A GEODESIC VOTING METHOD FOR THE SEGMENTATION OF TUBULAR TREE AND CENTERLINES

Youssef Rouchdy[1,2] and Laurent D. Cohen[1,∗]

[1] CEREMADE, UMR 7534, Université Paris Dauphine, 75775 Paris Cedex 16, France
[2] Télécom ParisTech, CNRS LTCI, 75013 Paris, France

ABSTRACT

This paper presents a geodesic voting method to segment tree structures, such as cardiac or cerebral blood vessels. Many authors have used minimal cost paths, or similarly geodesics relative to a weight potential P, to find a vessel between two end points. Our goal focuses on the use of a set of such geodesic paths for finding a tubular tree structure, using minimal interaction. This work adapts the geodesic voting method that we have introduced for the segmentation of thin tree structures to the segmentation of centerlines and tubular trees. The original approach of geodesic voting consists in computing geodesics from a set of end points scattered in the image to a given source point. The target structure corresponds to image points with a high geodesic density. Since the potential takes low values on the tree structure, geodesics will locate preferably on this structure and thus the geodesic density should be high. Geodesic voting method gives a good approximation of the localization of the tree branches, but it does not allow to extract the tubular aspect of the tree. Furthermore, geodesic voting does not guarantee that the extracted tree corresponds to the centerline of the tree. Here, we introduce an explicit constraint that moves the high geodesic density to the centerline of the tree and simultaneously approximates the localization of the boundary of the tubular structure. We show results of the segmentation with this approach on 2D angiogram images. This approach can be extended to 3D images in a straight forward manner.

Index Terms— Geodesic voting, Fast Marching, minimal paths, tree structure segmentation, vessels segmentation.

1. INTRODUCTION

In this paper we present novel methods for the segmentation of tree structures. These methods are based on minimal paths with a metric designed from the images and can be applied to the segmentation of numerous structures, such as: microglia extensions; neurovascular structures; blood vessels; and pulmonary tree. The vascular tree is modelled as a tubular structure. We can classify the methods used to segment the vascular tree into three kind of approaches according to the method used to extract the tubular aspect of the tree: surface models; centerline based models; and 4D curve models. The first category extracts directly the surface of the vessel, see [1]. For the second approach, centerlines based models, centerlines are extracted first and a second process is required to segment the vessel surface, see [2]. The last approach, 4D curve model, consists in segmenting the vessel centerlines and surfaces simultaneously as a path in a (3D+radius) space [3, 4]. For a review of vessel segmentation methods, see [5, 6].

Minimal paths techniques were extensively used for extraction of tubular tree structures. These approaches are more robust than the region growing methods, particularly in the presence of local perturbations due to the presence of stenosed branches of the tree or imaging artefacts where the image information might be insufficient to guide the growing process. Several minimal path techniques have been proposed to deal with this problem [7, 8]. These techniques consist in designing a metric from the image in such a way that the tubular structures correspond to geodesic paths according to this metric. Solving the problem from the practical point of view consists of a front propagation from a source point within a vessel which is faster on the branches of the vascular tree. These methods required the definition by the user of a starting point (propagation source) and end points. Each end point allows to extract a branch of the tree as a minimal path from this point to the source point, the points located on the minimal path are very likely located on the vessel of interest. Few works have been devoted to reduce the interaction of the user in the segmentation of tree structure to the initialization of the propagation from a single point. Authors of [9] defined a stopping criteria from a medialness measure, the propagation is stopped when the medialness drops below a given threshold. This method might suffer from the same problem as the region growing, the medialness might drop below the given threshold in the presence of pathology or imaging artefacts. Wink et al. [10] proposed to stop the propagation when the geodesic distance reaches a certain value. However, this method is limited to the segmentation of a single vessel and the definition of the threshold of the geodesic distance is not straightforward. Cohen and Deschamps [11] defined this threshold from prior information about the total length of the

tree structure to be visited.

Li et al. [4] proposed a 4D curve model with a key point searching scheme to extract multi-branch tubular structures. The vascular tree is a set of 4D minimal paths, giving 3D centerlines and width. While this method has the advantage to segment vessel centerlines and surfaces simultaneously, it requires the definition of eight parameters. One point inside the tubular structure and the radius are used to initialize the Fast marching propagation, three parameters are used to set the Fast Marching potential and three distance parameters limit the propagation to the inside of the tubular structure to avoid leakage outside the tree. These last three parameters may require an important intervention of the user since they are crucial to extract the whole structure. If these distance parameters are not suitable, parts of the tree structure may be missed during the propagation.

