Geodesic methods for biomedical image segmentation

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Some joint works with F. Bennmansour, Y. Rouchdy, J. Mille, H. Li and A. Yezzi.
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Overview

- Minimal Paths, Fast Marching and Front Propagation
- Anisotropic Minimal Paths and Tubular model
- Finding contours as a set of minimal paths
- Application to 2D and 3D tree structures
- Geodesic Density for tree structures

Paths of minimal energy

Looking for a path along which a feature Potential P(x,y) is minimal

Example: a vessel
- Dark structure
- P = gray level

Input: Start point p1=(x1,y1)
- End point p2=(x2,y2)
- Image

Output: Minimal Path
Paths of minimal energy

Looking for a path along which a feature Potential $P(x,y)$ is minimal

$E(C)=\int_0^1 P(C(s)) ds$

example: cardiac ventricle contour $P =$ gradient based

Input : Start point $p_1=(x_1,y_1)$
End point $p_2=(x_2,y_2)$
Image
Output: Minimal Path

Minimal Paths: Eikonal Equation

STEP 1: search for the surface of minimal action $U$ of $p_1$ as the minimal energy integrated along a path between start point $p_1$ and any point $p$ in the image

Solution of Eikonal equation: $\nabla U = P$ on contours, dark structures, ...

Example $P=1$, Euclidean distance to $p_1$

In general, $U$ weighted geodesic distance to $p_1$

Potential $P>0$ takes lower values near interesting features:

STEP 2: Back-propagation from the end point $p_2$ to the start point $p_1$:

Simple Gradient Descent along $U_{p_1}$

$\frac{dC}{ds}(s) = -\nabla U_{p_1}(C(s))$ with $C(0)=p_2$. 
Minimal Path between p1 and p2

Step #1: U obtained by the FAST MARCHING ALGORITHM

\[
\begin{align*}
\|\nabla U_1(x)\| &= \hat{P}(x) \quad \text{pour} \ x \in \Omega \\
U_1(p_1) &= 0
\end{align*}
\]

L. D. Cohen, R. Kimmel
Global minimum for active contour models : a minimal path approach.
Minimal Path between $p_1$ and $p_2$

Step #1

\[
\begin{align*}
&\|\nabla \mathcal{U}(x)\| = \tilde{P}(x) \quad \text{pour } x \in \Omega \\
&\mathcal{U}(p_1) = 0
\end{align*}
\]

Step #2
gradient descent on $\mathcal{U}$ for extraction of minimal path $c_{p_1,p_2}^*$

\[
\begin{align*}
&\frac{\partial c_{p_1,p_2}(s)}{\partial s} = -\nabla \mathcal{U}(c_{p_1,p_2}(s)) \\
&c_{p_1,p_2}(0) = p_2
\end{align*}
\]

Minimal Path between $p_1$ and $p_2$

minimal path $c_{p_1,p_2} = \min_{\gamma \in \mathcal{C} p_1,p_2} \int \tilde{P}(s)ds$ is obtained by solving ODE:

\[
\begin{align*}
&\frac{\partial c_{p_1,p_2}(s)}{\partial s} = -\nabla \mathcal{U}(c_{p_1,p_2}(s)) \\
&c_{p_1,p_2}(0) = p_2
\end{align*}
\]

$\Rightarrow$ simple gradient descent on $\mathcal{U}$ from $p_2$ to $p_1$
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3D Minimal Path for tubular shapes in 2D

Centerline + width

2D in space, 1D for radius of vessel (Li, Yezzi 2007)

Figure 1. A tubular surface is presented as the envelope of a family of spheres with continuously changing center points and radii.
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Anisotropic Energy

Considers the local orientations of the structures

\[ E(C) = \int_{0}^{s} P(C(s), C'(s)) ds \]

\[ P(C(s), C'(s)) = \sqrt{C'(s)^T H(C(s)) C'(s)} \]

describes an infinitesimal distance along an oriented pathway \( C \), relative to a metric \( H \).

Anisotropic Energy: Eikonal Equation

\[ E(C) = \int_{0}^{s} \sqrt{C'(s)^T H(C(s)) C'(s)} ds \]

Start point \( C(0) = p_1 \):
\[ U_{p_1}(p) = \inf_{C(0)=p\in\mathcal{C}_1} E(C) \]

\[ \left\| \nabla U_{p_1}(p) \right\|_{H^{-1}(p)}^t = \sqrt{\nabla U_{p_1}^T H^{-1} \nabla U_{p_1}} = 1 \]

and \( U_{p_1}(p_1) = 0 \)

Anisotropic Energy: Gradient descent

\[ E(C) = \int_{0}^{s} \sqrt{C'(s)^T H(C(s)) C'(s)} ds \]

