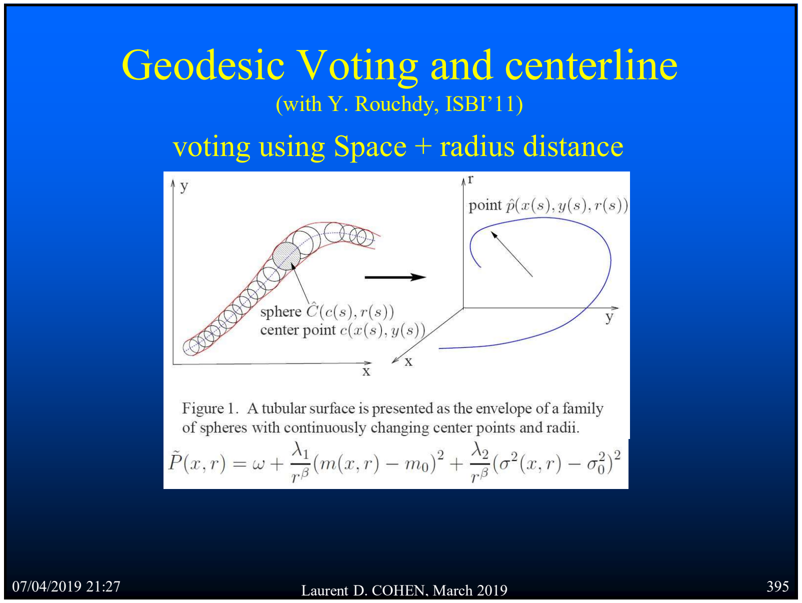

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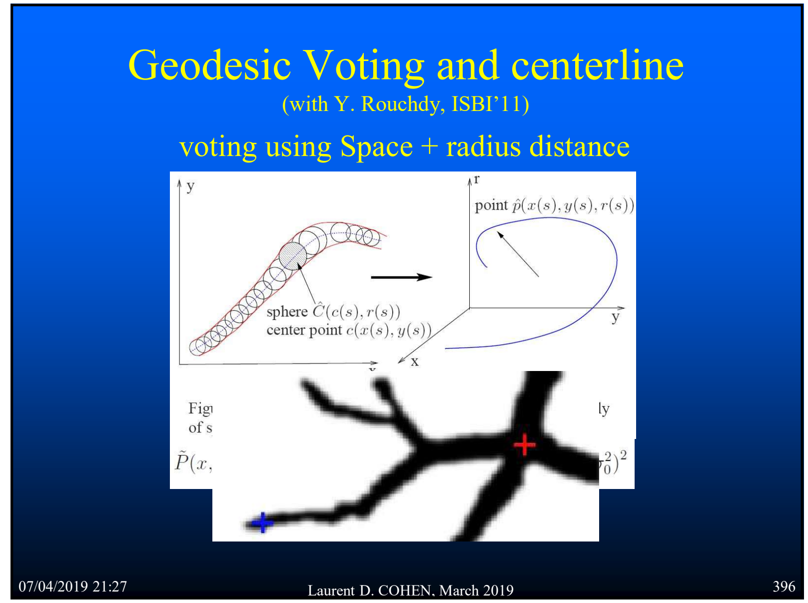

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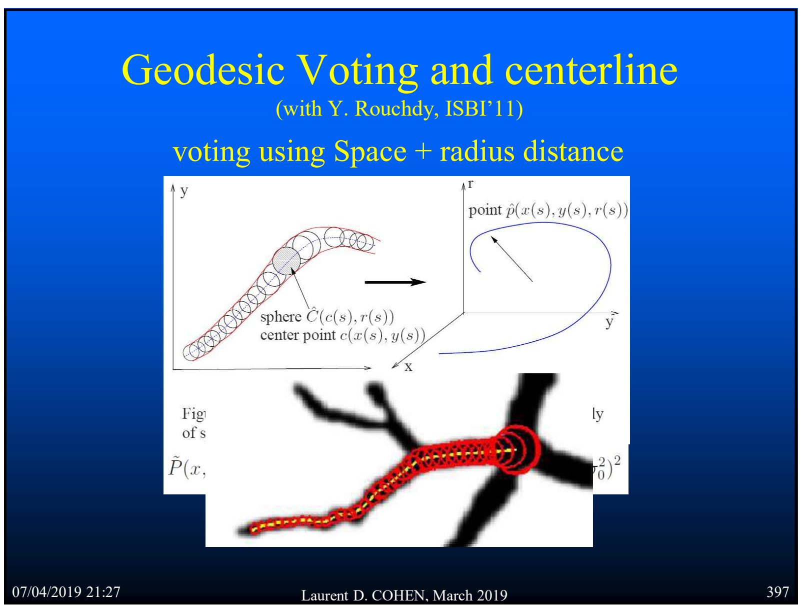

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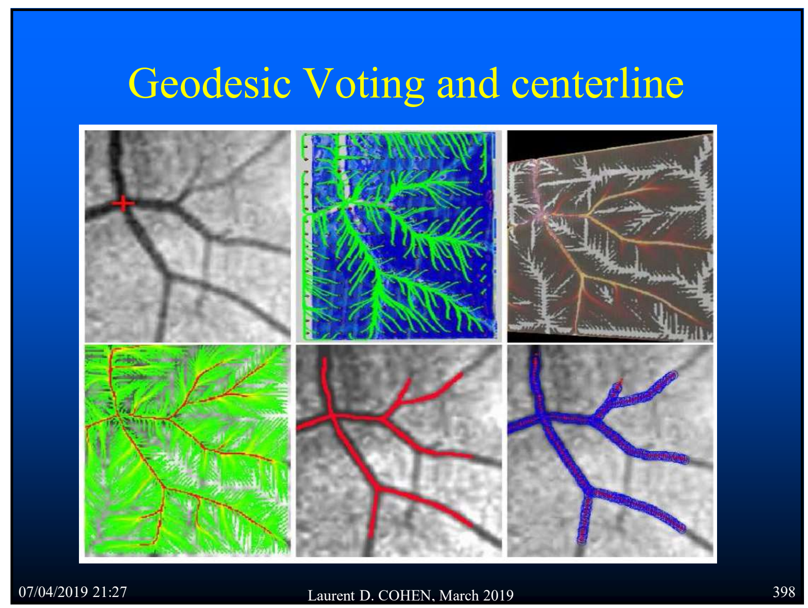