In this paper, we present a method to extract tree structures without using any *a priori* information. Furthermore, the user has to provide only a single point on the tree structure. The method is generic: it can be used to extract any type of tree structure in 2D as well as in 3D. It is based on the geodesic voting method introduced in [12, 13]. It consists in computing geodesics from a set of end points scattered in the image to a given source point. The target structure corresponds to image points with a high geodesic density. The geodesic density is defined at each pixel of the image as the number of geodesics that pass over this pixel. Since the potential takes low values on the tree structure, geodesics will locate preferably on this structure and thus the geodesic density should be high on the tree structure. While the original voting method allows to extract tree structures it does not permit to extract the walls of the vessels. Furthermore, the tree extracted with this approach does not necessarily corresponds to centerlines of the tree. Here, we introduce an explicit constraint to move the high densities in the geodesic voting method to the centerlines of the tree and simultaneously extract the walls of the tree. In Section 2, we present the tools needed in Section 3 to introduce the new geodesic voting method. In Sections 4, we applied our approach to the segmentation of vessels from 2D angiogram images.

2. BACKGROUND

2.1. Minimal paths

In the context of image segmentation Cohen and Kimmel proposed, in [14], a deformable model to extract contours between two points given by the user. The model is formulated as finding a geodesic for a weighted distance:

$$\min_{y} \int_{0}^{L} \big(w + P(y(s))\big)\mathrm{d}s, \tag{1}$$

the minimum is considered over all curves $y(s)$ traced on the image domain Ω that link the two end points, that is, $y(0) =$

x_0 and $y(L) = x_1$. The constant w imposes regularity on the curve. $P > 0$ is a potential cost function computed from the image, it takes lower values near the edges or the features. For instance $P(y(s)) = I(y(s))$ leads to darker lines while $P(y(s)) = g(||\nabla I||)$ leads to edges, where I is the image and g is a decreasing positive function.

To compute the solution associated to the source x_0 of this problem, [14] proposed a Hamiltonian approach: Find the geodesic weighted distance U that solves the Eikonal equation: $||\nabla U(x)|| = w + P(x), \ \forall x \in \Omega$. The ray y is subsequently computed by back-propagation from the end point x_1 by solving the Ordinary Differential Equation (ODE): $y'(s) = -\nabla U(y)$. To solve the Eikonal equation, we use the Fast Marching algorithm introduced in [15]. The idea behind the Fast Marching algorithm is to propagate the wave in only one direction, starting with the smaller values of the action map U and progressing to the larger values using the upwind property of the scheme. Therefore, the Fast Marching method permits to solve the Eikonal in complexity $O\big(n\log(n)\big)$, for details see [14, 15].

2.2. Geodesic Voting for segmentation of tree structures

We have introduced in [12, 13] a new concept to segment a tree structure from only one point given by the user in the tree structure. This method consists in computing the geodesic density from a set of geodesics extracted from the image.

Assume you are looking for a tree structure for which a potential cost function has been defined as above and has lower values on this tree structure. First we provide a starting point x_0 roughly at the root of the tree structure and we propagate a front in the whole image with the Fast Marching method, obtaining the minimal action U.

Then assume you consider an end point anywhere in the image. Backtracking the minimal path from the end point you will reach the tree structure somewhere and stay on it till the start point is reached. So a part of the minimal path lies on some branches of the tree structure. The idea of this approach is to consider a large number of end points $\{x_k\}_{k=1}^{N}$ on the image domain, and analyze the set of minimal paths y_k obtained. For this we consider a voting scheme along the centerlines. When backtracking each path, you add 1 to each pixel you pass over. At the end of this process, pixels on the tree structure will have a high vote since many paths have to pass over it. On the contrary, pixels in the background will generally have a low vote since very few paths will pass over them. The result of this voting scheme is what we can call the geodesic density. This means at each pixel the density of geodesics that pass over this pixel. The tree structure corresponds to the points with high geodesic density.

