Workshop on Functional Inequalities

June 4-6, 2025

Université Paris-Dauphine PSL

Room A (2nd floor)

Organized by CEREMADE, Laboratoire Jacques-Louis Lions (Sorbonne Université) and Fondation Sciences Mathématiques de Paris.

Speakers

- **Matteo Bonforte** (Universidad Autónoma de Madrid)
- **Jose Antonio Carrillo** (University of Oxford)
- **Djalil Chafai** (Ecole Normale Supérieure and Université Paris-Dauphine)
- **Dario Cordero-Erausquin** (Sorbonne Université)
- **Manuel del Pino** (University of Bath)
- **Charlotte Dietze** (LMU, Munich)
- **Antoine Henrot** (Université de Lorraine)
- **Gemei Liu** (ETH Zürich)
- **Michael Loss** (Georgia Tech University)
- **Idriss Mazari** (Université Paris-Dauphine)
- **Monica Musso** (University of Bath)
- **Jonas Peteranderl** (LMU, Munich)
- **Cyril Roberto** (Université Paris-Nanterre)
- **Sylvia Serfaty** (Sorbonne Université & Courant Institute New York)
- **Bruno Volzone** (Politecnico di Milano)

Organizers
Jean Dolbeault, Maria J. Esteban, Rupert Frank, Mathieu Lewin and Emmanuel Trélat.

PAGE OF THE WORKSHOP

REGISTRATION

PROGRAM

Wednesday June 4, 2025

09:00 - 9:50

Sharp Inequalities for Spinors
Michael Loss (Georgiatech)

While functional inequalities play an important role in mathematical physics, the focus was usually onSchr\"odinger operators with some notable exceptions like Hardy inequalities for Dirac operators.
In this talk I present some preliminary results for the sharp spinorial versionn of the Caffarelli-Kohn-Nirenberg inequalities, i.e., Sobolev inequalities with weights.
This is joint work with Jean Dolbeault, Maria Esteban and Rupert Frank.

Coffee break

09:50 - 10:20

10:20 - 11:10

Delaunay-like compact equilibria in the liquid drop model.
Monica Musso (Bath)

The liquid drop model was originally introduce by Gamow in 1928 to model atomic nuclei. The model describes the competition between surface tension (which keeps the nuclei together) and Coulomb force (which corresponds to repulsion among the protons). Equilibrium shapes correspond to sets in the 3-dimensional Euclidean space which satisfies an equation that links the mean curvature of the boundary of the set to the Newtonian potential of the set.
In this talk I will present the construction of toroidal surfaces, modelled on a family of Delaunay surfaces, with large volume which provide new equilibrium shapes for the liquid drop model. This work is in collaboration with M. del Pino and A. Zuniga.

11:10 - 12:00

Small mass, non-minimizing critical regions in the liquid Drop Model
Manuel del Pino (University of Bath)

I the liquid drop model, we present the construction of a critical configuration for the associated variational energy and a sufficiently small prescribed mass.
The region presented is axially symmetric, resembling two similar spheres connected by a small catenoidal neck. This solution is presumably the end point of a bifurcation branch of axially symmetric solutions to the variational problem, with the prescribed mass m as its parameter, as m goes to zero. This is a joint work with Rupert Frank and Monica Musso.

12:00 - 14:00

Lunch break

14:00 - 14:50

Quantitative inequalities in optimal control theory and convergence of thresholding schemes
Idriss Mazari-Fouquer (Ceremade)

We will give an overview of recent progress in the study of quantitative inequalities for optimal control problems. In particular, we will show how they can be used to obtain convergence results for thresholding schemes, which are of great importance in the simulation of optimal control problems.

This is a joint work with A. Chambolle and Y. Privat.

14:50 - 15:20

Coffee break

15:20 - 16:10

Gilles Pisier's approach to Gaussian concentration, isoperimetry, and Poincaré-type inequalities
Bruno Volzone (Politecnico di Milano)

In this talk we discuss a natural extension of Gilles Pisier's approach to the study of measure concentration, isoperimetry, and Poincar´e-type inequalities. This approach allows to explore counterparts of various results about Gaussian measures in the class of rotationally invariant probability distributions on Euclidean spaces, including multidimensional Cauchy measures.
These results are the object of a joint project with S. Bobkov.

16:10 - 17:00

Global minimizers of Interaction Energies
Jose Carrillo (Oxford)

I will review the existence and uniqueness of global minimizers for interaction energy functionals. Euler-Lagrange equations in the infinity wasserstein distance will be discussed. Based on linear convexity/concavity arguments, qualitative properties of

the global minimizers will also be treated. Anisotropic singular potentials appearing in dislocations will be shown to have rich qualitative properties with loss of dimension and ranges of explicit minimizers.
This talk will be based on several works in collaboration with Ruiwen Shu (University of Georgia).

