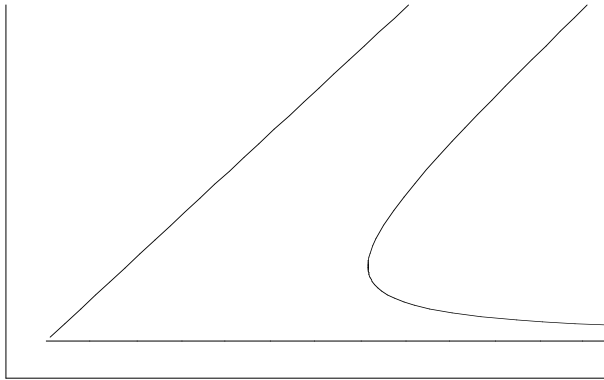


Multiple bubbling for the exponential nonlinearity in the slightly supercritical case

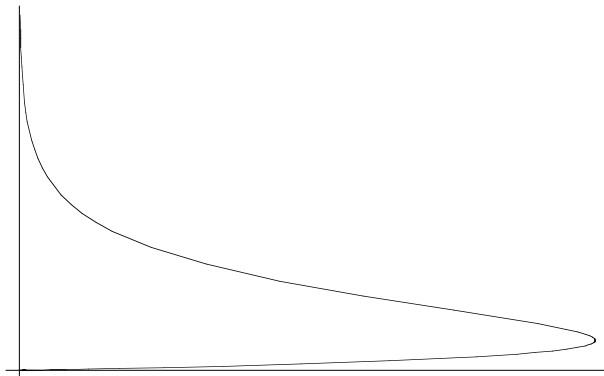
Manuel del Pino, Jean Dolbeault, Monica Musso

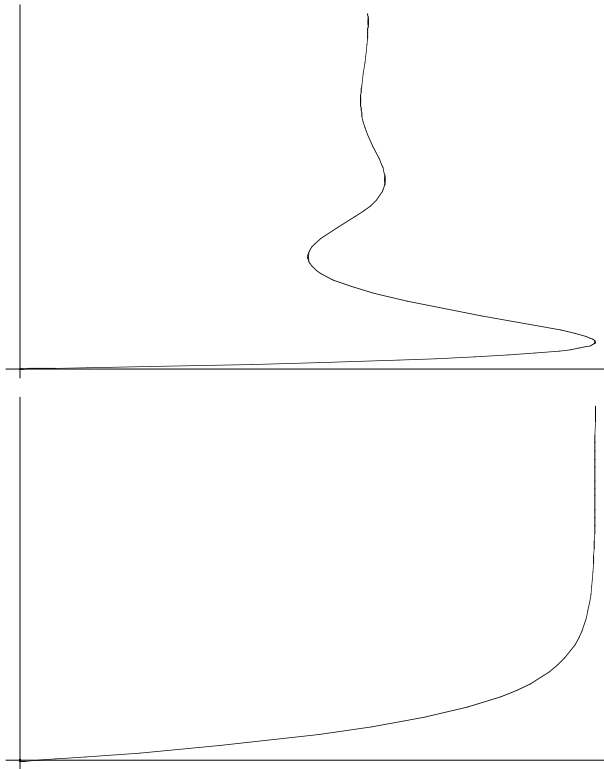
Range of the parameters

```
ParametricPlot[{{ $\frac{p(p+3)}{p-1}$ , p}, {p, p}, {3 p - 2.3, 1}},  
{p, 1.1, 10}, PlotRange -> {{0, 15}, {0, 10}}, Ticks -> None];
```



```
F[x0_, y0_, e_, smin_, smax_, tic_, Dsply_, PR_] := ParametricPlot[  
  Evaluate[{x[s], y[s]} /. NDSolve[{x'[s] == x[s] (y[s] + 2), y'[s] == -e y[s] - x[s],  
    x[0] == x0, y[0] == y0}, {x, y}, {s, smin, smax}]], {s, smin, smax},  
  PlotPoints -> 100, DisplayFunction -> Dsply, Ticks -> tic, PlotRange -> PR];  
  
GO[η_, n_, smax_] := ParametricPlot[  
  Evaluate[{x[s], 2 s - Log[ $\frac{x[s]}{2 \eta}$ ]} /. NDSolve[{x'[s] == x[s] (y[s] + 2), y'[s] ==  
    -(n - 2) y[s] - x[s], x[0] == 2 η, y[0] == -(n - 2) * η}, {x, y}, {s, -1, smax}]],  
  {s, -1, smax}, PlotPoints -> 40, PlotRange -> All, Ticks -> None];  
GO[0.01, 2, 7];  
GO[0.01, 3, 13];  
GO[0.000001, 12, 12];
```