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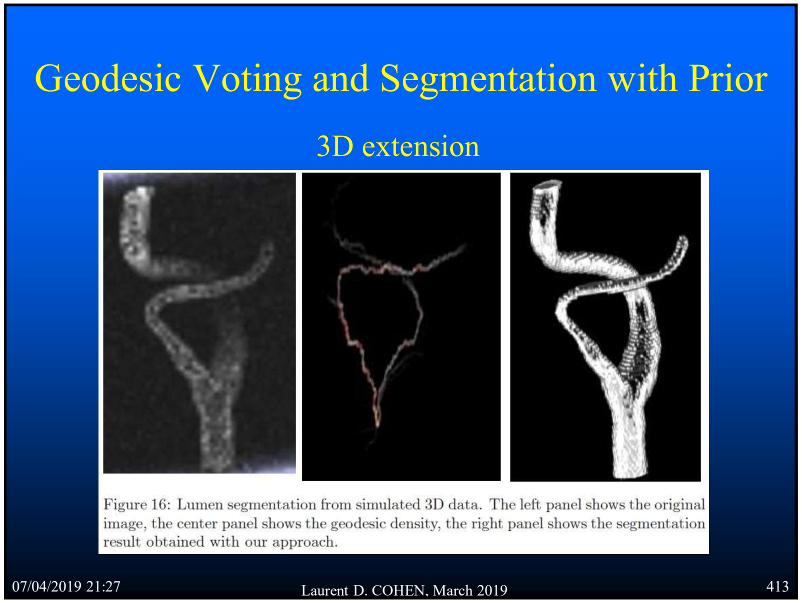

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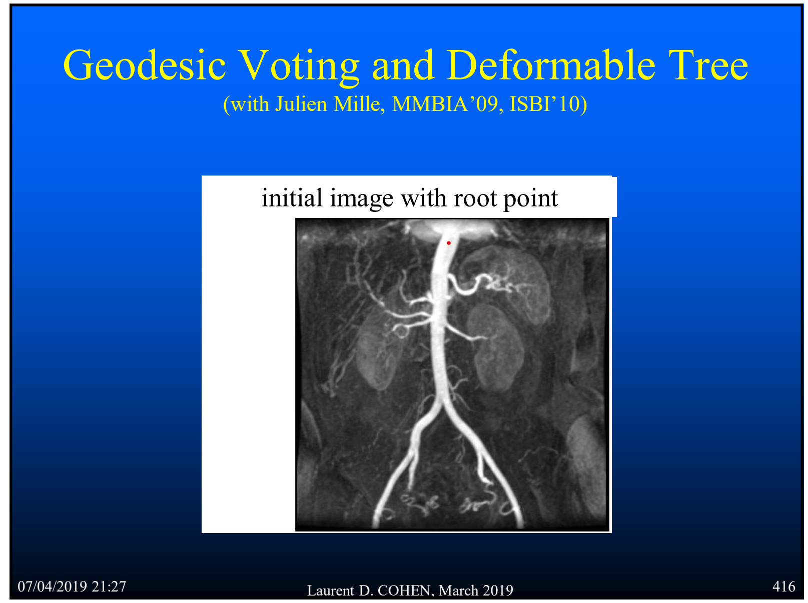

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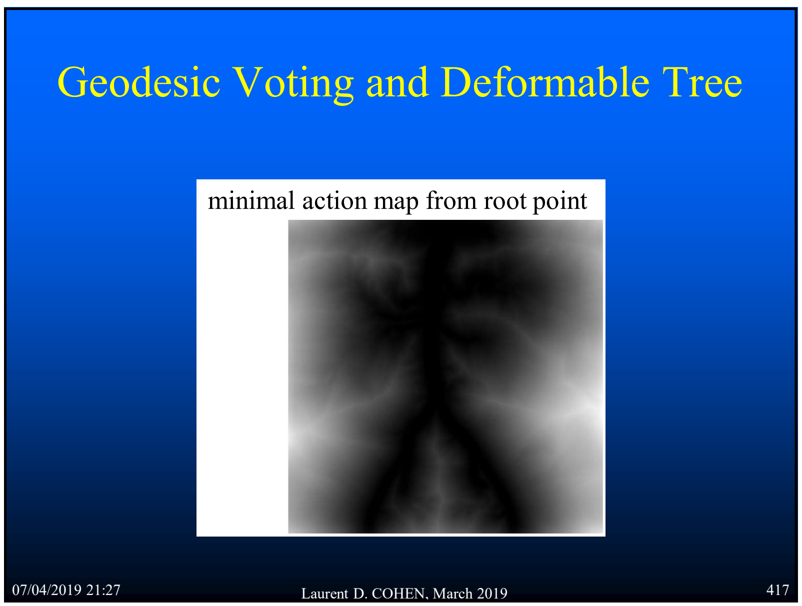