Thursday June 5, 2025

09:00 - 9:50

Hardy-Sobolev interpolation inequalities
Charlotte Dietze (LMU Munich)

We derive a family of interpolation estimates which improve Hardy's inequality and cover the Sobolev critical exponent. We also determine all optimizers among radial functions in the endpoint case and discuss open questions on nonrestricted optimizers.
This is joint work with Phan Thành Nam.

Coffee break

09:50 - 10:20

10:20 - 11:10

Minimizing the first eigenvalue of systems
Antoine Henrot *(Nancy) In this talk, we ask whether a Faber-Krahn type inequality is true for the first eigenvalue of classical systems in physics: Stokes, Lam\'e (linear elasticity) or Maxwell. We will give partial results showing that the ball is not always the minimizer, by contrast with the scalar case of the Laplacian.*
This is a joint work with Antoine Lemenant, Idriss Mazari and Yannick Privat

11:10 - 12:00

The sharp sigma_2 - curvature inequality on the sphere in quantitative form
Jonas Peteranderl (LMU Munich)

In case a sharp functional inequality admits optimizers, we are interested in improving the inequality by adding terms that involve a distance to the set of optimizers. As is well known, among all metrics on the sphere that are conformal to the standard metric and have positive scalar curvature, the total sigma_2 -curvature, normalized by the volume, is uniquely (up to Möbius transformations) minimized by the standard metric. Formulating this problem as a functional inequality in the conformal factor, we show that if a metric almost minimizes, then it is almost the standard metric (up to Möbius transformations). Moreover, we obtain a refined inequality with optimal exponents for two different notions of distance to the set of minimizers.
The talk is based on joint work with Rupert Frank.

12:00 - 14:00

Lunch break

14:00 - 14:50

Heating Legendre and Laplace
Dario Cordero-Erausquin (Paris Sorbonne)

We will report on some recent progress on sharp integral inequalities for the Laplace and Legendre transforms, related to classical inequalities in convex geometry such as the Blaschke-Santalo inequality.
These inequalities will be established by proving monotonicity along heat or Fokker-Plank flow. In particular, we will explain how duality (in the form of the Legendre transform) interacts

with heat flow.
Monotonicity along heat flow will then rely on the variance Brascamp-Lieb inequality.

14:50 - 15:20 Coffee break

15:20 - 16:10 **On the Entropy Power of Sums of Dependent Random Variables**
Cyril Roberto (Paris Nanterre & CNRS)

I'll present a version of the entropy power inequality for dependent random vectors, after Takano, Johnson and Rioul. A notable consequence is that an entropy power inequality stated using conditional entropies holds for random vectors whose joint density is log-supermodular. I'll also show that log-supermodularity is stable under convolution.

16:10 - 17:00 **Sharp stability for the Sobolev inequality in absence of bubbling.**
Gemei Liu (ETH Zurich)

I will provide an introduction to the quantitative stability of the Sobolev inequality in terms of the Sobolev energy and the Euler-Lagrange equations. Then, I will then present a recent joint work with Yi Zhang that extends the sharp stability estimate of critical from the known case $p = 2$ to the full range $1 < p < n$, assuming no bubbling. The main challenge is that the p-Sobolev operator becomes non-linear, which requires us to construct new vectorial inequalities and corresponding spectrum gap inequalities.

Friday June 6, 2025

09:50 - 10:20 Coffee break

10:20 - 11:10 **Aspects of the Cutoff Phenomenon for Diffusions**
Djalil Chafaï (Paris Dauphine)

The cutoff phenomenon, conceptualized in the context of finite Markov chains, states that for certain evolution equations, started from a point, the distance towards a long time equilibrium may become more and more abrupt in high dimensional state spaces and for certain choices of initial conditions. This can be seen as a critical competition between trend to equilibrium and initial condition. This talk is about the cutoff phenomenon for a few classes of linear and nonlinear diffusions.
This is about joint works with Jeanne Boursier, and Cyril Labbé, with Max Fathi, and with Max Fathi and Nikita Simonov.

11:10 - 12:00 **Mean-field limits for Coulomb-type dynamics via the modulated energy method**
Sylvia Serfaty (Paris Sorbonne & Courant Institute New York)

We consider a system of N points in singular interaction of Coulomb or Riesz type, evolving by gradient flow or conservative flow (such as the point vortex system in 2D) with or without noise. We describe the convergence to the mean-field limit by a modulated energy method, that relies on a functional inequality of commutator estimate type. We also discuss the question of obtaining global-in-time convergence and its connection with modulated log-Sobolev inequalities.