# MAGNETIC INTERPOLATION INEQUALITIES, GROUND STATES OF MAGNETIC NONLINEAR SCHRÖDINGER EQUATIONS AND SYMMETRY

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In presence of a magnetic field, interpolation inequalities of Gagliardo-Nirenberg type can be reduced to the non-magnetic case using the diamagnetic inequality. Such nonlinear problems have very interesting spectral consequences, as they allow to estimate ground state energies of magnetic Schrödinger operators in terms of norms of the potential. In order to get optimal estimates, one has to study the solution of an Euler-Lagrange equation which is the ground state of a nonlinear Schrödinger (NLS) equation with magnetic field. When the magnetic field has simple symmetry properties, it is natural to ask whether the solution of NLS inherits of this symmetry or not, in which case a symmetry breaking phenomenon occurs. Various cases can be considered (constant magnetic fields and Aharonov-Bohm magnetic fields, models in various dimensions, bifurcations from symmetric to non-symmetric solutions) as well as applications to, for instance, magnetic Hardy inequalities in dimensions two and three. A major difficulty is that solutions are complex valued and the phase of the solution of NLS makes the problem much harder than for real valued versions of NLS. As an introduction, the reader is invited to refer to the slides of a lecture on *Symmetry of the ground state of the nonlinear Schrödinger equation in presence of a magnetic field* that can be found at

https://www.ceremade.dauphine.fr/~dolbeaul/Lectures/files/Alghero-18-9-2019.pdf

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▷ D. BONHEURE, J. DOLBEAULT, M. ESTEBAN, A. LAPTEV, AND M. LOSS, *Inequalities involving Aharonov-Bohm magnetic potentials in dimensions 2 and 3*, arXiv:1902.06454.

This paper is devoted to the symmetry and symmetry breaking properties of a 2-dimensional magnetic Schrödinger operator involving an Aharonov-Bohm magnetic vector potential. We investigate the symmetry properties of the optimal potential for the corresponding magnetic Keller-Lieb-Thirring inequality. We prove that this potential is radially symmetric if the intensity of the magnetic field is below an explicit threshold, while symmetry is broken above a second threshold corresponding to a higher magnetic field. The method relies on the study of the magnetic kinetic energy of the wave function and

*Date*: February 26, 2020.

amounts to study the symmetry properties of the optimal functions in a magnetic Hardy-Sobolev interpolation inequality. We give a quantified range of symmetry by a non-perturbative method. To establish the symmetry breaking range, we exploit the coupling of the phase and of the modulus and also obtain a quantitative result.

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▷ D. BONHEURE, J. DOLBEAULT, M. J. ESTEBAN, A. LAPTEV, AND M. LOSS, *Symmetry results in two-dimensional inequalities for Aharonov-Bohm magnetic fields*, Communications in Mathematical Physics, (2019).

This paper is devoted to interpolation inequalities of Gagliardo-Nirenberg type associated with Schrödinger operators involving Aharonov-Bohm magnetic potentials and related magnetic Hardy inequalities in dimensions 2 and 3. The focus is on symmetry properties of the optimal functions, with explicit ranges of symmetry and symmetry breaking in terms of the intensity of the magnetic potential.

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▷ J. DOLBEAULT, M. J. ESTEBAN, A. LAPTEV, AND M. LOSS, *Magnetic rings*, Journal of Mathematical Physics, 59 (2018), p. 051504.

We study functional and spectral properties of perturbations of the operator $-(\partial_s + i\,a)^2$ in $\mathrm{L}^2(\mathbb{S}^1)$. This operator appears when considering the restriction to the unit circle of a two dimensional Schrödinger operator with the Bohm-Aharonov vector potential. We prove a Hardy-type inequality on $\mathbb{R}^2$ and, on $\mathbb{S}^1$, a sharp interpolation inequality and a sharp Keller-Lieb-Thirring inequality.

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▷ J. DOLBEAULT, M. J. ESTEBAN, A. LAPTEV, AND M. LOSS, *Interpolation inequalities and spectral estimates for magnetic operators*, Annales Henri Poincaré, 19 (2018), pp. 1439–1463.

We prove magnetic interpolation inequalities and Keller-Lieb-Thirring estimates for the principal eigenvalue of magnetic Schrödinger operators. We establish explicit upper and lower bounds for the best constants and show by numerical methods that our theoretical estimates are accurate.

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