Start point \( C(0) = p_1 \):
\[ U_{p_1}(p) = \inf_{C(0)=p\in\mathcal{C}_1} E(C) \]

\[ C'(s) = -H^{-1}(C(s)) \nabla U_{p_1}(C(s)) \]

and \( U_{p_1}(p_1) = 0 \)

Anisotropic Energy: includes Isotropic case

\[ E(C) = \int_{0}^{s} \sqrt{C'(s)^T H(C(s)) C'(s)} ds \]

Start point \( C(0) = p_1 \):
\[ H(p) = P^e(p) I_d \]

\[ \left\| \nabla U_{p_1}(p) \right\| = P \]

\[ C''(t) = -\nabla U_{p_1}(C(t)) \]
Anisotropy and Geodesics

Tensor eigen-decomposition:
\[ H(x) = \lambda_1(x)e_1(x)e_1(x)^T + \lambda_2(x)e_2(x)e_2(x)^T \quad \text{with} \quad 0 < \lambda_1 \leq \lambda_2 \]
\[ \{ \eta \in \mathbb{R}^n \mid \eta^T H(x) \eta \leq 1 \} \]

\[ \lambda_2(x)^{-\frac{1}{2}} e_2(x) \]
\[ \lambda_1(x)^{-\frac{1}{2}} e_1(x) \]

Geodesics tend to follow \( e_1(x) \).

Orientation dependent Energy
Optimally Oriented Flux (OOF)

Orientation dependent Energy
Orientation-dependent Energy

Orientation-Dependent Energy
(with Benmansour, CVPR’09, IJCV’10)

\[ E(C) = \int_{C} P(C(s), C'(s)) ds \]

Considers the local orientations of the structures.

Examples of 3D Minimal Paths for tubular shapes in 2D
2D in space, 1D for radius of vessel

Examples of 4D Minimal Paths for tubular shapes in 3D
Examples of 4D Minimal Paths for tubular shapes in 3D

3D in space, 1D for radius of vessel

Motivation 1:
How to use minimal paths for a closed contour

Motivation 2:
Global Minimum is not always the best. This needs intermediate points.

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Motivation 2: Global Minimum is not always the best. This needs intermediate points.

Finding a closed contour by growing minimal paths and adding keypoints

Finding a closed contour by growing minimal paths and adding keypoints

<table>
<thead>
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<th>Potential ( P )</th>
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Finding a closed contour by growing minimal paths and adding keypoints

Potential $\mathcal{P}$  Euclidean path length $\mathcal{L}$  Minimal action map $\mathcal{U}$  Voronoi partition $\mathcal{V}$

Finding a closed contour by growing minimal paths

Finding a contour between two points by growing minimal paths

(a)  (b)
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Finding a contour between two points by growing minimal paths

Minimal Paths for tubular shapes

Examples of 3D Minimal Paths for tubular shapes in 2D

2D in space, 1D for radius of vessel
Growing minimal paths for tree structure with the keypoints approach

Given a source point and a maximum length parameter $\lambda$

Stopping criterion: when the front reaches all Harris points

Keypoints and 3D Minimal Paths for tubular shapes in 2D
(with Li and Yezzi, MICCAI'09)

Fig. 1. The entire multi-branch structure extraction is reduced to finding structures between all adjacent key point pairs. The 4D path length $D$ between each key point pair is equal to $d_{\text{step}}$. For easier visualization, the same concept is illustrated here using circles instead of spheres.

2D in space, 1D for radius of vessel
Keypoints and 3D Minimal Paths for tubular shapes in 2D

Keypoints and 4D Minimal Paths for tubular shapes in 3D

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Fig. 3: Segmentation results via the proposed method on another 2D projection angiogram image. Panels from left to right show the initial point and the detected iterative key points and the detected vessel surfaces.
Geodesic Density: Biological image

Geodesic Density: Biological image

Geodesic Density: Shading Zone Problem

Figure 2: Shading zones. Left panel: extraction of geodesics from the boundary of the domain. The green cross represents the source point from which the propagation is started and the pink lines represent paths extracted from the image border to the source point. The paths are superimposed on the image; only 10 percent of the paths extracted are shown in the figure. Center panel: geodesic density superimposed on the image, density is shown as transparent when equal to 0. Right panel: shading zones. The colored regions correspond to the extracted shading zones (obtained by morphological operations on the geodesic density); these are zones without vote.