The set of end points for which you consider the geodesics can be defined through different choices. This could be all pixels over the image domain, random points, scattered points according to some criterion, or simply the set of points on the

boundary of the image domain, see [13]. We define the voting score or the geodesic density at each pixel p of the image by

$$\mu(p) = \sum_{k=1}^{N} \delta_p(y_k) \qquad (2)$$

where the function $\delta_p(y)$ returns 1 if the path y crosses the pixel p, else 0. Once the geodesic voting is made, the tree structure is obtained by a simple thresholding of the geodesic density μ. As shown in Figure 1, the contrast between the background and the tree is large and the threshold can be chosen easily. We used for all experiments the following value

$$Th = \frac{\max(\text{geodesic density})}{100} \qquad (3)$$

as threshold to extract the tree structure using the voting maps. Figure 1 (panel: second row on the right) shows the effect of the threshold on the overlap ratio that measures the similarity between the the manually segmented data A and the segmentation result B[1]. This figure shows that the threshold can be chosen in a large range that contains the threshold Th, given by the equation (3).
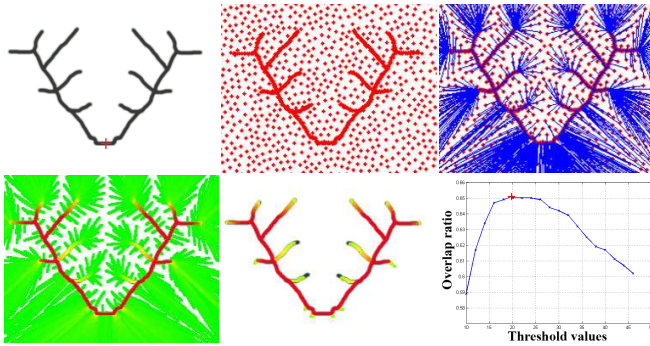
Fig. 1. Geodesic voting. First row: the left panel shows the synthetic tree, the red cross represents the root of the tree; the center panel shows the farthest points (see [13]); the right panel shows in blue the geodesics extracted from the farthest points to the root. Second row: the left panel shows the geodesic density; the center panel shows the geodesic density after thresholding; the right panel plots the effect of the variation of the threshold on the overlap ratio, the red square represents the value Th (given by the equation (3)).

---

[1]The overlap ratio is defined by the relation:

$$O(A, B) = \frac{2|A \cap B|}{|A| + |B|}, \qquad (4)$$

where $|A|$ and $|B|$ are respectively the number of the foreground voxels in the image A and B. $|A \cap B|$ is the number of voxels in the shared regions (intersection of the foreground of the two images).

3. GEODESIC VOTING FOR TUBULAR TREE SEGMENTATION

The geodesic voting method described in the previous section gives a good approximation of the localization of each branch of the tree. In this section we introduce a constraint that ensures that the segmented tree approximates well the centerlines of the tree and we adapt the geodesic voting method to segment the walls of the tubular tree structure. The idea is to perform the geodesic voting with a potential that integrates an extra-dimension used to measure the distance from the centerline to the walls of the vessels. The potential proposed by [3] incorporates this measure. More precisely, this potential is defined by $\tilde{P} : (x, r) \in \Omega \times [0, r_{max}] \longrightarrow \tilde{P}(x, r)$. It incorporates the full set of image values within the sphere of center x and radii r and it is designed in such a way that the whole sphere lies inside the desired object and is as large as possible so that it is tangential to the boundary of the object. The extension of the minimal path extraction model (1) to the case of a potential with an extra-dimension is achieved by minimizing the following energy

$$\min_{c,r} \int_{\Omega} \left(\omega + \tilde{P}(c(s), r(s)) \right) ds. \qquad (5)$$

The minimization of this energy allows simultaneous approximation of the minimal path and the radii of the spheres tangents to the boundary of the tube with centers located along the minimal path. The computation of the path is achieved with the framework presented in the Section 2. The radii are considered as an extra spatial dimension: for a 2D image the propagation with Fast Marching is done in 3D, for 3D images the Fast Marching is performed in 4D, see [15].
Using the potential $\tilde{P}$ and a set of end points (x_k, r_k) (uniform grid) in the domain, we extract a set of geodesics y_k from which we compute the geodesic density $(x, r) \longrightarrow \mu(x, r)$ given by the equation (2). In this case the geodesic voting map is a function of the spatial dimension and also of the radii of the spheres $(x, r) \longrightarrow \mu(x, r)$. There are many ways to use this (3D+radius) geodesic density in order to extract the tree structure. Due to the lack of space we focus on the following spatial densities:

$$\tilde{\mu}_m(x) = \sum_{r=0}^{r_{max}} \mu(x, r), \quad \tilde{\mu}_s(x) = \max_{r \in [0, r_{max}]} \mu(x, r) \qquad (6)$$