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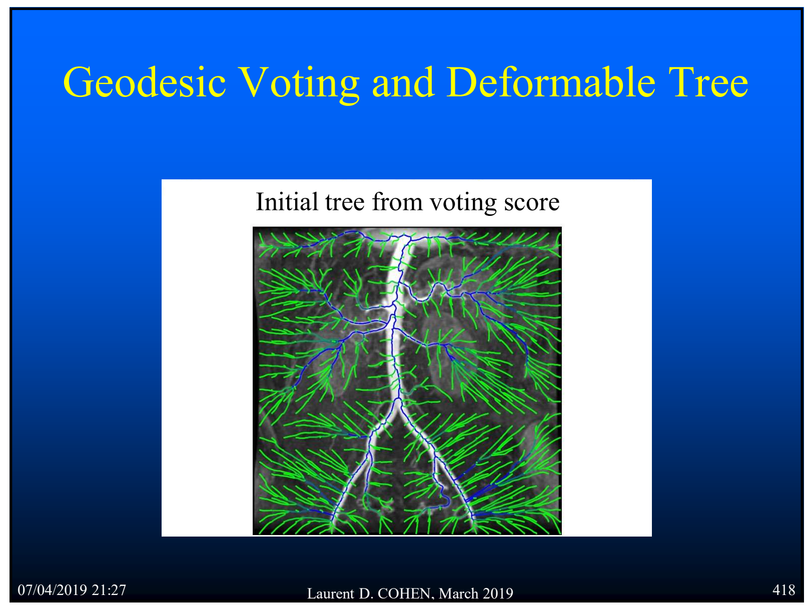

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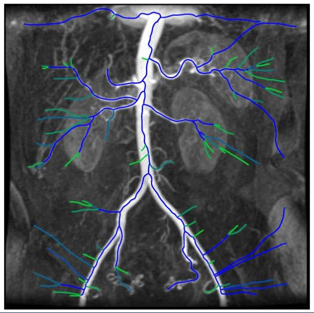
Geodesic Voting and Deformable Tree

Removing unsignificant segments by thresholding the geodesic voting



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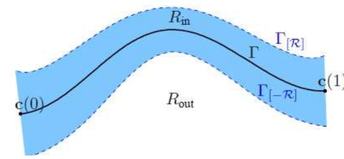
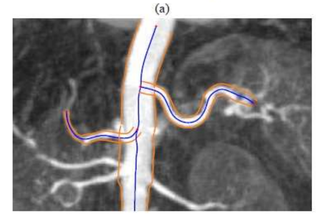
Geodesic Voting and Deformable Tree



$\Gamma_{[\mathcal{R}]}(u) = \Gamma(u) + \mathcal{R}(u)\mathbf{n}(u)$ et $\Gamma_{[-\mathcal{R}]}(u) = \Gamma(u) - \mathcal{R}(u)\mathbf{n}(u)$

(a)



(b)

Figure 2. Deformable band defined by medial curve and thickness (a), representation of the tree by discontinuous bands (b)

$$E_{smooth}[\Gamma, \mathcal{R}] = \int_{\Omega} \left\| \frac{d\Gamma}{du} \right\| + \left(\frac{d\mathcal{R}}{du} \right)^2 du$$

$$E_{region}[\Gamma, \mathcal{R}] = \iint_{R_{in}} g_{in}(\mathbf{x}) d\mathbf{x} + \iint_{R_{out}} g_{out}(\mathbf{x}) d\mathbf{x}$$

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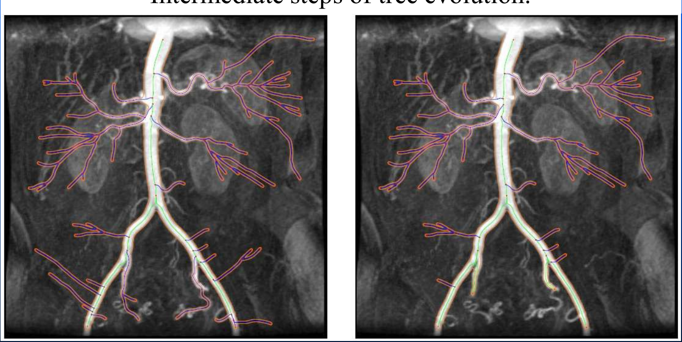
Geodesic Voting and Deformable Tree

Intermediate steps of tree evolution.



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Geodesic Voting and Deformable Tree

final step of tree evolution.

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Geodesic Voting and Deformable Tree

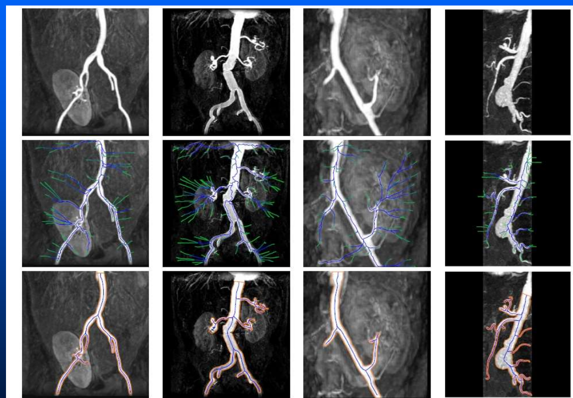

Figure 5. Original images (top row), trees after thresholding on voting score (middle row), final trees with boundary curve (bottom row)

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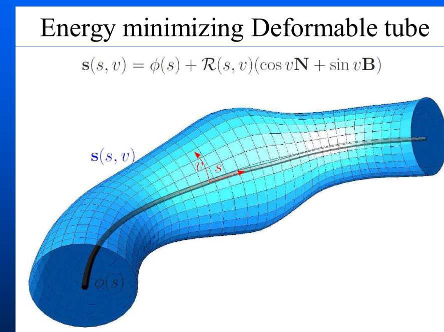
Geodesic Voting and Deformable Tree



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Geodesic Voting and Deformable Tree

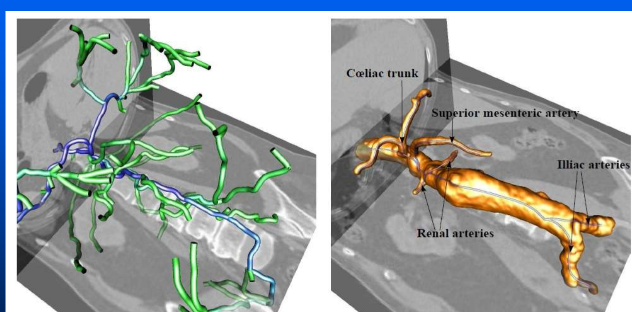

Figure 6. Tree after thresholding on voting score (left) and final tree with boundary surface (right)

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Geodesic Minimal Paths

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Application to Virtual Endoscopy
(with Philips Research France)

Endoscopy : camera inside the body to explore
Virtual Endoscopy: simulates endoscopy, by giving a trajectory in a volumic image like 3D CT or MRI, and by image synthesis along the trajectory
Non-invasive Technique : useful for teaching and real endoscopy preparation.
Allows to visit in regions that endoscopy cannot reach.

