Multiplicity results for the assigned Gauss curvature problem in R^2 (1 / 5)

Jean Dolbeault, Maria Esteban,
and Gabriella Tarantello

Note that all computations are done for the nonlinearity: e^u

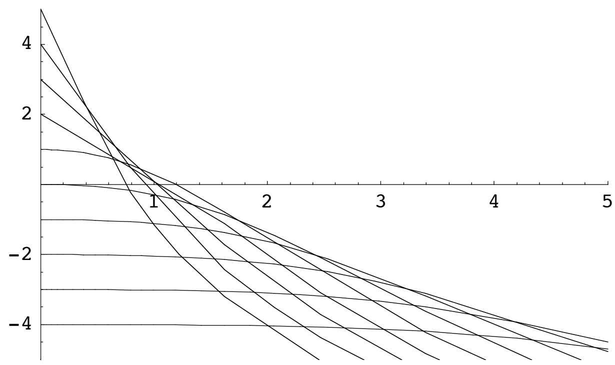
```
Off[NDSolve::"mxst"]
Off[NDSolve::"nlnum"]
Off[Power::"infy"]
Off[∞::"indet"]
```

```
η = 10^-4;
ϵ[a_, ξ_] := Min[ξ ⅇ^-a, 10^-6]
rmax = 40;
```

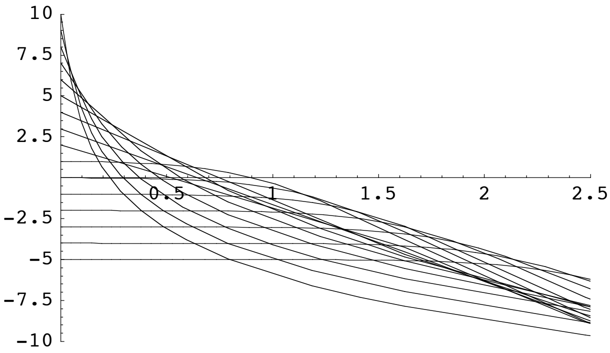
■ Plot of the solution as a function of r for various values of a and N

```
Pltu[a_, N_, DF_] := Module[{δ = ϵ [a, η]},
  Plot[u[s] /. NDSolve[{u'[r] == v[r], u[δ] == a - 1/4 ⅇ^a δ^2, v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^u[r],
      v[δ] == -1/2 ⅇ^a δ , m'[r] == r (1 + r^2)^N ⅇ^u[r], m[δ] == 1/2 ⅇ^a δ^2},
    {u, v, m}, {r, δ, rmax}], {s, δ, rmax}, DisplayFunction → DF]]
```

```
Show[Table[Pltu[a, 1, Identity], {a, -5, 5}],
  DisplayFunction -> $DisplayFunction, PlotRange → {{0, 5}, {-5, 5}}];
```



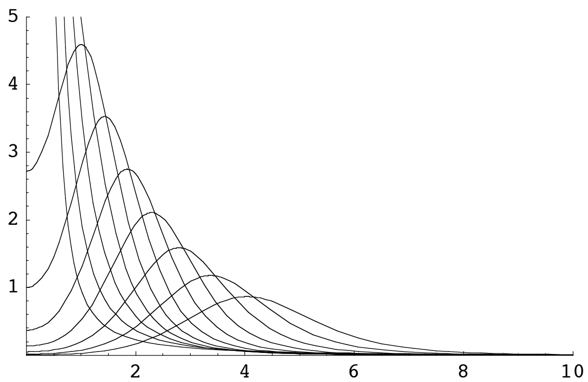
```
Show[Table[Pltu[a, 4, Identity], {a, -5, 10}],
  DisplayFunction -> $DisplayFunction, PlotRange → {{0, 2.5}, {-10, 10}}];
```



■ Plot of the density as a function of r for various values of a and N

```
PltDensity[a_, N_, DF_] := Module[{δ = ϵ [a, η]}, Plot[(1 + s^2)^N ⅇ^u[s] /.
    NDSolve[{u'[r] == v[r], u[δ] == a - 1/4 ⅇ^a δ^2, v'[r] == - u'[r]/r - (1 + r^2)^N ⅇ^u[r],
      v[δ] == - 1/2 ⅇ^a δ, m'[r] == r (1 + r^2)^N ⅇ^u[r], m[δ] == 1/2 ⅇ^a δ^2}, {u, v, m}, {r, δ, rmax}],
   {s, δ, rmax}, DisplayFunction → DF, PlotPoints → 100]]
```

```
Show[Table[PltDensity[a, 2, Identity], {a, -5, 5}],
  DisplayFunction -> $DisplayFunction, PlotRange → {{0, 10}, {0, 5}}];
```

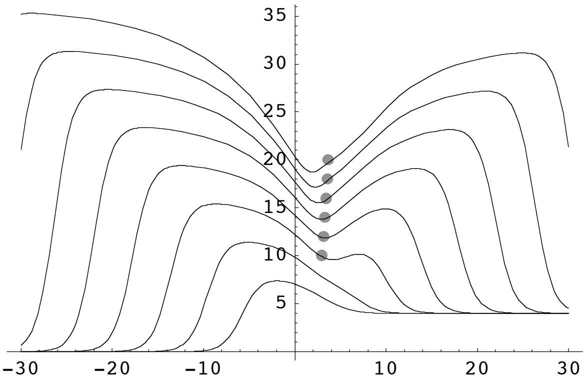


■ Plot of the limit of the density as r→∞ as a function of a and N

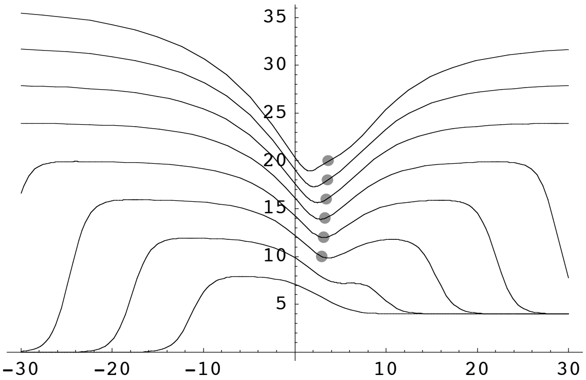
```
PltExlct[N_] := If[N > 2, ListPlot[{{Log[4 (N + 2)], 2 (N + 2)}},
   PlotStyle → {PointSize[0.02], GrayLevel[0.5]}, DisplayFunction → Identity], {}]
```

```
F[N_, amin_, amax_, DF_] :=
 Show[{PltExlct[N], Plot[Module[{δ = ϵ[a, η]}, m[rmax] /. NDSolve[
      {u'[r] == v[r], u[δ] == a - 1/4 ⅇ^a δ^2, v'[r] == - u'[r]/r - (1 + r^2)^N ⅇ^u[r], v[δ] == - 1/2 ⅇ^a δ,
       m'[r] == r (1 + r^2)^N ⅇ^u[r], m[δ] == 1/2 ⅇ^a δ^2}, {u, v, m}, {r, δ, rmax}]],
    {a, amin, amax}, DisplayFunction → Identity]}, DisplayFunction → DF]
```

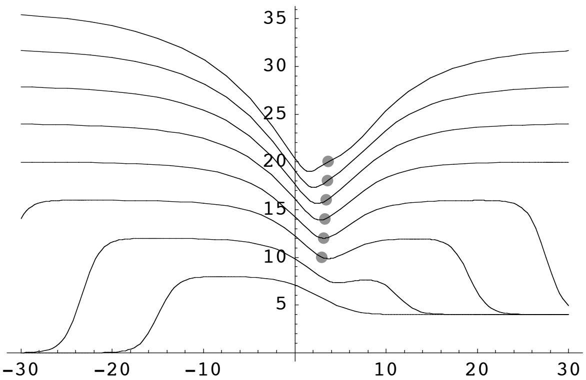
```
rmax = 10;
Show[Table[F[N, -30, 30, Identity], {N, 1, 8}],
  DisplayFunction -> $DisplayFunction, PlotRange → All];
```




```
rmax = 40;
Show[Table[F[N, -30, 30, Identity], {N, 1, 8}],
  DisplayFunction -> $DisplayFunction, PlotRange → All];
```

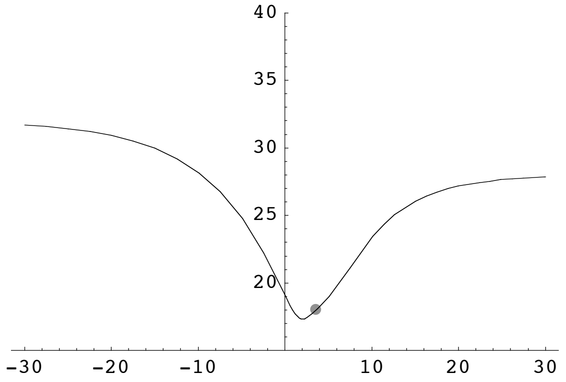



```
rmax = 100;
Show[Table[F[N, -30, 30, Identity], {N, 1, 8}],
  DisplayFunction -> $DisplayFunction, PlotRange → All];
```