Geodesic Density: Shading Zone Problem

Different solutions proposed:
- Multiple source points
- Second step in each shading zone
- Transport Equation
- Adaptive Voting: End points adaptively scattered
Geodesic Density: Shading Zone Problem

Different solutions proposed:
- Multiple source points
- Second step in each shading zone
- End points scattered in the image

Geodesic Density: Real Biological image

Not sensitive to the source point location

Geodesic Density: Biological image
Geodesic Density: Biological image

Geodesic Density: Real example

Geodesic Density: from the image boundary

Geodesic Density: all points

Boundary: 4*256 end points

From all points: 256*256 end points
Geodesic Density: adaptive voting

Adaptive voting: 1000 end points

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Geodesic Voting and centerline
(with Y. Rouchdy, ISBI'11)

voting using Space + radius distance

Figure 1. A tubular surface is presented as the envelope of a family of spheres with continuously changing center points and radii.

\[ P(x, r) = \omega + \frac{1}{\mu_1}(r(x, r) - m_0)^2 + \frac{1}{\mu_2}(r(x, r) - m_2)^2 \]
Geodesic Voting and centerline

voting using Space + radius distance

\[ \tilde{\mu}_a(x) = \sum_{r=0}^{r_{\text{max}}} \mu(x, r), \quad \tilde{\mu}_m \doteq (x) = \max_{r \in [0, r_{\text{max}}]} \mu(x, r) \]
Geodesic Voting and centerline

voting using Space + radius distance

Figure 8: Comparison of the original voting method and our approach. From left to right: in blue the manual segmentation of the centerlines of the tree; results obtained by the original voting method (overlap ratio $\Omega = 0.41$); the geodesic density $\hat{\rho}_B$ obtained by our approach; the density $\hat{\rho}_B$ after thresholding (overlap ratio $\Omega = 0.75$).

Geodesic Voting and Segmentation with Prior

Level set method either edge based or region based

Level set initialized with result of voting

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  - Using Geodesic voting as a prior shape

Level set initialized with result of voting
Geodesic Voting and Segmentation with Prior
(with Y. Rouchdy, SSVM'11)

Chan Vese Energy including shape Prior given by \( \Phi \)

\[
\mathcal{V}(\phi, c_1, c_2) = \int_\Omega \left( \lambda_1 (u_0 - c_1)^2 H_\epsilon(\phi) + \lambda_2 (u_0 - c_2)^2 (1 - H_\epsilon(\phi)) + \mu \delta_\epsilon(\phi) \right) |\nabla \phi| \, dx,
\]

\[
E_{\Phi}(\phi, c_1, c_2) = \mathcal{V}(\phi, c_1, c_2) + \gamma \int_\Omega (\phi - \hat{\phi})^2 2 \sigma^2 \delta_\epsilon(\phi) \, dx.
\]

Figure 10: Geodesic voting segmentation of vessels from a 2D retinal image. The left panel shows in red the voting tree on the image; the second panel show the voting tree obtained by thresholding the geodesic density; the third panel shows in red the voting tree after morphological dilatation; the right panel shows the segmentation result obtained with the geodesic voting method presented in Section 4.2.2 (GVP).

Figure 11: Blood vessels segmentation using the GVP and GVR methods from one of the twelve cropped retinal images given in Table 3. The left panel shows the original image; the second panel shows in blue the manual segmentation; the third panel shows the segmentation result obtained with the GVR method; the right panel shows the segmentation result obtained with the GVP method.
Geodesic Voting and Segmentation with Prior

Chan Vese Energy including shape Prior given by \( \phi \)

Figure 12: Blood vessels segmentation using the GVP and GVR methods from one of the twelve cropped retinal images given in Table 3. The left panel shows the initial image; the second panel shows in blue the maximal segmentation; the third panel shows in red the segmentation obtained with GVP method; the right panel shows the segmentation obtained with our GVR method.

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  - Using Geodesic voting as a prior shape
  - Using Geodesic voting as initial deformable tree

Geodesic Voting and Segmentation with Prior

3D extension

Figure 16: Lemon segmentation from simulated 3D data. The left panel shows the original image, the center panel shows the geodesic density, the right panel shows the segmentation result obtained with our approach.
Geodesic Voting and Deformable Tree
(with Julien Mille, MMBIA’09, ISBI’10)

- Initial image with root point
- Minimal action map from root point
- Initial tree from voting score
- Removing insignificant segments by thresholding the geodesic voting
Geodesic Voting and Deformable Tree

Intermediate steps of tree evolution.

Final step of tree evolution.
Geodesic Voting and Deformable Tree

Energy minimizing Deformable tube

$ s(x,v) = \phi(x) + R(x,v)(\cos(N + \sin(\beta)) )$

16/07/2012 12:11  Laurent COHEN, May 2012

Conclusion

- Minimally interactive tools for vessels and vascular tree segmentation (tubular branching structures)
- User provides only one initial point and sometimes second end point or stopping parameter
- Fast and efficient propagation algorithm
- Models may include orientation and scale of vessels
- Voting approach as a powerful tool to find the structure, which can be completed with other approach.

16/07/2012 12:11  Laurent COHEN, May 2012

Thank you !

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