Problem: Long and difficult to define trajectory by hand in a complex volume like colon.
Solution : We proposed (long ago) a method building automatically the trajectory and the tubular surface to visit, by giving only one starting point.

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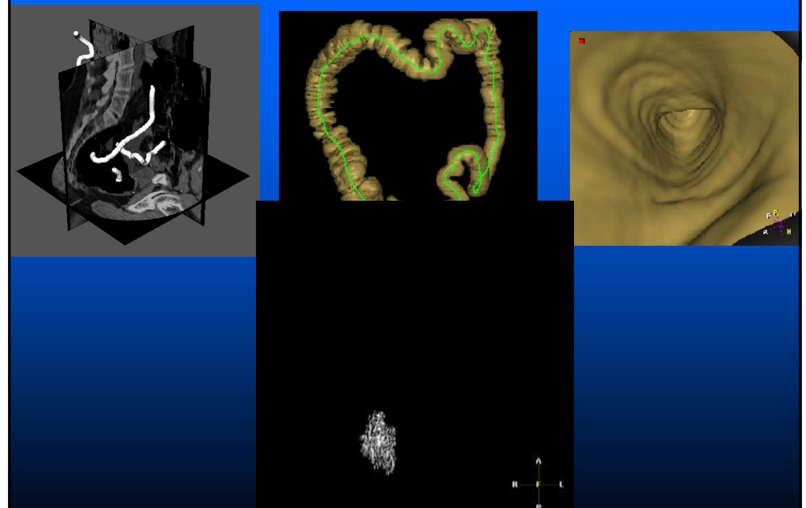
3D minimal paths for virtual endoscopy.



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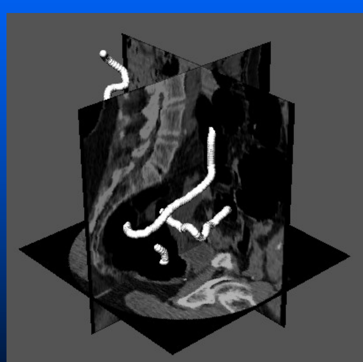
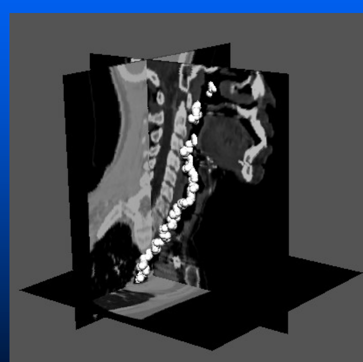
Examples of 3D minimal paths



Colon 3D CT



Trachea 3D CT

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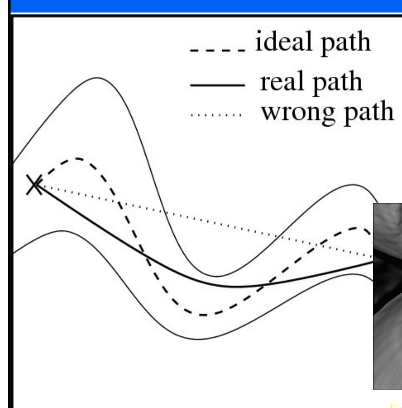
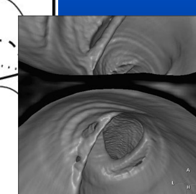
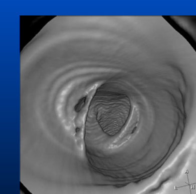
Centering the Minimal path



If the potential is amlmost constant inside the structure: minimal path is the shortest path and close to walls.



Endoscopic view not centered



Endoscopic view centered

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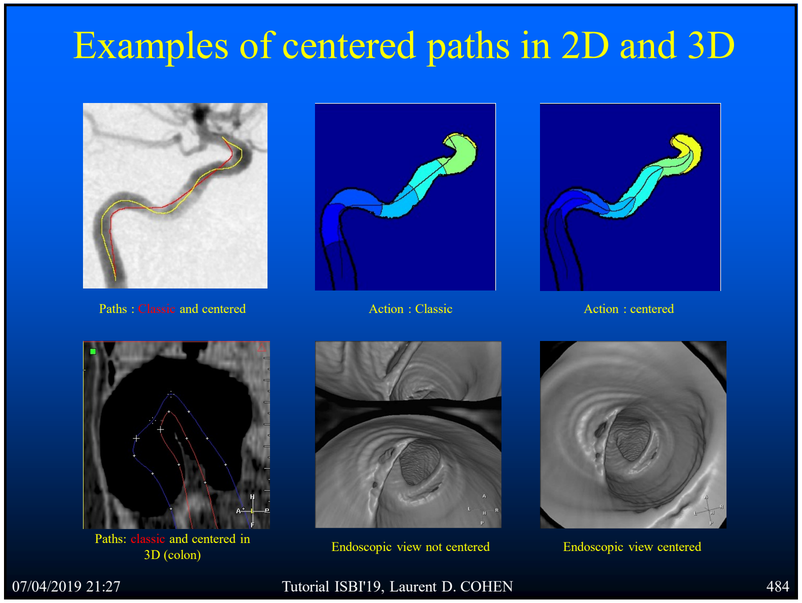