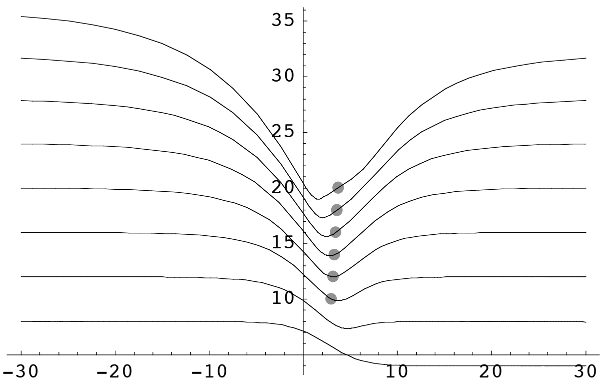
▪ FIGURE 1:

```
rmax = 10^7;
Show[F[7, -30, 30, Identity],
  DisplayFunction -> $DisplayFunction, PlotRange → {All, {15, 40}}];
```



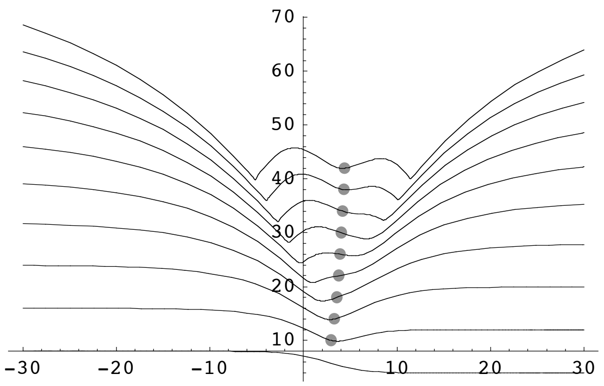
▪ FIGURE 2:

```
rmax = 10^7;
Show[Table[F[N, -30, 30, Identity], {N, 1, 8}],
  DisplayFunction -> $DisplayFunction, PlotRange → All];
```



▪ FIGURE 3:

```
rmax = 10^7;
Show[Table[F[N, -30, 30, Identity], {N, 1, 20, 2}],
  DisplayFunction -> $DisplayFunction, PlotRange → All];
```

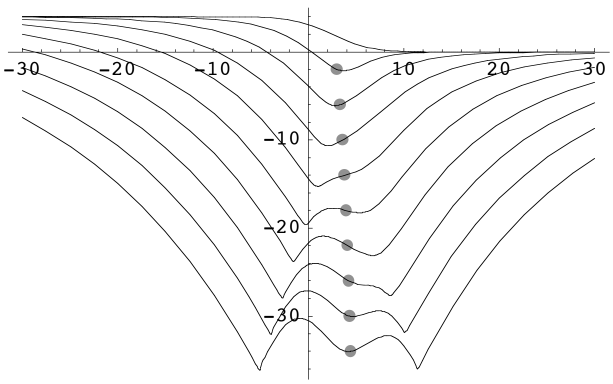


▪ Plot of the limit (translated of -4N) of the density as r→∞ as a function of a and N

```
PltExlcttrans[N_] := If[N > 2, ListPlot[{{Log[4 (N + 2)], 2 (2 - N)}},
   PlotStyle → {PointSize[0.02], GrayLevel[0.5]}, DisplayFunction → Identity], {}]

Ftrans[N_, amin_, amax_, DF_] :=
 Show[{PltExlcttrans[N], Plot[Module[{δ = ϵ[a, η]}, m[rmax] - 4 N /. NDSolve[
      {u'[r] == v[r], u[δ] == a - 1/4 ⅇ^a δ^2, v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^u[r], v[δ] == -1/2 ⅇ^a δ,
       m'[r] == r (1 + r^2)^N ⅇ^u[r], m[δ] == 1/2 ⅇ^a δ^2}, {u, v, m}, {r, δ, rmax}]],
    {a, amin, amax}, DisplayFunction → Identity]}, DisplayFunction → DF]
```

▪ FIGURE 4:

```
rmax = 10^7;
Show[Table[Ftrans[N, -30, 30, Identity], {N, 1, 20, 2}],
  DisplayFunction -> $DisplayFunction, PlotRange → All];
```

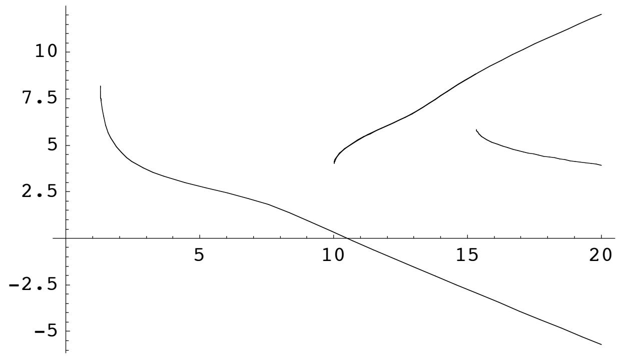


▪ Location of the minima of α(a) as a function of N and curve min α(a) / 4N as a function of N

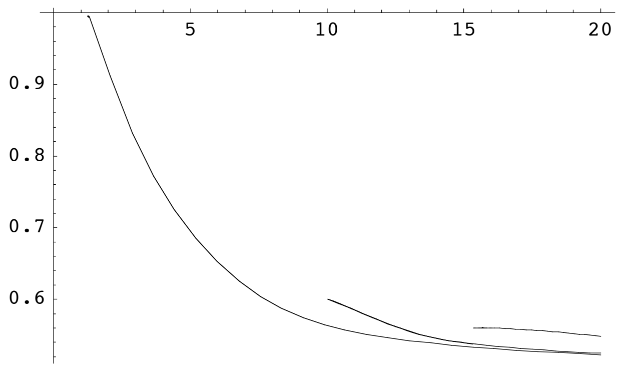
```
ξ = 10^-6;

Mass[N_, amin_, amax_, a_] :=
 Module[{δ = ϵ[a, η]}, m[rmax] /. NDSolve[{u'[r] == v[r], u[δ] == a - 1/4 ⅇ^a δ^2,
     v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^u[r], v[δ] == -1/2 ⅇ^a δ, m'[r] == r (1 + r^2)^N ⅇ^u[r],
     m[δ] == 1/2 ⅇ^a δ^2}, {u, v, m}, {r, δ, rmax}]][[1]]

MinIterDicho[NN_, amin_, amax_, a_, f_, s_] :=
 If[a > amax + 0.1 || a < amin - 0.1, {N[a], 4 NN}, If[Abs[s] < ξ, {N[a], f}, Module[{M =
      Mass[NN, amin, amax, a]},
    If[M < f, MinIterDicho[NN, amin, amax, a + s, M, s],
     MinIterDicho[NN, amin, amax, a - s / 2, M, -s / 2]]]]]

SearchMin[N_, amin_, ainit_, amax_, s_] :=
 Module[{x = Mass[N, amin, amax, ainit]}, MinIterDicho[N, amin, amax, ainit + s, x, s]]
```

■ FIGURE 5:

```
Show[Plot[SearchMin[x, -30, 1.5, 30, 0.1][[1]],
   {x, 1.27, 1.3}, DisplayFunction → Identity],
  Plot[SearchMin[x, -30, 5 - x / 2, 30, -0.25][[1]], {x, 1.3, 20},
   DisplayFunction → Identity], Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[1]],
   {x, 10.01, 15.325}, DisplayFunction → Identity],
  Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[1]], {x, 15.335, 20},
   DisplayFunction → Identity], Plot[SearchMin[x, -30, -5 + x, 30, 0.5][[1]],
   {x, 10.01, 20}, DisplayFunction → Identity], DisplayFunction -> $DisplayFunction];
```

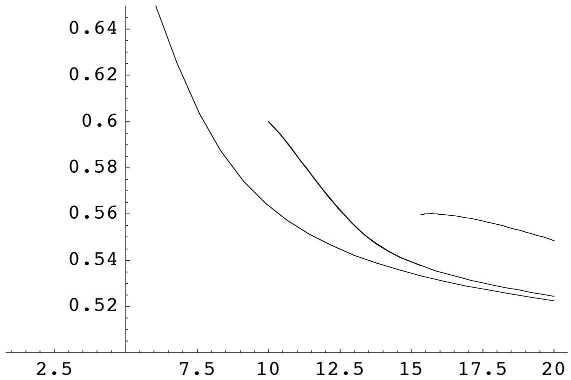



```
Show[Plot[SearchMin[x, -30, 1.5, 30, 0.1][[2]] / (4 x), {x, 1.27, 1.3},
   DisplayFunction → Identity], Plot[SearchMin[x, -30, 5 - x / 2, 30, -0.25][[2]] / (4 x),
   {x, 1.3, 20}, DisplayFunction → Identity],
  Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[2]] / (4 x),
   {x, 10.01, 15.325}, DisplayFunction → Identity],
  Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[2]] / (4 x), {x, 15.335, 20},
   DisplayFunction → Identity], Plot[SearchMin[x, -30, -5 + x, 30, 0.5][[2]] / (4 x),
   {x, 10.01, 20}, DisplayFunction → Identity], DisplayFunction -> $DisplayFunction];
```



