Refined graph convergence

\[ u(x, t) \]

Figure 1: For \( t > 0 \) large enough, the two cases \( s(t) \leq c_M \) and \( s(t) > c_M \) are possible.
Special solutions

Figure 2: The $N$-wave solution corresponding to $U_0(\xi) = \frac{q}{q-1} \frac{1}{\xi^{q-1}} \mathbb{1}_{[0,1]}(\xi)$ for various $\tau > 0$, in case $q = \frac{3}{2}$.
Figure 3: The solution corresponding to \( U_0(\xi) = \kappa_0 \mathbb{I}_{[a_0,b_0]}(\xi) \xi^{q-1} + h \mathbb{I}_{[b_0,c_0]}(\xi) \) is plotted here for various \( \tau > 0 \), in case \( q = \frac{3}{2}, a_0 = 0, b_0 = \frac{1}{2}, c_0 = 1, h = \frac{1}{2} \) and \( \kappa_0 \) such that \( \int U_0(\xi) \, d\xi = 1 \).
Figure 4: The solution with \( U_0(\xi) = 1_{[0,1]}(\xi) \) in case \( q = \frac{3}{2} \). This corresponds to the limit situation (in the second case) for which \( b_0 = 0 \) at \( \tau = 0 \) and \( \kappa(\tau) (b(\tau))^{1/(q-1)} = h \) for any \( \tau \in (0, \tau_0) \).
General solutions

\[ U(\xi, \tau) \]

\[ \tau = 0 \quad \tau = 0.4 \quad \tau = 0.8 \quad \tau = 1.2 \quad \tau = 1.6 \]

\[ \xi \]

Figure 5: A typical solution.
Figure 6: Upper and lower solutions.
Figure 7: Left: initial data. Right: for some $\tau > 0$ large enough.