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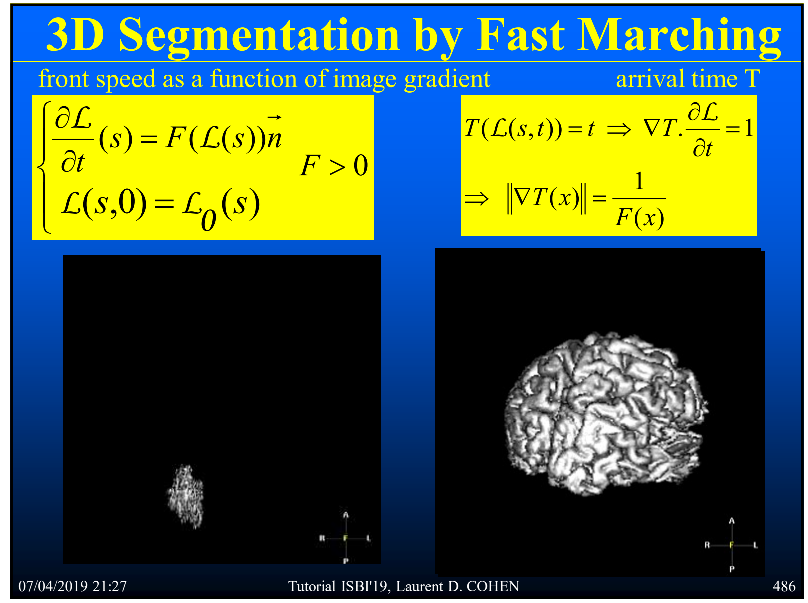

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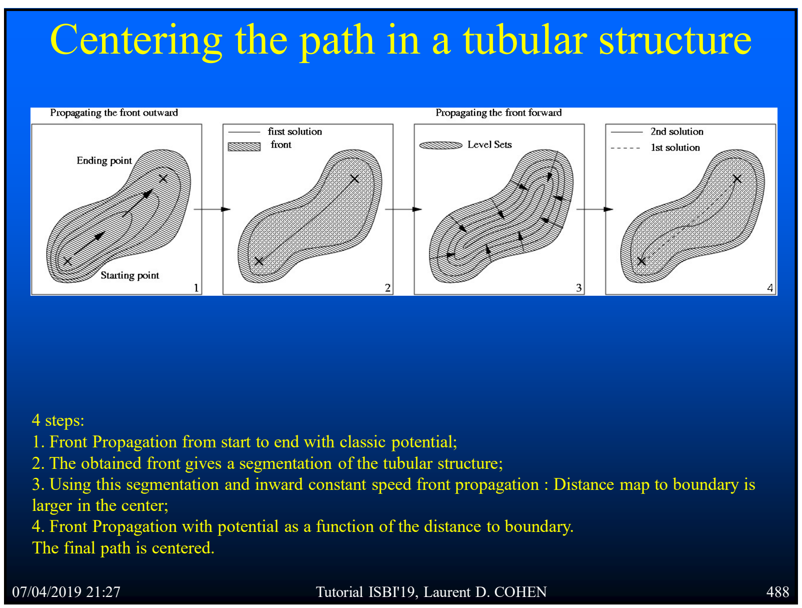

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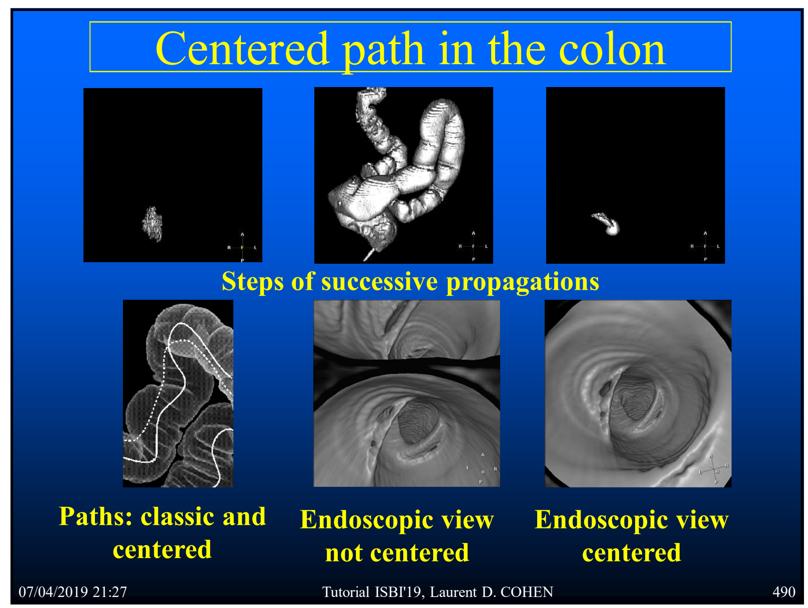

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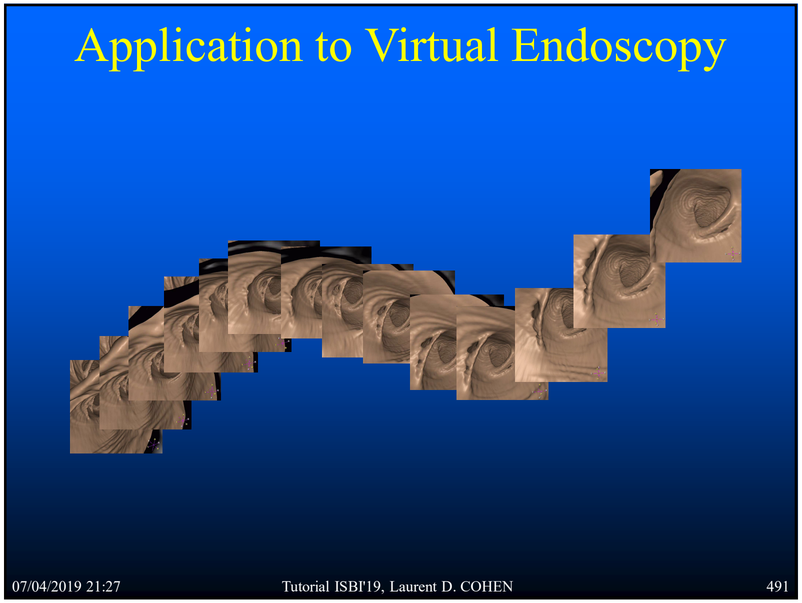