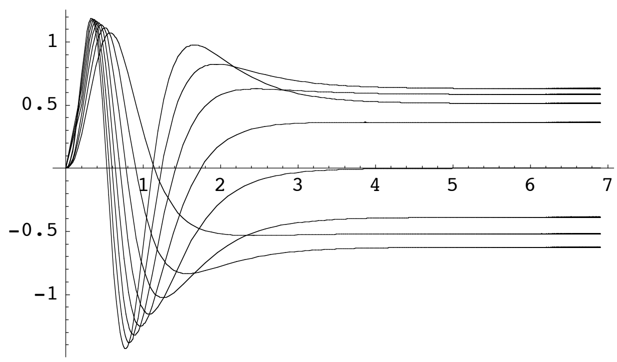
■ FIGURE 6:

```
Show[Plot[SearchMin[x, -30, 1.5, 30, 0.1][[2]] / (4 x), {x, 1.27, 1.3},
   DisplayFunction → Identity], Plot[SearchMin[x, -30, 5 - x / 2, 30, -0.25][[2]] / (4 x),
   {x, 1.3, 20}, DisplayFunction → Identity],
  Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[2]] / (4 x),
   {x, 10.01, 15.325}, DisplayFunction → Identity],
  Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[2]] / (4 x), {x, 15.335, 20},
   DisplayFunction → Identity], Plot[SearchMin[x, -30, -5 + x, 30, 0.5][[2]] / (4 x),
   {x, 10.01, 20}, DisplayFunction → Identity],
  DisplayFunction -> $DisplayFunction, PlotRange -> {All, {0.5, 0.65}}];
```

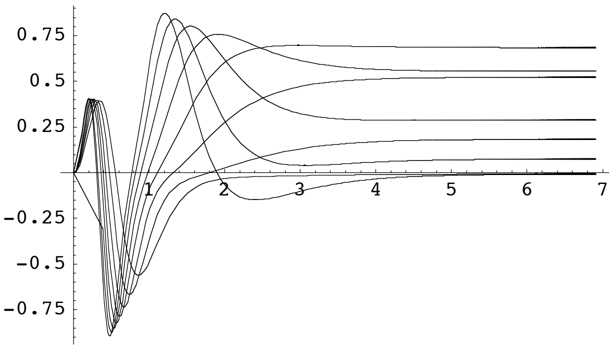


■ A study of the variation of α at a=2(N+2)

```
η = 10^-4;

F[smax_, N_, DS_] :=
 ParametricPlot[{Log[s + 1], -s w[s]} /. NDSolve[{w'[r] == -(1/r) w[r] - (4 (N + 2))/(1 + r^2)^2 v[r],
     v'[r] == w[r], v[η] == 1 - (N + 2) η^2, w[η] == -2 (N + 2) η}, {v, w}, {r, η, s}],
  {s, η, smax}, PlotRange → All, PlotPoints → 1000, DisplayFunction → DS]

Show[Table[F[1000, N, Identity], {N, 1, 8}], DisplayFunction -> $DisplayFunction];
```



```
F2[smax_, N_, DS_] := ParametricPlot[
  {Log[s + 1], -s w2[s]} /. NDSolve[{w'[r] == -1/r w[r] - (4 (N + 2))/(1 + r^2)^2 v[r], v'[r] == w[r],
     v[η] == 1 - (N + 2) η^2, w[η] == -2 (N + 2) η, w2'[r] == -1/r w2[r] - (4 (N + 2))/(1 + r^2)^2 (v[r]^2 + v2[r]),
     v2'[r] == w2[r], v2[η] == -(N + 2) η^2, w2[η] == -2 (N + 2) η}, {v, w, v2, w2}, {r, η, s}],
  {s, η, smax}, PlotRange → All, PlotPoints → 1000, DisplayFunction → DS]

Show[Table[F2[1000, N, Identity], {N, 1, 8}], DisplayFunction -> $DisplayFunction];
```




```
Value[s_, N_] := {-s w[s], -s w2[s]} /.
  NDSolve[{w'[r] == -1/r w[r] - (4 (N + 2))/(1 + r^2)^2 v[r], v'[r] == w[r], v[η] == 1 - (N + 2) η^2,
     w[η] == -2 (N + 2) η, w2'[r] == -1/r w2[r] - (4 (N + 2))/(1 + r^2)^2 (v[r]^2 + v2[r]), v2'[r] == w2[r],
     v2[η] == -(N + 2) η^2, w2[η] == -2 (N + 2) η}, {v, w, v2, w2}, {r, η, s}][[1]]

Table[Table[Value[10^k, N][[1]], {k, 1, 4}], {N, 1, 8}]
Table[Table[Value[10^k, N][[2]], {k, 1, 4}], {N, 1, 8}]
```

```
{{-0.533282, -0.519153, -0.518298, -0.518284},
 {-0.711464, -0.626805, -0.62479, -0.624761},
 {-0.516131, -0.390376, -0.388184, -0.388155},
 {-0.115306, -0.00119999, -0.0000124936, -6.1403×10^-7},
 {0.310414, 0.362732, 0.362112, 0.362096}, {0.628069, 0.588684, 0.586108, 0.586064},
 {0.769142, 0.635356, 0.631318, 0.631255}, {0.723434, 0.517824, 0.513267, 0.513201}}

{{-0.0218764, -0.008657, -0.00730824, -0.00726577},
 {0.0760127, 0.177593, 0.182539, 0.18266}, {0.388792, 0.51933, 0.52273, 0.522802},
 {0.678607, 0.687482, 0.685754, 0.685721}, {0.696482, 0.559331, 0.556178, 0.556151},
 {0.442239, 0.286287, 0.287705, 0.287809}, {0.0996408, 0.067561, 0.0751088, 0.0753368},
 {-0.148552, -0.0188572, -0.00903341, -0.00879915}}
```

Multiplicity results for the assigned Gauss curvature problem in R^2 (2/5)

Jean Dolbeault, Maria Esteban,
and Gabriella Tarantello

Note that all computations are done for the nonlinearity: e^{2u} (adapted from the file with nonlinearity e^u)

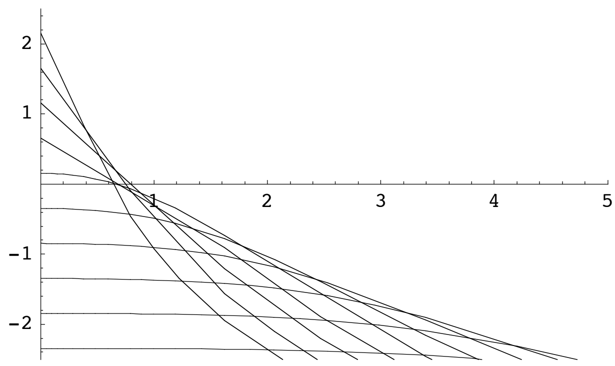
```
Off[NDSolve::"mxst"]
Off[NDSolve::"nlnum"]
Off[Power::"infy"]
Off[∞::"indet"]
```

```
η = 10^-4;
ϵ[a_, ξ_] := Min[ξ e^(-2 a), 10^-6]
rmax = 40;
shift = Log[2] / 2;
```

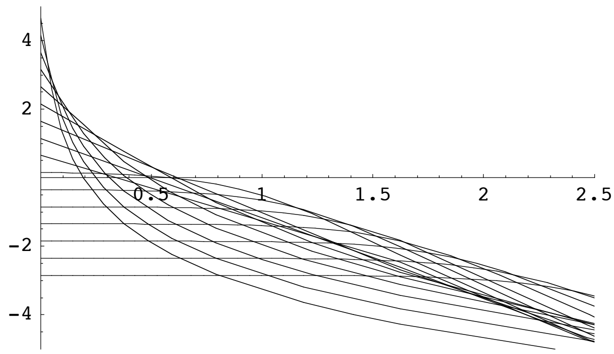
■ Plot of the solution as a function of r for various values of a and N

```
Pltu[a_, N_, DF_] :=
 Module[{δ = ϵ [a, η]}, Plot[u[s] /. NDSolve[{u'[r] == v[r], u[δ] == a - 1/4 e^(2 a) δ^2,
      v'[r] == -u'[r]/r - (1 + r^2)^N e^(2 u[r]), v[δ] == -1/2 e^(2 a) δ , m'[r] == r (1 + r^2)^N e^(2 u[r]),
      m[δ ] == 1/2 e^(2 a) δ^2}, {u, v, m}, {r, δ, rmax}], {s, δ, rmax}, DisplayFunction → DF]]
```

```
Show[Table[Pltu[a, 1, Identity], {a, -2.5 - shift, 2.5 - shift, 0.5}],
  DisplayFunction -> $DisplayFunction, PlotRange → {{0, 5}, {-2.5, 2.5}}];
```



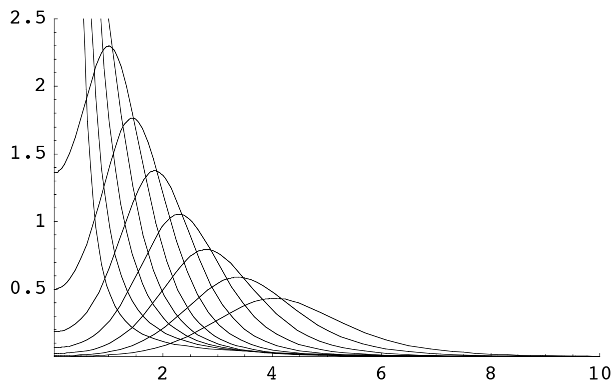
```
Show[Table[Pltu[a, 4, Identity], {a, -2.5 - shift, 5 - shift, 0.5}],
  DisplayFunction -> $DisplayFunction, PlotRange → {{0, 2.5}, {-5, 5}}];
```