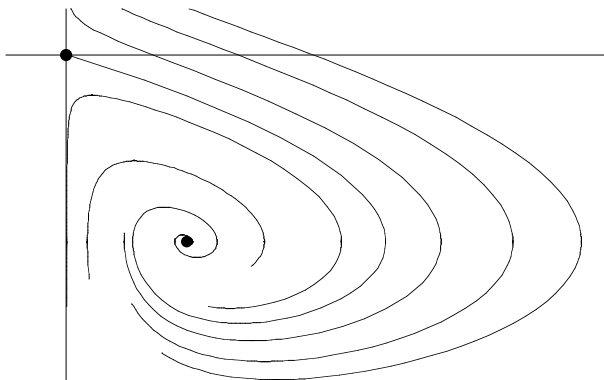


Supercritical Case

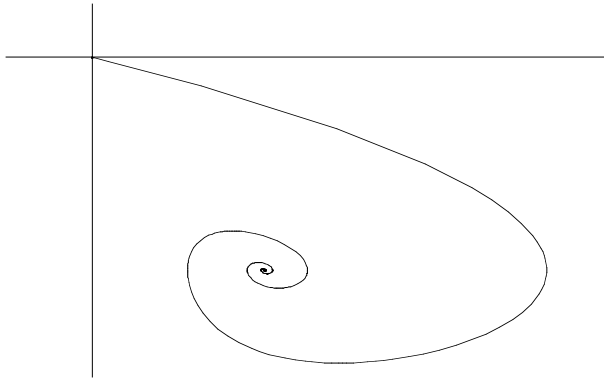
```

T = {{0.01, 0.01, 0.5, -2, 100}, {0.001, 1, 0.5, -2, 7}, {0.01, -1, 0.5, -2, 5.7},
      {0.2, -2.5, 0.5, 0.1, 5}, {3.7, -2, 0.5, -2, 2}, {3.5, -3, 0.5, -2, 1.1}};
Off[General::"spell1"]
Pplus[p_, e_] := {pp-1 e, -pp-1}
Tble = Table[F[T[[i]][[1]], T[[i]][[2]], T[[i]][[3]], T[[i]][[4]],
              T[[i]][[5]], None, Identity, {{-0.5, 3.5}, {-3.5, 1}}, {i, 1, 6}];
Pts = Show[Graphics[{PointSize[0.02], {Point[{0, 0}], Point[Pplus[2, 0.5]]}},
              DisplayFunction -> Identity, PlotRange -> {{-0.5, 3.5}, {-3.5, 1}}, Axes -> None]];
Show[{Tble, Pts}, DisplayFunction -> $DisplayFunction,
      PlotRange -> {{-0.5, 4.5}, {-3.5, 0.5}}, Ticks -> None];

```

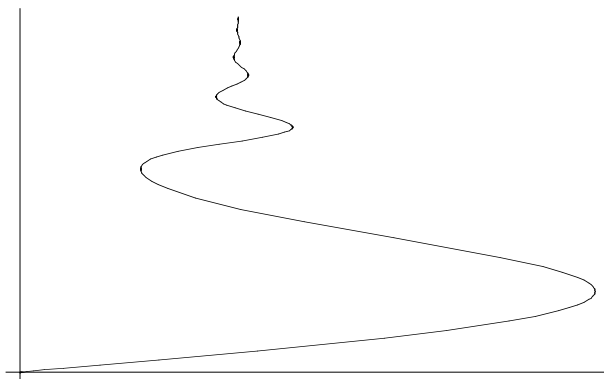


```
F[0.01, -0.01 * 2 / (2 + 0.5), 0.5, -2,
  100, None, $DisplayFunction, {{-0.5, 3}, {-3, 0.5}}];
```



■ Plot in a logarithmic scale: $\text{Log}[1+u[0]]$

```
G[η_, ε_, smax_, p_, PR_] := ParametricPlot[
  Evaluate[{x[s], Log[1 + p s - Log[ $\frac{x[s]}{\eta}$ ]]} /. NDSolve[{x'[s] == x[s] (y[s] + p),
    y'[s] == -ε y[s] - x[s], x[0] == η, y[0] == -p * η / (p + ε)}, {x, y}, {s, -1, smax}]],
  {s, -1, smax}, PlotPoints → 100, PlotRange → PR, Ticks → None];
G[0.01, 0.5, 30, 2, All];
```