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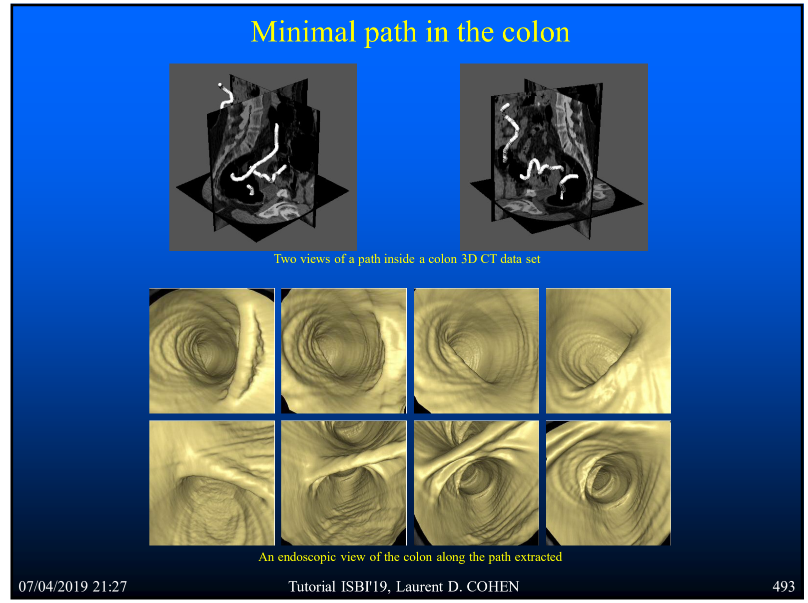

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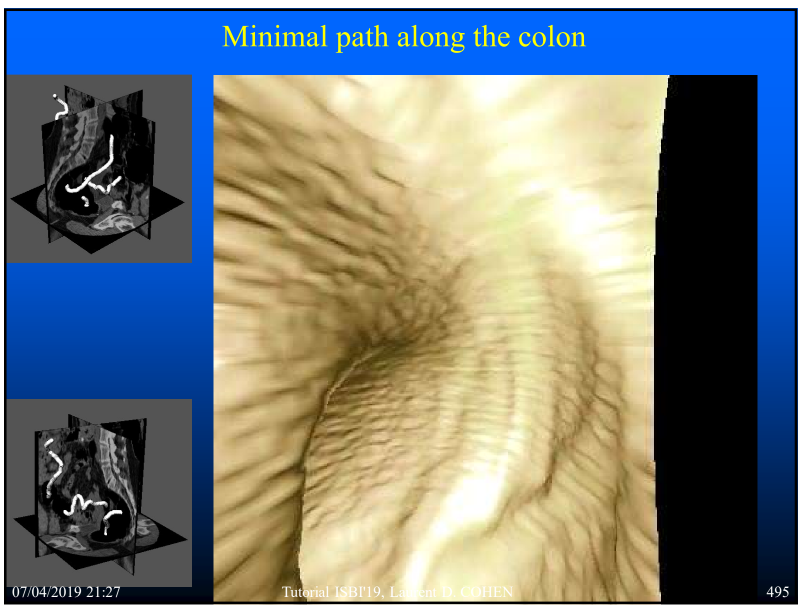

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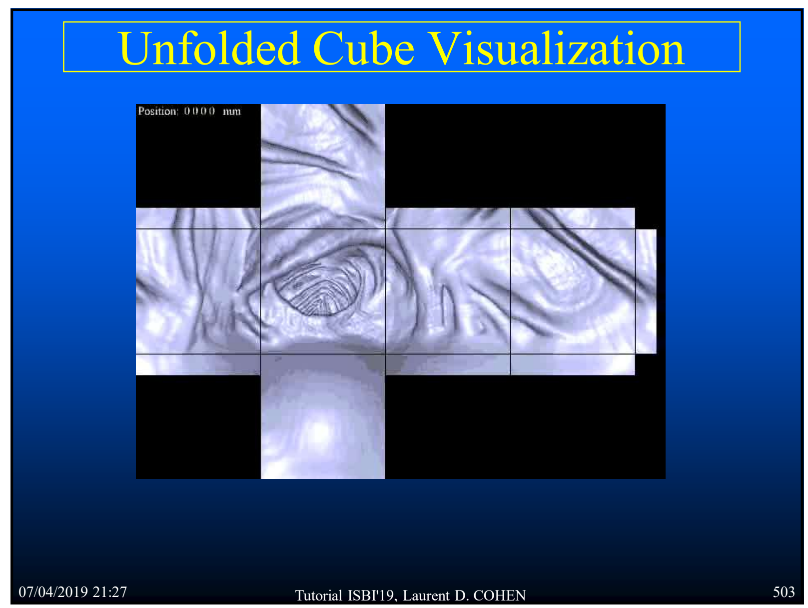

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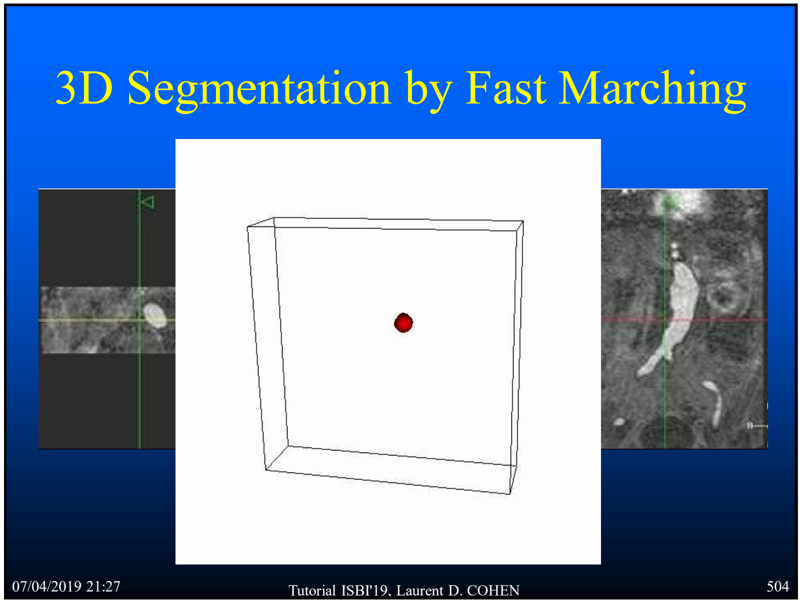