■ Plot of the density as a function of r for various values of a and N

```
PltDensity[a_, N_, DF_] :=
 Module[{δ = ϵ [a, η]}, Plot[(1 + s^2)^N ⅇ^(2 u[s]) /. NDSolve[{u'[r] == v[r],
      u[δ] == a - 1/4 ⅇ^(2 a) δ^2, v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^(2 u[r]), v[δ] == -1/2 ⅇ^(2 a) δ,
      m'[r] == r (1 + r^2)^N ⅇ^(2 u[r]), m[δ] == 1/2 ⅇ^(2 a) δ^2}, {u, v, m}, {r, δ, rmax}],
   {s, δ, rmax}, DisplayFunction → DF, PlotPoints → 100]]
```

```
Show[Table[PltDensity[a, 2, Identity], {a, -2.5 - shift, 2.5 - shift, 0.5}],
  DisplayFunction -> $DisplayFunction, PlotRange → {{0, 10}, {0, 2.5}}];
```

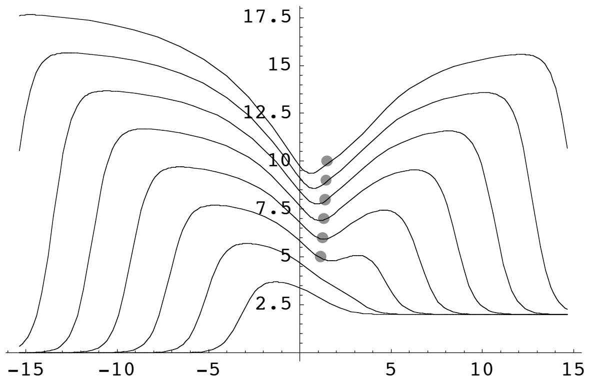


■ Plot of the limit of the density as r→∞ as a function of a and N

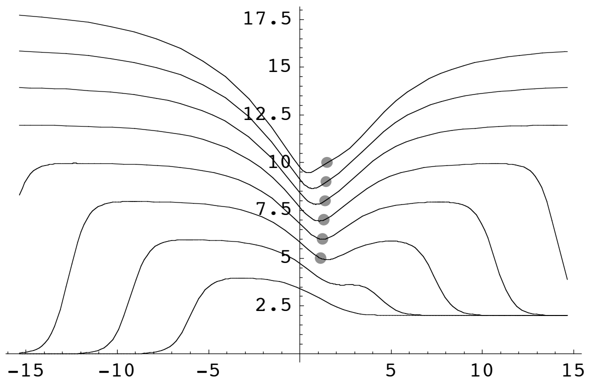
```
PltExlct[N_] := If[N > 2, ListPlot[{{1/2 Log[2 (N + 2)], N + 2}},
   PlotStyle → {PointSize[0.02], GrayLevel[0.5]}, DisplayFunction → Identity], {}]
```

```
F[N_, amin_, amax_, DF_] :=
 Show[{PltExlct[N], Plot[Module[{δ = ϵ[a, η]}, m[rmax] /. NDSolve[{u'[r] == v[r],
        u[δ] == a - 1/4 ⅇ^(2 a) δ^2, v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^(2 u[r]), v[δ] == -1/2 ⅇ^(2 a) δ,
        m'[r] == r (1 + r^2)^N ⅇ^(2 u[r]), m[δ] == 1/2 ⅇ^(2 a) δ^2}, {u, v, m}, {r, δ, rmax}]],
    {a, amin, amax}, DisplayFunction → Identity]}, DisplayFunction → DF]
```

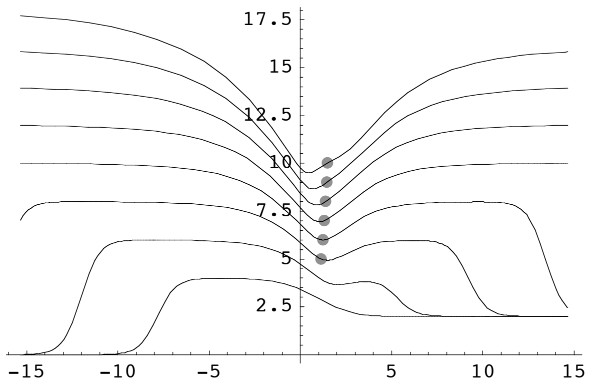
```
rmax = 10;
Show[Table[F[N, -15 - shift, 15 - shift, Identity], {N, 1, 8}],
  DisplayFunction -> $DisplayFunction, PlotRange → All];
```



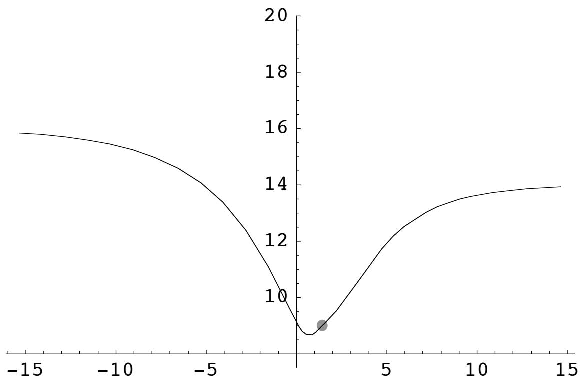

```
rmax = 40;
Show[Table[F[N, -15 - shift, 15 - shift, Identity], {N, 1, 8}],
  DisplayFunction -> $DisplayFunction, PlotRange → All];
```




```
rmax = 100;
Show[Table[F[N, -15 - shift, 15 - shift, Identity], {N, 1, 8}],
  DisplayFunction -> $DisplayFunction, PlotRange → All];
```

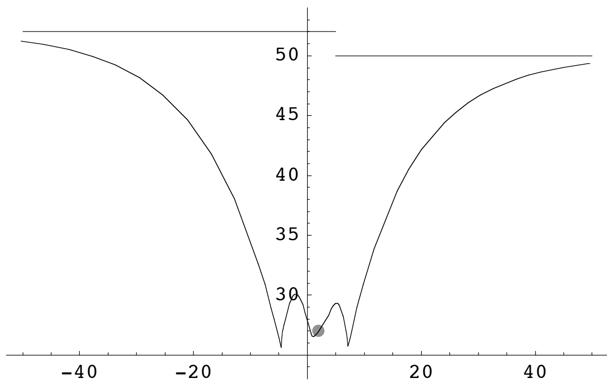


```
rmax = 10^7;
Show[F[7, -15 - shift, 15 - shift, Identity],
  DisplayFunction -> $DisplayFunction, PlotRange → {All, {7.5, 20}}];
```



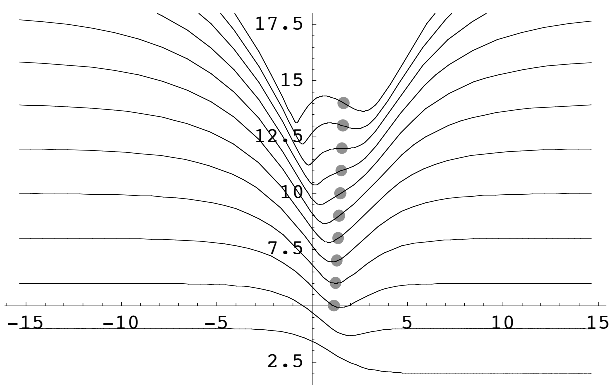
▪ FIGURE 1:

```
rmax = 10^4.5;
Show[ListPlot[{{5, 50}, {50, 50}}, DisplayFunction → Identity, PlotJoined → True],
  ListPlot[{{-50, 52}, {5, 52}}, DisplayFunction → Identity, PlotJoined → True],
  F[25, -50 - shift, 50 - shift, Identity],
  DisplayFunction -> $DisplayFunction, PlotRange → {All, {23, 54}}];
```



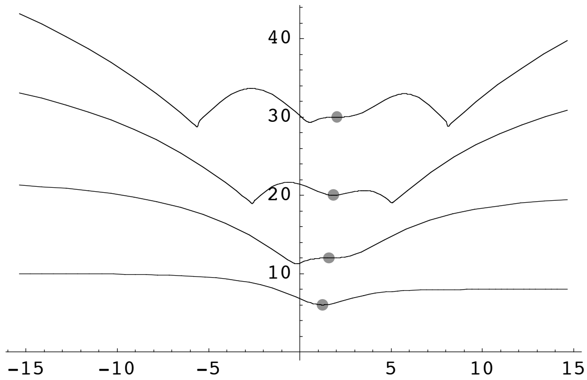
▪ FIGURE 2:

```
rmax = 10^7;
Show[Table[F[N, -15 - shift, 15 - shift, Identity], {N, 1, 12}],
  DisplayFunction -> $DisplayFunction, PlotRange → {All, {1.5, 18}}];
```