The thresholded density, μ_m or μ_s, approximates the centerlines of the tree. To get the distance from the centerlines to the walls vessels, we compute the radii, for each point x with $\tilde{\mu}_m(x) >$ threshold, by evaluating $\tilde{r}(x) = \arg \max_{r \in [0, r_{max}]} \tilde{\mu}(x, r)$. The map $\{\tilde{\mu}_m, \tilde{r}\}$ or $\{\tilde{\mu}_s, \tilde{r}\}$ allows to extract vessels walls and the centerlines. Figure 2 compares the original geodesic voting [12, 13], described in Section 2.2, and the results obtained with our approach. The new geodesic voting method with an extra dimension (2D+radius)

gives the best results in terms of the overlap ratio O given by Equation (4). The potential used in this experiment is described in the next section and given by the equation (7).
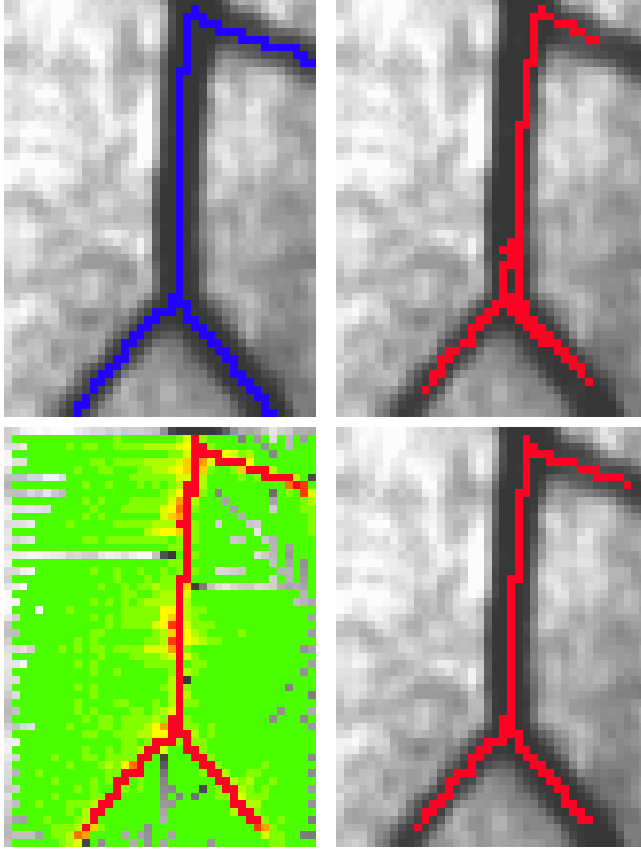
Fig. 2. Comparison of the original voting method and our approach. First row: the left panel shows in blue the manual segmentation of the centerlines of the tree; the right panel shows the results obtained by the original voting method (overlap ratio O = 0.41); Second row: the left panel shows the geodesic density $\tilde{\mu}_m$ obtained by our approach; the right panel shows the density $\tilde{\mu}_m$ after thresholding (overlap ratio O = 0.75).

4. EXPERIMENTS ON 2D DATA

In this section, we applied the geodesic voting method presented in the previous section to segment vessels from 2D retinal images provided by DRIVE (Digital Retinal Images for Vessel Extraction) [16]. The following potential was used:

$$\tilde{P}(x,r) = \omega + \frac{\lambda_1}{r^\beta}\big(m(x,r)-m_0\big)^2 + \frac{\lambda_2}{r^\beta}\big(\sigma^2(x,r)-\sigma_0^2\big)^2 \quad (7)$$

where m and σ^2 are the mean and the variance respectively of the sphere and are defined by:
$$m(x,r) = \big(\textstyle\int_{\mathcal{B}(x,r)} I(s)\mathrm{d}s\big)/\big(\int_{\mathcal{B}(x,r)} \mathrm{d}s\big) \text{ and } \sigma^2(x,r) =$$

$\big(\int_{\mathcal{B}(x,r)}(I(s) - m(s,r))^2\mathrm{d}s\big)/\big(\int_{\mathcal{B}(x,r)} \mathrm{d}s\big)$, m_0 and σ_0^2 represent the mean and the variance of the starting point; β is a real positive constant. This potential was studied in [3] for $\beta = 2$. However, optimal results can be obtained with $\beta < 2$. This potential satisfies the conditions described in the previous section. Other choices for the potential are presented and discussed in [3].