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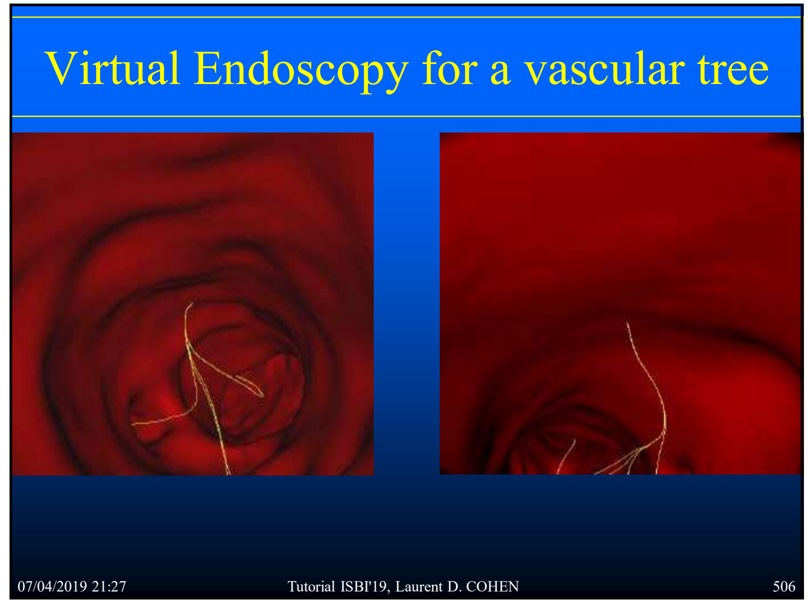

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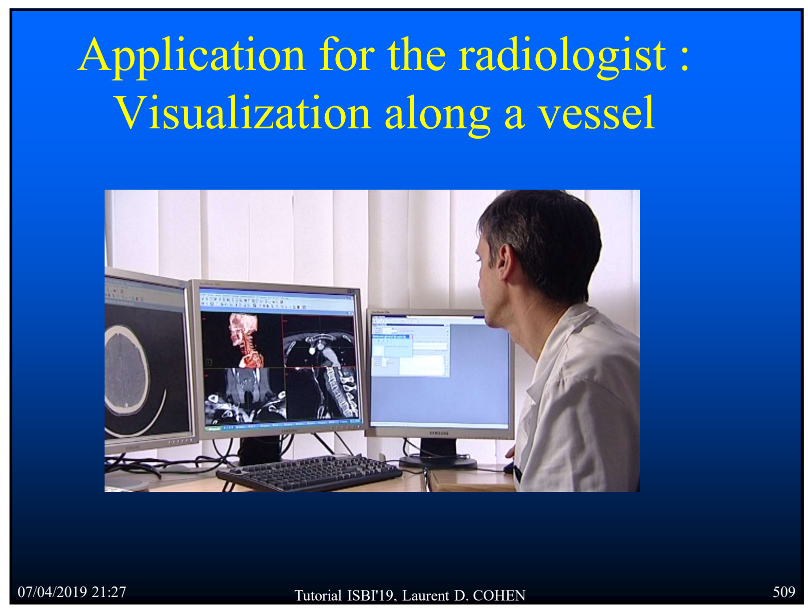

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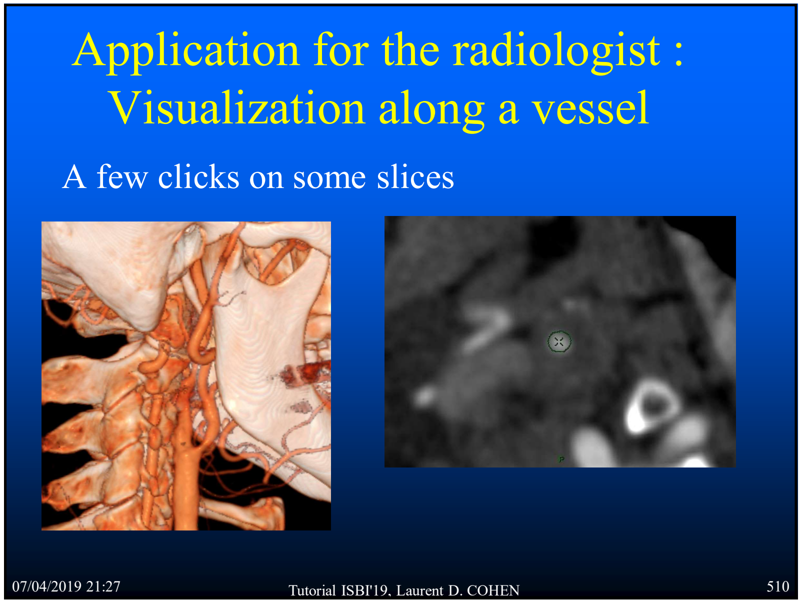