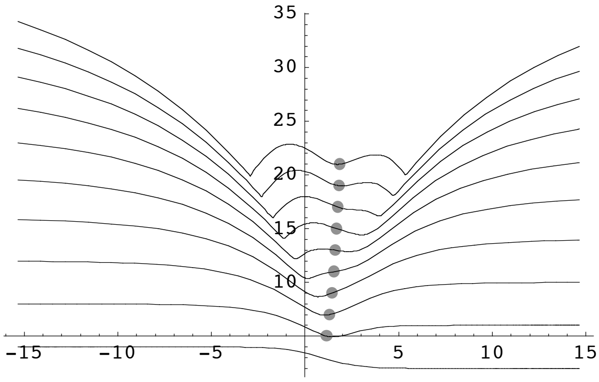


▪ FIGURE 3:

```
rmax = 10^7;
Tble = Table[k (k + 1) - 2, {k, 2, 5}];
Show[Table[F[Tble[[k - 1]], -15 - shift, 15 - shift, Identity], {k, 2, 5}],
  DisplayFunction -> $DisplayFunction];
```



```
rmax = 10^7;
Show[Table[F[N, -15 - shift, 15 - shift, Identity], {N, 1, 20, 2}],
  DisplayFunction -> $DisplayFunction, PlotRange → All];
```

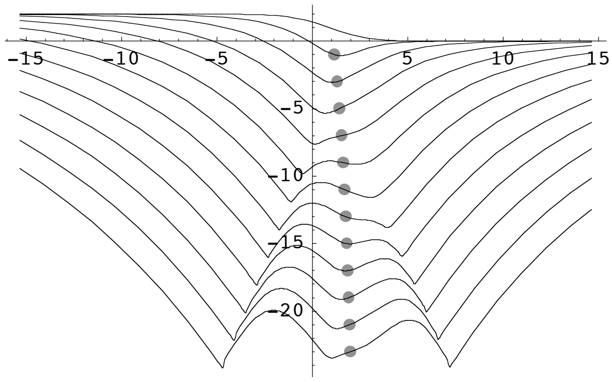


▪ Plot of the limit (translated of -2N) of the density as r→∞ as a function of a and N

```
PltExlcttrans[N_] := If[N > 2, ListPlot[{{1/2 Log[2 (N + 2)], 2 - N}},
   PlotStyle → {PointSize[0.02], GrayLevel[0.5]}, DisplayFunction → Identity], {}]

Ftrans[N_, amin_, amax_, DF_] :=
 Show[{PltExlcttrans[N], Plot[Module[{δ = ϵ[a, η]}, m[rmax] - 2 N /.
      NDSolve[{u'[r] == v[r], u[δ] == a - 1/4 ⅇ^(2 a) δ^2, v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^(2 u[r]), v[δ] ==
         -1/2 ⅇ^(2 a) δ, m'[r] == r (1 + r^2)^N ⅇ^(2 u[r]), m[δ] == 1/2 ⅇ^(2 a) δ^2}, {u, v, m}, {r, δ, rmax}]],
    {a, amin, amax}, DisplayFunction → Identity]}, DisplayFunction → DF]
```

▪ FIGURE 4:

```
rmax = 10^7;
Show[Table[Ftrans[N, -15 - shift, 15 - shift, Identity], {N, 1, 25, 2}],
  DisplayFunction -> $DisplayFunction, PlotRange → All];
```

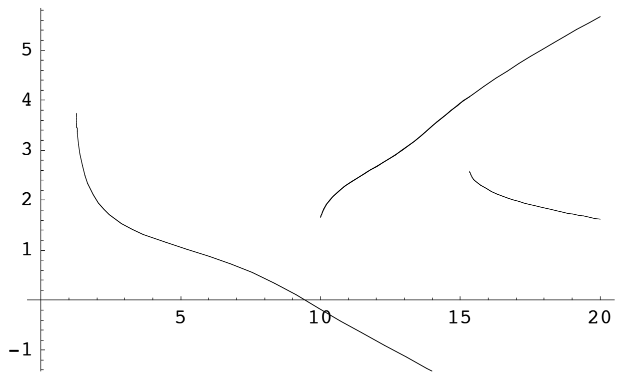


▪ Location of the minima of α(a) as a function of N and curve min α(a) / 2N as a function of N

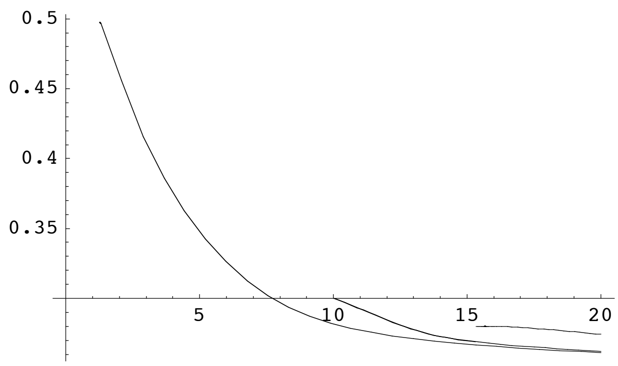
```
ξ = 10^-6;

Mass[N_, amin_, amax_, a_] :=
 Module[{δ = ϵ[a, η]}, m[rmax] /. NDSolve[{u'[r] == v[r], u[δ] == a - 1/4 ⅇ^a δ^2,
     v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^u[r], v[δ] == -1/2 ⅇ^a δ, m'[r] == r (1 + r^2)^N ⅇ^u[r],
     m[δ] == 1/2 ⅇ^a δ^2}, {u, v, m}, {r, δ, rmax}]][[1]]

MinIterDicho[NN_, amin_, amax_, a_, f_, s_] := If[a > amax + 0.1 || a < amin - 0.1,
  {N[a / 2 - shift], 2 NN}, If[Abs[s] < ξ, {N[a / 2 - shift], f / 2}, Module[{M =
      Mass[NN, amin, amax, a]},
    If[M < f, MinIterDicho[NN, amin, amax, a + s, M, s],
     MinIterDicho[NN, amin, amax, a - s / 2, M, -s / 2]]]]]

SearchMin[N_, amin_, ainit_, amax_, s_] :=
 Module[{x = Mass[N, amin, amax, ainit]}, MinIterDicho[N, amin, amax, ainit + s, x, s]]
```

■ FIGURE 5:

```
P5 = Show[Plot[SearchMin[x, -30, 1.5, 30, 0.1][[1]],
     {x, 1.27, 1.3}, DisplayFunction → Identity],
    Plot[SearchMin[x, -30, 5 - x / 2, 30, -0.25][[1]], {x, 1.3, 20},
     DisplayFunction → Identity], Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[1]],
     {x, 10.01, 15.325}, DisplayFunction → Identity],
    Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[1]], {x, 15.335, 20},
     DisplayFunction → Identity], Plot[SearchMin[x, -30, -5 + x, 30, 0.5][[1]],
     {x, 10.01, 20}, DisplayFunction → Identity], DisplayFunction -> $DisplayFunction];
```

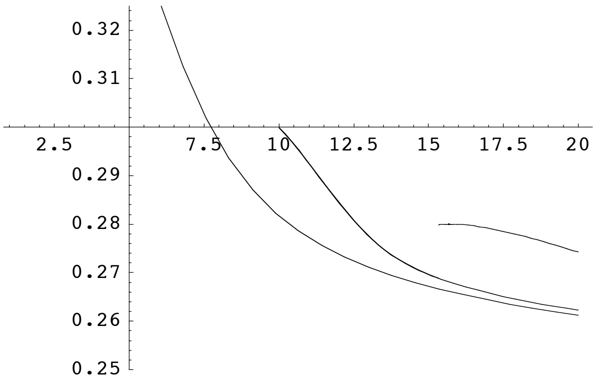



```
Show[Plot[SearchMin[x, -30, 1.5, 30, 0.1][[2]] / (4 x), {x, 1.27, 1.3},
    DisplayFunction → Identity], Plot[SearchMin[x, -30, 5 - x / 2, 30, -0.25][[2]] / (4 x),
    {x, 1.3, 20}, DisplayFunction → Identity],
   Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[2]] / (4 x),
    {x, 10.01, 15.325}, DisplayFunction → Identity],
   Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[2]] / (4 x), {x, 15.335, 20},
    DisplayFunction → Identity], Plot[SearchMin[x, -30, -5 + x, 30, 0.5][[2]] / (4 x),
    {x, 10.01, 20}, DisplayFunction → Identity], DisplayFunction -> $DisplayFunction];
```



■ FIGURE 6:

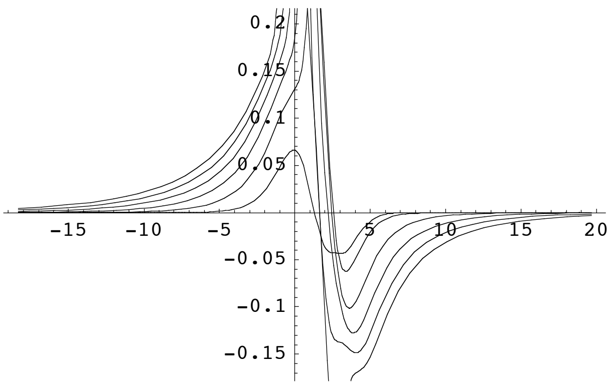
```
Show[Plot[SearchMin[x, -30, 1.5, 30, 0.1][[2]] / (4 x), {x, 1.27, 1.3},
   DisplayFunction → Identity], Plot[SearchMin[x, -30, 5 - x / 2, 30, -0.25][[2]] / (4 x),
   {x, 1.3, 20}, DisplayFunction → Identity],
  Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[2]] / (4 x),
   {x, 10.01, 15.325}, DisplayFunction → Identity],
  Plot[SearchMin[x, -30, Log[4 (x + 2)], 30, 0.1][[2]] / (4 x), {x, 15.335, 20},
   DisplayFunction → Identity], Plot[SearchMin[x, -30, -5 + x, 30, 0.5][[2]] / (4 x),
   {x, 10.01, 20}, DisplayFunction → Identity],
  DisplayFunction -> $DisplayFunction, PlotRange -> {All, {0.5 / 2, 0.65 / 2}}];
```