In Figures 3 and 2, we show experiments on 2D Angiogram retinal images. After performing the geodesic voting with the potential P_r given by the equation (7) we compute the map $\{\tilde{\mu}_m, \tilde{r}\}$. The starting point was chosen as the root of the tree, $r_{\max} = 4$, $\lambda_1 = \lambda_2 = 10$, $w = 0.01$ and $\beta = 2$. As the end points were chosen as a uniform grid, the spatial starting point can be chosen anywhere within the tree. However, the starting radii should be chosen carefully to get an optimal segmentation. In our experiments we obtained good estimation of these parameters by testing different values following the study presented in [3]. These parameters can be optimized and automatized for a given class of images.

5. CONCLUSION AND FUTURE WORK

In this paper we have presented a new method for the segmentation of tree structures, these methods are adapted to segment automatically the centerline and the walls of a tree from a single point given by the user, no a priori information about the tree is required. In contrast, the methods present in the literature for the segmentation of tree structures are not fully automatic and require prior information of the tree to be segmented. We have applied our approach to segment tubular tree structures from 3 images from the DRIVE data. The results are satisfying in terms of the overlap ratio O (O > 0.75). The next step is to extend our approach to 3D and to validate it on a large data set.

6. REFERENCES

[1] A. F. Frangi, W. J. Niessen, R. M. Hoogeveen, T. van Walsum, and M. A. Viergever, "Model-based quantitation of 3D magnetic resonance angiographic images," *IEEE Trans. Med. Imaging*, vol. 18, no. 10, pp. 946–956, 1999.

[2] S. Bouix, K. Siddiqi, and A. Tannenbaum, "Flux driven automatic centerline extraction," *Medical Image Analysis*, vol. 9, no. 3, pp. 209–221, 2005.

[3] H. Li and A. Yezzi, "Vessels as 4D curves: Global minimal 4D paths to extract 3D tubular surfaces and centerlines," *IEEE Trans. Med. Imaging*, vol. 26, pp. 1213–1223, 2007.

[4] H. Li, A. J. Yezzi, and L. D. Cohen, "3D multi-branch tubular surface and centerline extraction with 4d iterative key points," in *MICCAI (1)*, 2009, pp. 1042–1050.
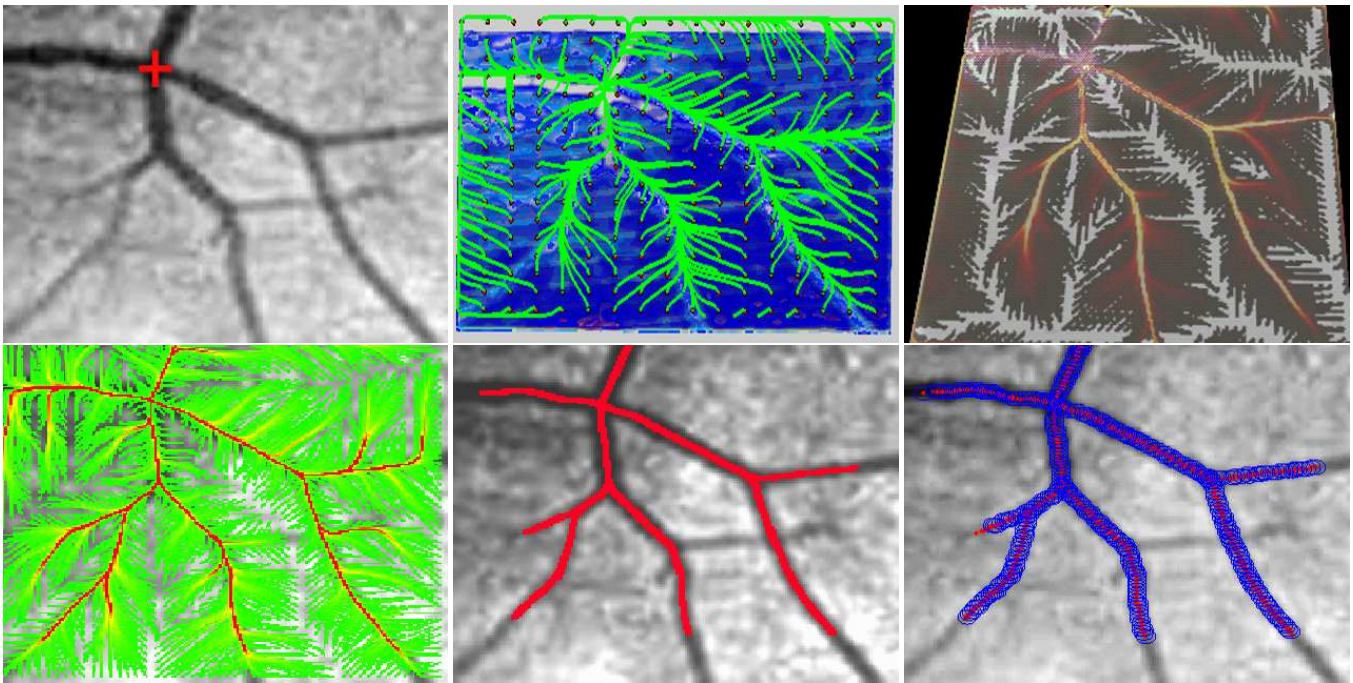
Fig. 3. Vessel segmentation of an angiogram 2D image with the geodesic voting method. First row: the left shows a 2D angiogram image, the red cross indicates the source point; the center panel shows, in (2D+radius) domain, in green the paths extracted from a uniform grid to the source point; the right panel shows the density computed in (2D+radius) domain, yellow color corresponds to high density and brown to low density. Second row: the left panel shows the geodesic density $\tilde{\mu}_m$, given by the equation (6), red color corresponds to high density, yellow color to medium, and green color to low density; the center panel shows in red the density $\tilde{\mu}_m$ after thresholding; the right panel shows in blue the extraction result of the tubular structure obtained by thresholding the map $\{\tilde{\mu}_m, \tilde{r}\}$.

