## Refined graph convergence



Figure 1: For t > 0 large enough, the two cases  $s(t) \leq c_M$ and  $s(t) > c_M$  are possible.

## Special solutions



Figure 2: The N-wave solution corresponding to  $U_0(\xi) = \frac{q}{q-1}\xi^{\frac{1}{q-1}} \mathbb{1}_{[0,1]}(\xi)$  for various  $\tau > 0$ , in case  $q = \frac{3}{2}$ .



Figure 3: ThAe solution corresponding to th  $U_0(\xi) = \kappa_0 \mathbb{1}_{[a_0,b_0]}(\xi) \xi^{\frac{1}{q-1}} + h \mathbb{1}_{[b_0,c_0]}(\xi)$  is plotted here for various  $\tau > 0$ , in case  $q = \frac{3}{2}$ ,  $a_0 = 0$ ,  $b_0 = \frac{1}{2}$ ,  $c_0 = 1$ ,  $h = \frac{1}{2}$  and  $\kappa_0$  such that  $\int U_0(\xi) d\xi = 1$ .



Figure 4: The solution with  $U_0(\xi) = \mathbb{1}_{[0,1]}(\xi)$  in case  $q = \frac{3}{2}$ . This corresponds to the limit situation (in the second case) for which  $b_0 = 0$  at  $\tau = 0$  and  $\kappa(\tau) (b(\tau))^{1/(q-1)} = h$  for any  $\tau \in (0, \tau_0)$ .

## General solutions



Figure 5: A typical solution.



Figure 6: Upper and lower solutions.

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Figure 7: Left: initial data. Right: for some  $\tau > 0$  large enough.