■ Function J(a)

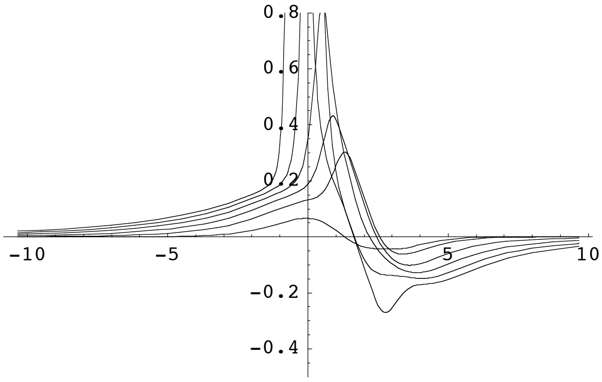
```
G[N_, amin_, amax_, DF_] := Plot[Module[{δ = ϵ[a, η]},
   J[rmax] /. NDSolve[{u'[r] == v[r], u[δ] == a - 1/2 ⅇ^(2 a) δ^2, v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^(2 u[r]),
      v[δ] == -ⅇ^(2 a) δ, m'[r] == r (1 + r^2)^N ⅇ^(2 u[r]), m[δ] == 1/2 ⅇ^(2 a) δ^2, φ'[r] == ψ[r],
      ψ'[r] == -ψ[r]/r - 2 (1 + r^2)^N ⅇ^(2 u[r]) φ[r], J'[r] == r (1 + r^2)^N ⅇ^(2 u[r]) Abs[φ[r]]^2 φ[r],
      φ[δ] == 1 - 1/2 ⅇ^(2 a) δ^2, ψ[δ] == -ⅇ^(2 a) δ, J[δ] == 1/2 ⅇ^(2 a) δ^2},
     {u, v, m, φ, ψ, J}, {r, δ, rmax}]], {a, amin, amax}, DisplayFunction → DF]
```

```
η = 10^-4;
rmax = 10^7;
Show[Table[G[N, -18 - shift, 20 - shift, Identity], {N, 1, 11, 2}],
  DisplayFunction -> $DisplayFunction];
```



■ FIGURE 7:

```
rmax = 10^7;
Show[Table[G[N, -10 - shift, 10 - shift, Identity], {N, 1, 12, 2}],
  DisplayFunction -> $DisplayFunction, PlotRange → {All, {-0.5, 0.8}}];
```

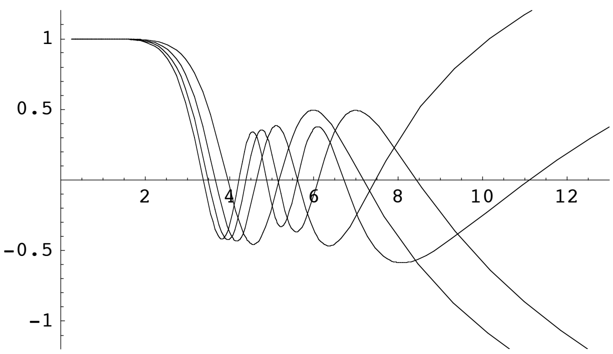


■ Plot of φ for a=$\frac{1}{2}$ Log[2 ($N+2$), N= 8, 16, 24, 32

```
H[N_, DF_] :=
 Plot[Module[{a = 1/2 Log[2 (N + 2)]}, Module[{δ = ϵ[a, η]}, φ[(s/5)^4] /. NDSolve[
     {u'[r] == v[r], u[δ] == a - 1/2 e^(2 a) δ^2, v'[r] == -u'[r]/r - (1 + r^2)^N e^(2 u[r]), v[δ] == -e^(2 a) δ,
      m'[r] == r (1 + r^2)^N e^(2 u[r]), m[δ] == 1/2 e^(2 a) δ^2, φ'[r] == ψ[r],
      ψ'[r] == -ψ[r]/r - 2 (1 + r^2)^N e^(2 u[r]) φ[r], J'[r] == r (1 + r^2)^N e^(2 u[r]) Abs[φ[r]]^2 φ[r],
      φ[δ] == 1 - 1/2 e^(2 a) δ^2, ψ[δ] == -e^(2 a) δ, J[δ] == 1/2 e^(2 a) δ^2},
     {u, v, m, φ, ψ, J}, {r, δ, rmax}]]], {s, 0.25, 20}, DisplayFunction → DF]
```

■ FIGURE 7bis:

```
Tb = {8, 16, 24, 32};
Show[Table[H[Tb[[k]], Identity], {k, 1, Length[Tb]}],
  DisplayFunction -> $DisplayFunction, PlotRange → {{0, 13}, {-1.2, 1.2}}];
```



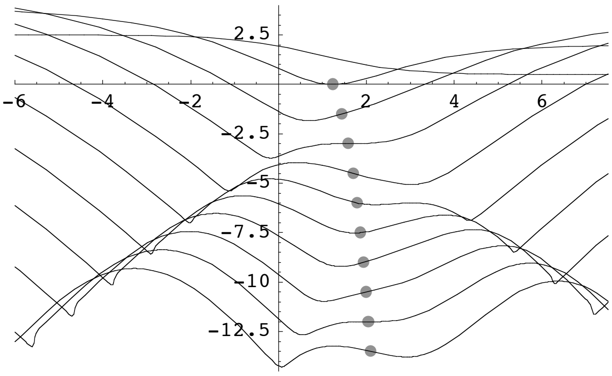
■ Plot of the limit (translated of -3N/2) of the density as r→∞ as a function of a and N

```
Off[General::"spell1"]
```

```
PltExlcttransN[N_] := If[N > 2, ListPlot[{{1/2 Log[2 (N + 2)], 2 - N / 2}},
   PlotStyle → {PointSize[0.02], GrayLevel[0.5]}, DisplayFunction → Identity], {}]
```

```
FtransN[N_, amin_, amax_, DF_] :=
 Show[{PltExlcttransN[N], Plot[Module[{δ = ε[a, η]}, m[rmax] - 3 N / 2 /.
      NDSolve[{u'[r] == v[r], u[δ] == a - 1/4 ⅇ^(2 a) δ^2, v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^(2 u[r]), v[δ] ==
         -1/2 ⅇ^(2 a) δ, m'[r] == r (1 + r^2)^N ⅇ^(2 u[r]), m[δ] == 1/2 ⅇ^(2 a) δ^2}, {u, v, m}, {r, δ, rmax}]],
    {a, amin, amax}, DisplayFunction → Identity]}, DisplayFunction → DF]
```

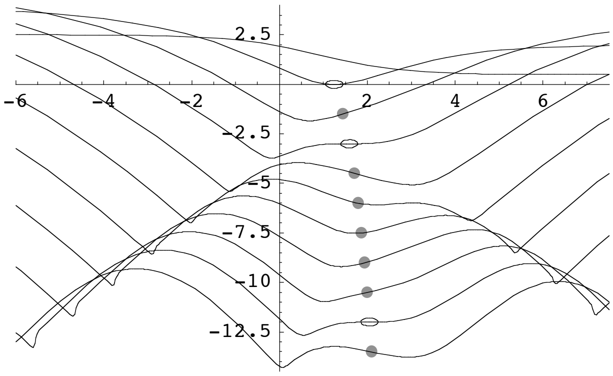
```
rmax = 10^7;
Show[Table[FtransN[N, -15 - shift, 15 - shift, Identity], {N, 1, 31, 3}],
  DisplayFunction -> $DisplayFunction, PlotRange → {{-6, 7.5}, {-14.5, 4}}];
```




```
PltExlcttransN[N_] :=
 If[N > 2, If[N == 4 || N == 10 || N == 18 || N == 28 || N == 54 || N == 88 || N == 130,
   Show[Graphics[Circle[{1/2 Log[2 (N + 2)], 2 - N / 2}, 0.2]], DisplayFunction → Identity],
   ListPlot[{{1/2 Log[2 (N + 2)], 2 - N / 2}},
    PlotStyle → {PointSize[0.02], GrayLevel[0.5]}, DisplayFunction → Identity]], {}]
```

▪ FIGURE 8:

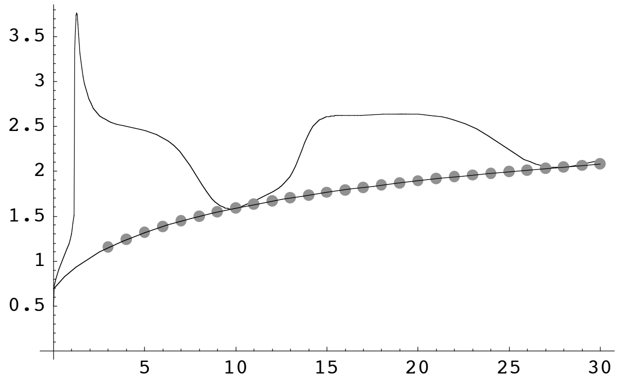
```
rmax = 10^7;
Show[Table[FtransN[N, -15 - shift, 15 - shift, Identity], {N, 1, 31, 3}],
  DisplayFunction -> $DisplayFunction, PlotRange → {{-6, 7.5}, {-14.5, 4}}];
```



▪ Plot of c_N (zero of the function J) as a function of N and α_N*

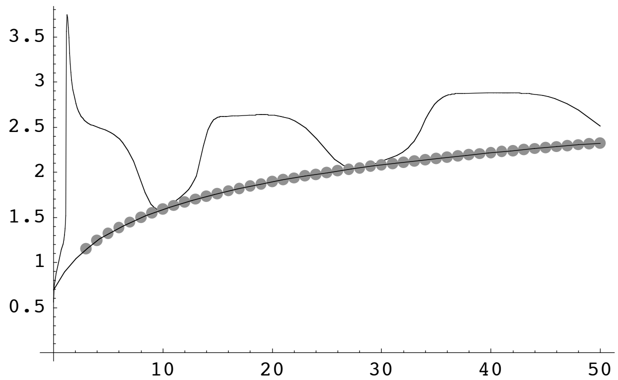
```
ξ = 10^-8;
rmax = 10^7;

IterDicho[a_, f_, s_, N_] :=
 Module[{M = Module[{δ = ε[a, η]}, J[rmax] /. NDSolve[{u'[r] == v[r], u[δ] == a - 1/2 e^(2 a) δ^2,
      v'[r] == -u'[r]/r - (1 + r^2)^N e^(2 u[r]), v[δ] == -e^(2 a) δ, m'[r] == r (1 + r^2)^N e^(2 u[r]),
      m[δ] == 1/2 e^(2 a) δ^2, φ'[r] == ψ[r], ψ'[r] == -ψ[r]/r - 2 (1 + r^2)^N e^(2 u[r]) φ[r],
      J'[r] == r (1 + r^2)^N e^(2 u[r]) Abs[φ[r]]^2 φ[r], φ[δ] == 1 - 1/2 e^(2 a) δ^2,
      ψ[δ] == -e^(2 a) δ, J[δ] == 1/2 e^(2 a) δ^2}, {u, v, m, φ, ψ, J}, {r, δ, rmax}]]},
  If[Abs[M[[1]]] < ξ || Abs[s] < ξ, {M[[1]], a, s}, If[M[[1]] * f < 0,
    IterDicho[a - s / 2, M[[1]], -s / 2, N], IterDicho[a + s, M[[1]], s, N]]]]

StartDicho[N_] := Module[{MM =
    Module[{δ = ε[1, η]}, Module[{MMM = J[rmax] /. NDSolve[{u'[r] == v[r], u[δ] == 0 - 1/2 δ^2,
         v'[r] == -u'[r]/r - (1 + r^2)^N e^(2 u[r]), v[δ] == -δ, m'[r] == r (1 + r^2)^N e^(2 u[r]),
         m[δ] == 1/2 δ^2, φ'[r] == ψ[r], ψ'[r] == -ψ[r]/r - 2 (1 + r^2)^N e^(2 u[r]) φ[r], J'[r] ==
          r (1 + r^2)^N e^(2 u[r]) Abs[φ[r]]^2 φ[r], φ[δ] == 1 - 1/2 δ^2, ψ[δ] == -δ, J[δ] == 1/2 δ^2},
        {u, v, m, φ, ψ, J}, {r, δ, rmax}]}, MMM]]}, IterDicho[1, MM[[1]], 1, N]]
```

```
Nmax = 30;

P9a = Plot[StartDicho[N][[2]], {N, 0, Nmax}, DisplayFunction → $DisplayFunction];

DotN[N_] := If[N > 2, ListPlot[{{N, 1/2 Log[2 (N + 2)]}},
   PlotStyle → {PointSize[0.02], GrayLevel[0.5]}, DisplayFunction → Identity], {}]

P9c = Plot[1/2 Log[2 (N + 2)], {N, 0, Nmax}, DisplayFunction → Identity];

P9b = Show[Table[DotN[N], {N, 0, Nmax}], P9c, DisplayFunction → Identity];

Show[P9a, P9b, DisplayFunction → $DisplayFunction];
```



■ FIGURE 9:

```
Nmax = 50;

P9a = Plot[StartDicho[N][[2]], {N, 0, Nmax}, DisplayFunction → Identity];

P9c = Plot[1/2 Log[2 (N + 2)], {N, 0, Nmax}, DisplayFunction → Identity];

P9b = Show[Table[DotN[N], {N, 0, Nmax}], P9c, DisplayFunction → Identity];

Show[P9a, P9b, DisplayFunction → $DisplayFunction];
```

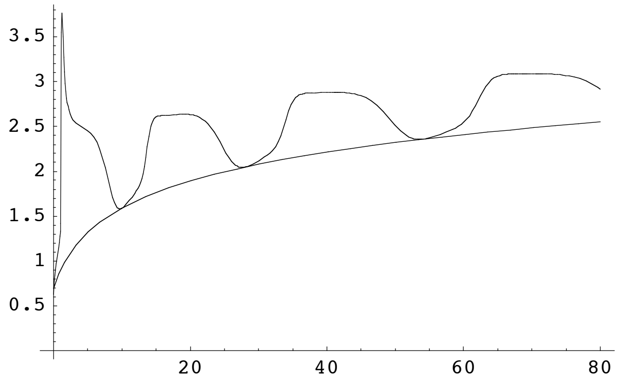


Nmax = 80;

P9a = Plot[StartDicho[N][[2]], {N, 0, Nmax}, DisplayFunction → Identity];

P9c = Plot[$\frac{1}{2}$ Log[2 (N + 2)], {N, 0, Nmax}, DisplayFunction → Identity];

Show[P9a, P9c, DisplayFunction → $DisplayFunction];

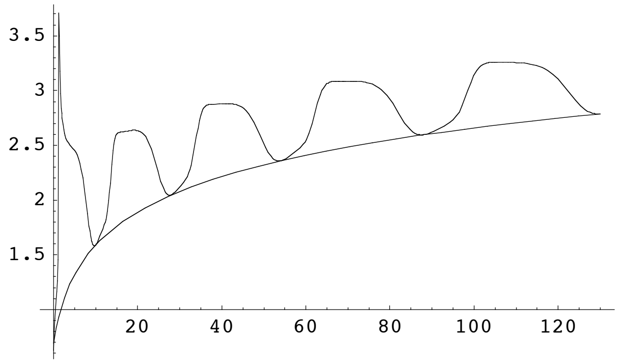


Nmax = 130;

P9a = Plot[StartDicho[N][[2]], {N, 0, Nmax}, DisplayFunction → Identity];

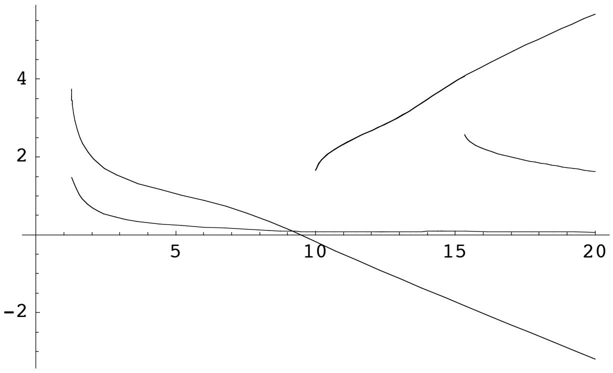
P9c = Plot[$\frac{1}{2}$ Log[2 (N + 2)], {N, 0, Nmax}, DisplayFunction → Identity];

Show[P9a, P9c, DisplayFunction → $DisplayFunction];



▪ FIGURE 10:

```
P10a = Plot[StartDicho[N][[2]] / (2 N), {N, 1.27, 20}, DisplayFunction -> Identity];
Show[P5, P10a];
```