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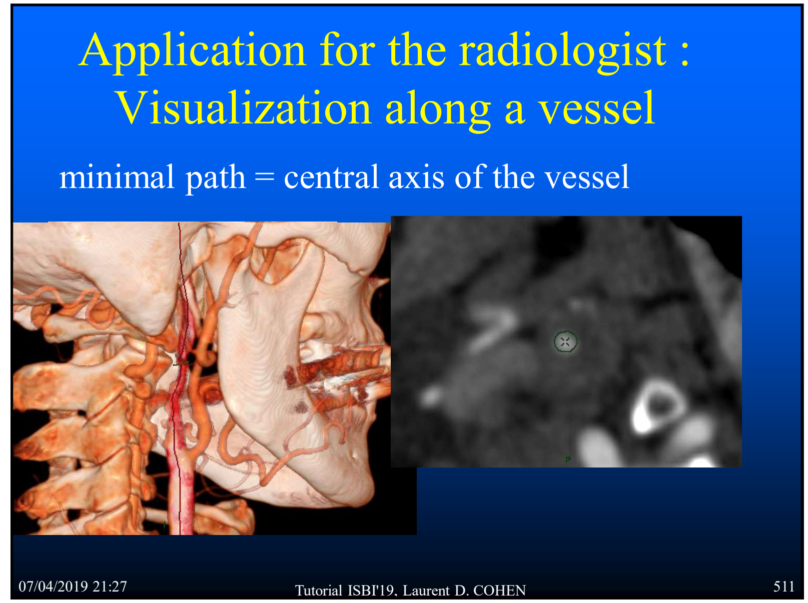

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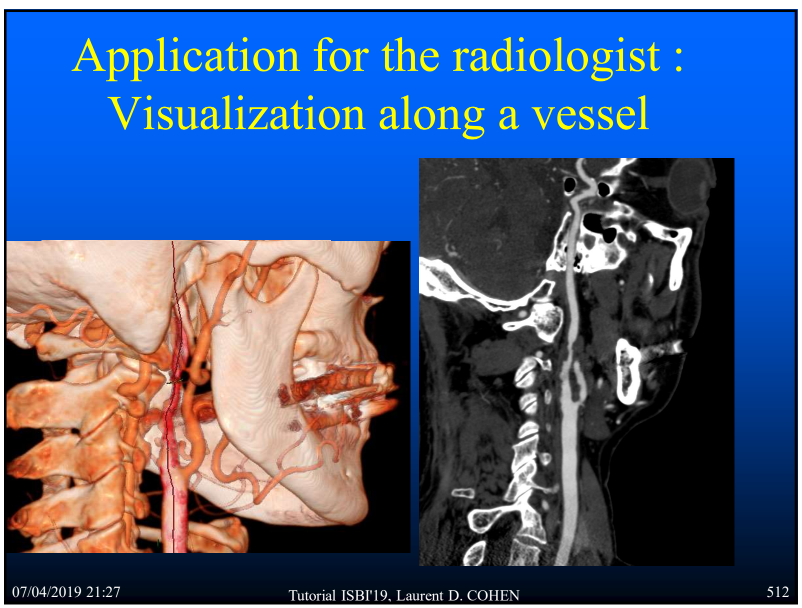

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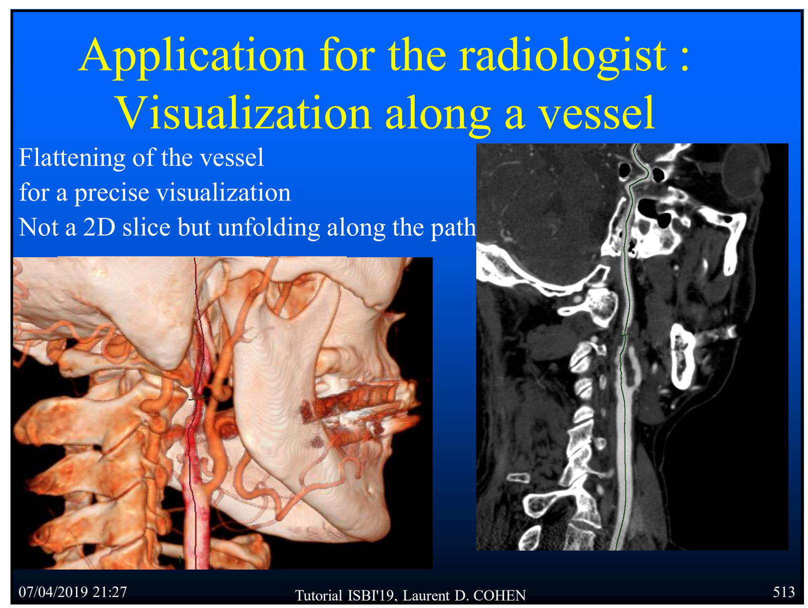

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Application for the radiologist : Visualization along a vessel

Flattening of the vessel for a precise visualization: we can turn around the central axis

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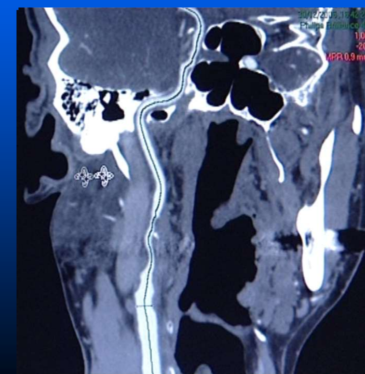
Application for the radiologist : Visualization along a vessel

Flattening of the vessel for a precise visualization: we can turn around the central axis



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Application for the radiologist : Visualization along a vessel

Flattening of the vessel for a precise visualization: we can turn around the central axis

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Conclusion

- Minimally interactive tools for segmentation
- User provides only one initial point and sometimes second end point or stopping parameter
- Fast and efficient propagation algorithm
- Models may include orientation, scale, curvature and region-based information
- Can reproduce minimization of all Active Contours
- Possible applications to segmentation of natural images as well using a set of geodesic paths

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Geodesic Methods in Biomedical Image Analysis

Laurent D. COHEN
Directeur de Recherche CNRS
CEREMADE, UMR CNRS 7534
Université Paris-9 Dauphine
Place du Maréchal de Lattre de Tassigny
75016 Paris, France
Cohen@ceremade.dauphine.fr
www.ceremade.dauphine.fr/~cohen

Some of this work has been in collaboration with R. Kimmel, T. Deschamps, R. Ardon, A. Yezzi, G. Peyré, F. Benmansour, Da Chen and J.M. Mirebeau.
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