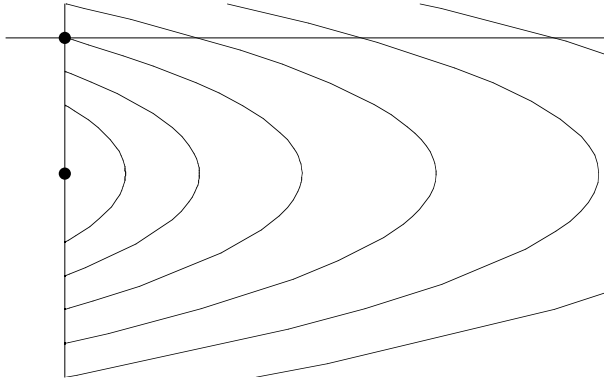


Critical Case

```

T = {{0.001, 0.001, 0, -2, 100}, {0.01, 1, 0, -2, 100}, {0.01, -1, 0, -2, 100},
      {0.01, 1.5, 0, -2, 100}, {0.01, -0.5, 0, -2, 100}, {0.01, 0.5, 0, -2, 100}};
Tble = Table[F[T[[i]][[1]], T[[i]][[2]], T[[i]][[3]], T[[i]][[4]],
              T[[i]][[5]], None, Identity, {{-0.5, 3.5}, {-5, 1}}], {i, 1, 6}];
Pts = Show[Graphics[{PointSize[0.02], {Point[{0, 0}], Point[Pplus[2, 0]]}},
                  DisplayFunction -> Identity, PlotRange -> {{-0.5, 3.5}, {-5, 1}}, Axes -> None]];
Show[{Tble, Pts}, DisplayFunction -> $DisplayFunction,
      PlotRange -> {{-0.5, 4.6}, {-5, 0.5}}, Ticks -> None];

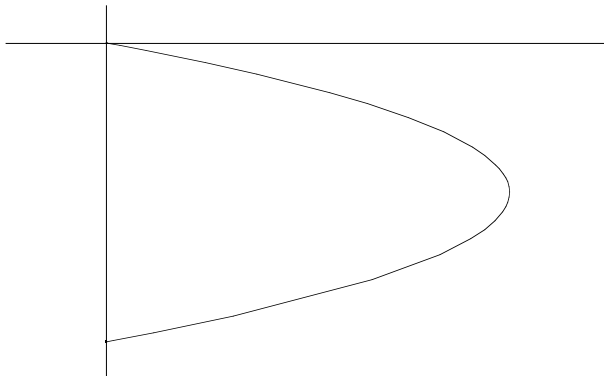
```



```

F[0.001, 0.001, 0, -2, 100, None, $DisplayFunction, {{-0.5, 2.5}, {-4.5, 0.5}}];

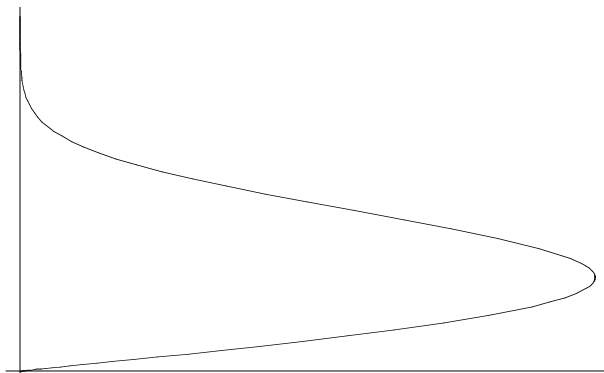
```



```

G[0.01, 0, 10, 2, Automatic];