[5] C. Kirbas and F. Quek, "A review of vessel extraction techniques and algorithms," *ACM Comput. Surv.*, vol. 36, no. 2, pp. 81–121, 2004.

[6] D. Lesage, E. D. Angelini, I. Bloch, and G. Funka-Lea, "A review of 3D vessel lumen segmentation techniques: Models, features and extraction schemes," *Medical image analysis*, vol. 13, no. 6, pp. 819–845, 2009.

[7] T. Deschamps and L. D. Cohen, "Fast extraction of minimal paths in 3D images and applications to virtual endoscopy," *Medical Image Analysis*, vol. 5, no. 4, pp. 281 – 299, 2001.

[8] B. B. Avants and J. P. Williams, "An adaptive minimal path generation technique for vessel tracking in CTA CE-MRA volume images," in *MICCAI 2000*, London, UK, 2000, pp. 707–716, Springer-Verlag.

[9] M. A. Gülsün and H. Tek, "Robust vessel tree modeling," in *MICCAI (1)*, 2008, pp. 602–611.

[10] O. Wink, W. J. Niessen, B. Verdonck, and M. A. Viergever, "Vessel axis determination using wave front propagation analysis," in *MICCAI 2001*, London, UK, 2001, pp. 845–853, Springer-Verlag.

[11] L. D. Cohen and T. Deschamps, "Segmentation of 3D tubular objects with adaptive front propagation and minimal tree extraction for 3D medical imaging," *Math. Models Methods Appl. Sci.*, vol. 10, no. 4, 2007.

[12] Y. Rouchdy and L. D. Cohen, "Image segmentation by geodesic voting. application to the extraction of tree structures from confocal microscope images," in *The 19th International Conference on Pattern Recognition*, Tampa, Florida, 2008, pp. 1–5.

[13] Y. Rouchdy and L. D. Cohen, "The shading zone problem in geodesic voting and its solutions for the segmentation of tree structures. application to the segmentation of microglia extensions," in *Computer Vision and Pattern Recognition Workshop*, Miami, Florida, 2009, pp. 66–71, IEEE Computer Society.

[14] L. D. Cohen and R. Kimmel, "Global minimum for active contour models: A minimal path approach," *International Journal of Computer Vision*, vol. 24, no. 1, pp. 57–78, 1997.

[15] J. A. Sethian, *Level set methods and fast marching methods*, Cambridge University Press, 1999.

[16] J.J. Staal, M.D. Abramoff, M. Niemeijer, M.A. Viergever, and B. van Ginneken, "Ridge based vessel segmentation in color images of the retina," *IEEE Trans. Med. Imaging*, vol. 23, no. 4, pp. 501–509, 2004.