Deformable tree models for 2D and 3D branching structures extraction

Julien MILLE and Laurent D. COHEN
CEREMADE, CNRS UMR 7534, Université Paris Dauphine
75775 Place du Maréchal de Lattre de Tassigny, Paris, France
(mille, cohen)@ceremade.dauphine.fr

Minimal paths

The minimal path approach by Cohen and Kimmel [1] aims at finding curve \( C \) of minimal length \( L \) in a Riemannian space endowed with an isotropic metric:

\[
L(C) = \int P(C(s)) ds
\]

In our case, the minimal path is used to define an initial centerline of tree segments. Its intensity should be almost uniform and close to the intensity of seed point \( x_i \):

\[
P(x) = w + ((1 - w) - (w - 1))
\]

The minimal action map \( \delta \) of origin \( x_i \) satisfies the Eikonal equation \( |\nabla \delta| = P \) and can thus be computed by the Fast Marching method [5]. The geodesic path linking \( x_i \) to \( x_j \) is built by back-propagation on \( U \).

When several geodesics are extracted, they tend to merge at different junction points. The geodesic voting score [4], i.e. the number of geodesics passing through this point, gives significant information on the structure.

Building the initial tree

Several geodesic paths are extracted from points evenly sampled on a grid. In order to detect junctions, we check for the presence of an already traced path at each gradient descent step.

The deforming band

The deforming band [3], inspired from the ribbon snake model [2], is devoted to the recovery of tubular structures. The band is defined by open curve \( \Gamma \) and radius function \( R \).

Curve \( \Gamma \) plays the role of the medial axis. The inner region \( R_{in} \) of width \( 2R \) is bounded by curves \( \Gamma_{in} \) and \( \Gamma_{out} \), constructed by translating \( \Gamma \) along normal \( n \):

\[
\Gamma_{in} = \Gamma_{out} = \Gamma + R n(s)
\]

We minimize energy functional \( E \) with respect to \( \Gamma \) and \( R \):

\[
E(\Gamma, R) = \omega E_{smooth}(\Gamma, R) + (1 - \omega) E_{data}(\Gamma, R)
\]

which implies to solve two coupled Euler-Lagrange equations:

\[
\frac{\delta E_{smooth}(\Gamma, R)}{\delta \Gamma} = 0 \quad \text{and} \quad \frac{\delta E_{smooth}(\Gamma, R)}{\delta R} = 0
\]

The regularizing term \( E_{smooth} \) maintains smoothness on curve and radius function:

\[
E_{smooth}(\Gamma, R) = \int \delta|\nabla \phi| + R^2 \phi dx
\]

Since the structure of interest should satisfy an intensity homogeneity criterion, the data term is as follows:

\[
E_{data}(\Gamma, R) = \int |\delta \phi| + \int R^2 \phi dx
\]

where \( \delta \phi(x) = (f(x) - \mu_{true})^2 \) and \( \mu_{true} = (f(x) - \mu_{data})^2 \), with average intensities inside and outside the entire tree structure.

The band is implemented as a polygonal line of vertices \( p_i \) with associated radii \( R_{i} \). Vertices positions and corresponding radii are updated by gradient descent of the Euler-Lagrange equations.

Extension to 3D: the deformable tube

Let \( \delta \) be the curve axis and \( R \) the radius function. Tangent \( T \), normal \( N \) and binormal \( B \) define an orthogonal coordinate system sweeping along the curve. They are used to define varying cross-sections orthogonal to the curve:

\[
s(x, v) = \delta(x) + R(x, v) (\cos v N + \sin v B)
\]

We assume that a smooth axis curve and a smoothly varying radius will yield a smooth surface. The regularizing term is:

\[
E_{smooth}[\delta, R] = \int |\delta \phi| + \int R^2 \phi dx
\]

where \( E_{data} \) is expressed in a similar way as in the 2D model.

The tube is implemented as a polygonal line of \( n \) vertices \( p_i \), each vertex being endowed with \( m \) angular positions with associated radii \( R_{i} \). Radii are updated using gradient descent of the energy. Vertex coordinates are computed as centroids of angular positions when all radii have been updated.

\[
R_{i}^{\text{new}} = R_{i}^{\text{old}} - \Delta t \frac{\delta E}{\delta R_{i}^{\text{old}}} \phi|_{R_{i}^{\text{old}}}
\]

Vertices positions are smoothed afterward.

Evolution of the tree structure

A hierarchical structure of bands is built by assigning a deformable cylinder model to each segment. Segments are endowed with two status variables: ACTIVE and STABLE. Evolution of the tree is achieved by repeating the following steps:

- Select the longest convex sequence of segments, such that the last segment is not STABLE
- Mark all segments in the sequence as ACTIVE
- Evolve ACTIVE segments according to gradient descent
- Update average intensities \( \mu_{true} \) and \( \mu_{data} \)
- Mark all segments in the sequence as STABLE and disable their ACTIVE status. Remove insignificant segments

To obtain a continuous representation, the boundary is refined after the deformation using an active contour or surface. For the 3D tree, we build a closed curve \( C \) by scanning the tree according to a depth-first algorithm. For the 3D tree model, we implement the boundary surface as a triangulated mesh.

Results

The 2D deformable tree model is tested on a set of MR angiography images, whereas the 3D tree is applied on a CT volume data. False segments remaining after vote thresholding are not critical regarding the final segmenta-

References


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