```



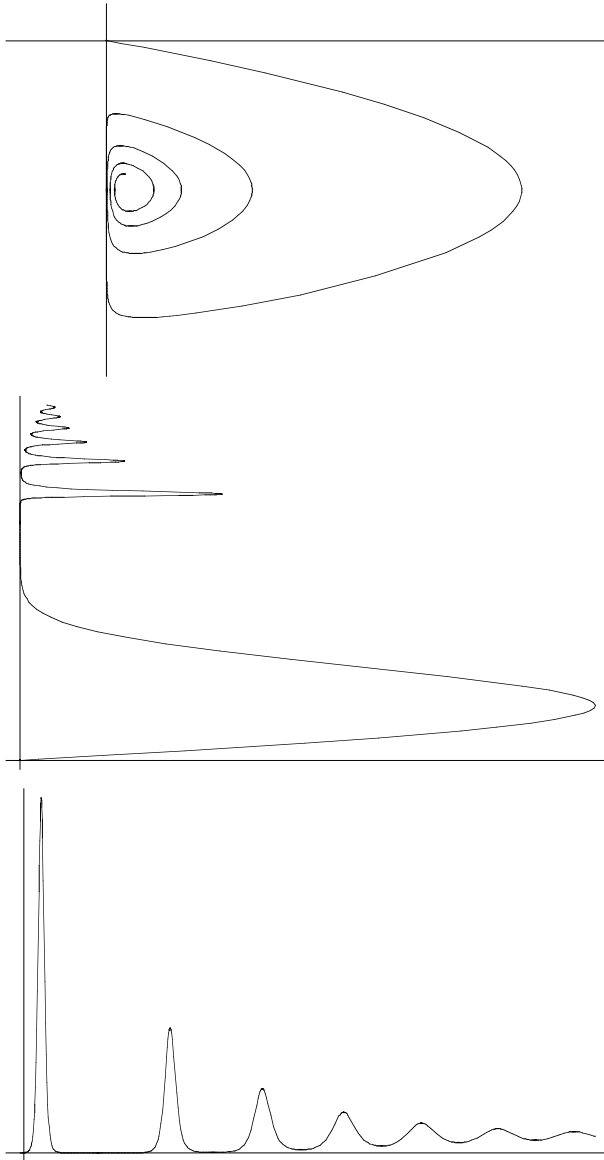
Multi-Bubbles

```

H[η_, ε_, smax_, p_] :=
  Plot[Evaluate[x[s] /. NDSolve[{x'[s] == x[s] (y[s] + p), y'[s] == -ε y[s] - x[s],
    x[0] == η, y[0] == -p * η / (p + ε)}, {x, y}, {s, -1, smax}]],
    {s, -1, smax}, PlotPoints → 100, PlotRange → All, Ticks → None];

F[0.001, 0.001, 0.05, -2, 100, None, $DisplayFunction, {{-0.5, 2.5}, {-4.5, 0.5}}];
G[0.001, 0.05, 150, 2, All];
H[0.001, 0.05, 150, 2];

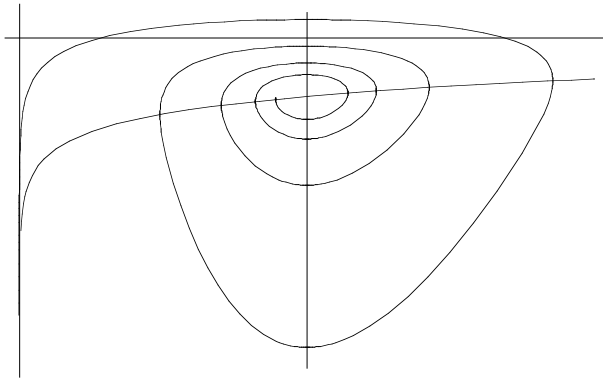
```



```

Fnew[x0_, y0_, e_, smin_, smax_, tic_, Dsply_, PR_] :=
  ParametricPlot[Evaluate[{-y[s], Log[x[s]]} /. NDSolve[{x'[s] == x[s] (y[s] + 2),
    y'[s] == -e y[s] - x[s], x[0] == x0, y[0] == y0}, {x, y}, {s, smin, smax}]],
    {s, smin, smax}, PlotPoints -> 100, DisplayFunction -> Dsply,
    Ticks -> tic, PlotRange -> PR];
FnewPlot = Fnew[0.001, 0.001, 0.05, -2, 100, True, Identity, All];
Critical = Plot[Log[0.05 u], {u, 0.01, 4},
  PlotRange -> {{-0.2, 4}, {-13, 1}}, Ticks -> None, DisplayFunction -> Identity];
Show[{FnewPlot, Critical, Graphics[Line[{2, -13}, {2, 1}]}]},
  DisplayFunction -> $DisplayFunction, Ticks -> None];

```



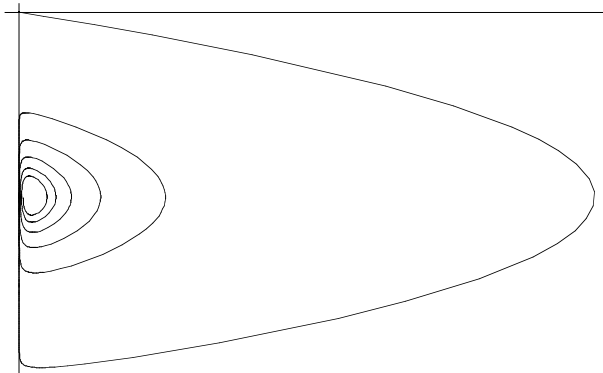
Asymptotic regime for Multi-Bubbles ?

```

RF[p_, η_, e_, smin_, smax_, tic_, Dsply_, PR_, PP_] := ParametricPlot[
  Evaluate[{x[s], y[s]} /. NDSolve[{x'[s] == x[s] (y[s] + p), y'[s] == -e y[s] - x[s],
    x[0] == η, y[0] == -p * η / (p + e)}, {x, y}, {s, smin, smax}]], {s, smin, smax},
  PlotPoints -> PP, DisplayFunction -> Dsply, Ticks -> tic, PlotRange -> PR];

RF[5, 0.0001, 0.05, -2, 100, None, $DisplayFunction, All, 100];

```



```

RF[5, 0.0001, 0.035, -2, 50, None, $DisplayFunction, All, 1000];

```