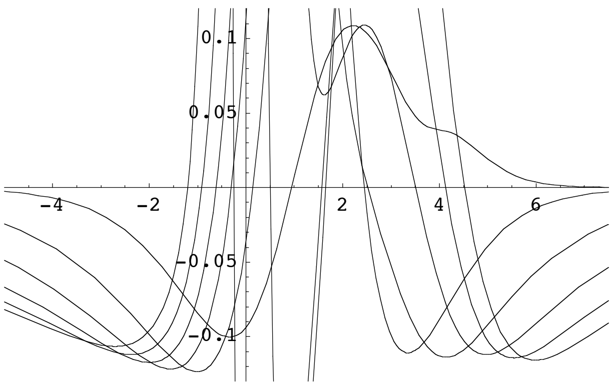


▪ Function K(a)

```
GG[N_, amin_, amax_, DF_] :=
 Plot[Module[{δ = ϵ[a, η]}, K[rmax] /. NDSolve[{u'[r] == v[r], u[δ] == a - 1/2 ⅇ^(2 a) δ^2,
     v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^(2 u[r]), v[δ] == -ⅇ^(2 a) δ, m'[r] == r (1 + r^2)^N ⅇ^(2 u[r]),
     m[δ] == 1/2 ⅇ^(2 a) δ^2, φ'[r] == φ1[r], φ1'[r] == -φ1[r]/r - 2 (1 + r^2)^N ⅇ^(2 u[r]) φ[r],
     ψ'[r] == ψ1[r], ψ1'[r] == -ψ1[r]/r - 2 (1 + r^2)^N ⅇ^(2 u[r]) (2 φ[r]^2 + ψ[r]),
     K'[r] == r (1 + r^2)^N ⅇ^(2 u[r]) (2 φ[r]^2 + ψ[r]), φ[δ] == 1 - 1/2 ⅇ^(2 a) δ^2,
     φ1[δ] == -ⅇ^(2 a) δ, ψ[δ] == -1/2 ⅇ^(2 a) δ^2, ψ1[δ] == -ⅇ^(2 a) δ, K[δ] == ⅇ^(2 a) δ^2},
    {u, v, m, φ, φ1, ψ, ψ1, K}, {r, δ, rmax}]], {a, amin, amax}, DisplayFunction → DF]
```

▪ FIGURE 11:

```
η = 10^-4;
rmax = 10^7;
Show[Table[GG[N, -18 - shift, 20 - shift, Identity], {N, 1, 11, 2}],
  DisplayFunction -> $DisplayFunction, PlotRange → {{-5, 7.5}, {-0.13, 0.12}}];
```



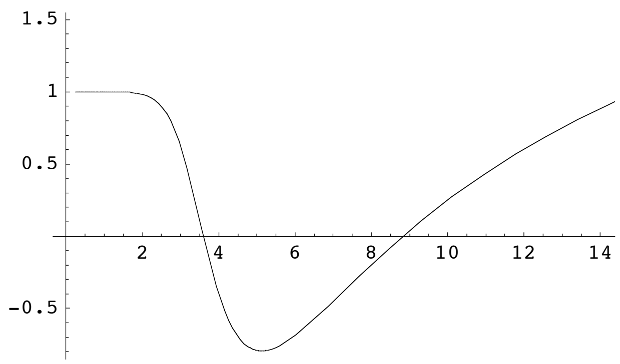
Multiplicity results for the assigned Gauss curvature problem in R^2 (3/5)

Jean Dolbeault, Maria Esteban,
and Gabriella Tarantello

Note that all computations are done for the nonlinearity: e^{2u}
The goal is to compute the function J and K for a =a^*_N in terms of N

Plot of φ

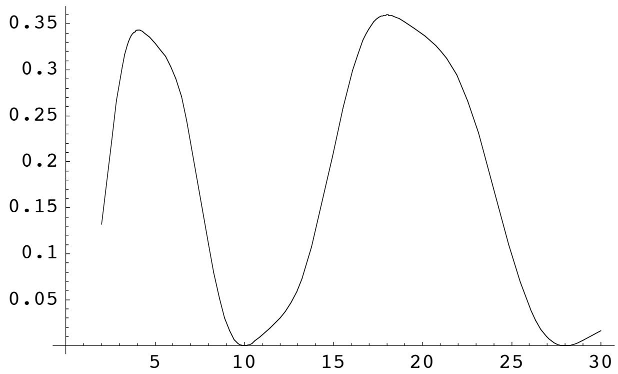
```
Off[NDSolve::"mxst"]
Off[NDSolve::"nlnum"]
Off[Power::"infy"]
Off[∞::"indet"]

η = 10^-4;
ϵ[a_, ξ_] := Min[ξ ⅇ^(-2 a), 10^-6]
rmax = 40;
shift = Log[2] / 2;

rmax = 10^7;

H[N_, DF_, A_] := Plot[Module[{a = A}, Module[{δ = ϵ[a, η]}, φ[(s/5)^4] /.
    NDSolve[{u'[r] == v[r], u[δ] == a - 1/2 ⅇ^(2 a) δ^2, v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^(2 u[r]),
      v[δ] == -ⅇ^(2 a) δ, m'[r] == r (1 + r^2)^N ⅇ^(2 u[r]), m[δ] == 1/2 ⅇ^(2 a) δ^2, φ'[r] == ψ[r],
      ψ'[r] == -ψ[r]/r - 2 (1 + r^2)^N ⅇ^(2 u[r]) φ[r], J'[r] == r (1 + r^2)^N ⅇ^(2 u[r]) Abs[φ[r]]^2 φ[r],
      φ[δ] == 1 - 1/2 ⅇ^(2 a) δ^2, ψ[δ] == -ⅇ^(2 a) δ, J[δ] == 1/2 ⅇ^(2 a) δ^2},
     {u, v, m, φ, ψ, J}, {r, δ, rmax}]]], {s, 0.25, 20}, DisplayFunction → DF]

H[1.2, $DisplayFunction, 2];
```



Plot of J (a^*_N) as a function of N

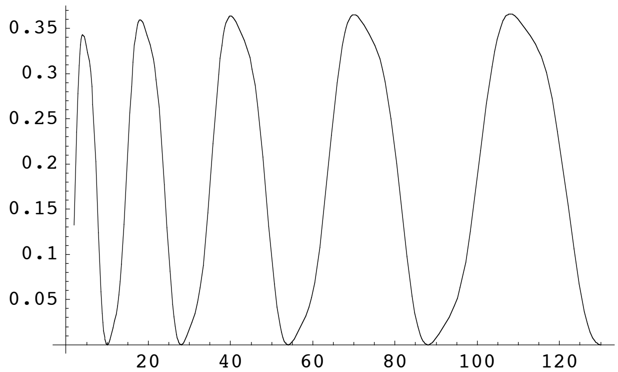
```
JPlot[Nmax_] :=
 Plot[Module[{a = 1/2 Log[2 (N + 2)]}, Module[{δ = ϵ[a, η]}, J[rmax] /. NDSolve[
     {u'[r] == v[r], u[δ] == a - 1/2 ⅇ^(2 a) δ^2, v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^(2 u[r]), v[δ] == -ⅇ^(2 a) δ,
      m'[r] == r (1 + r^2)^N ⅇ^(2 u[r]), m[δ] == 1/2 ⅇ^(2 a) δ^2, φ'[r] == ψ[r],
      ψ'[r] == -ψ[r]/r - 2 (1 + r^2)^N ⅇ^(2 u[r]) φ[r], J'[r] == r (1 + r^2)^N ⅇ^(2 u[r]) Abs[φ[r]]^2 φ[r],
      φ[δ] == 1 - 1/2 ⅇ^(2 a) δ^2, ψ[δ] == -ⅇ^(2 a) δ, J[δ] == 1/2 ⅇ^(2 a) δ^2},
     {u, v, m, φ, ψ, J}, {r, δ, rmax}]]], {N, 2, Nmax}]
```

```
JPlot[30];
```



▪ FIGURE 12:

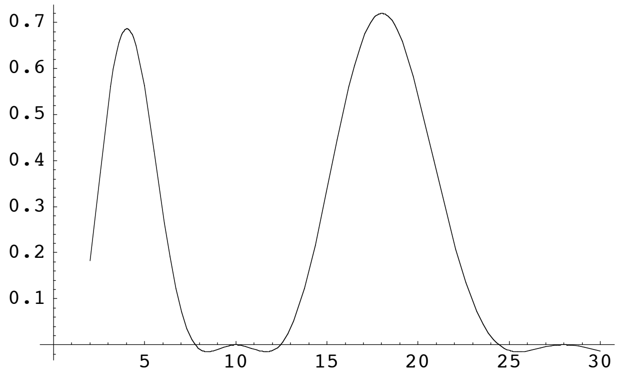
```
JPlot[130];
```



Plot of K (a^*_N) as a function of N

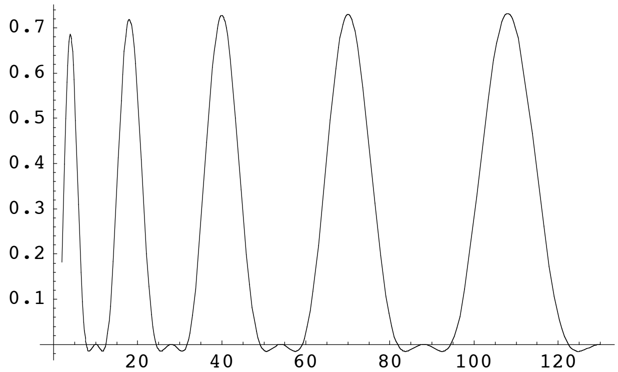
```
KPlot[Nmax_] := Plot[Module[{a = 1/2 Log[2 (N + 2)]},
   Module[{δ = ϵ[a, η]}, K[rmax] /. NDSolve[{u'[r] == v[r], u[δ] == a - 1/2 e^(2 a) δ^2,
      v'[r] == -u'[r]/r - (1 + r^2)^N e^(2 u[r]), v[δ] == -e^(2 a) δ, m'[r] == r (1 + r^2)^N e^(2 u[r]),
      m[δ] == 1/2 e^(2 a) δ^2, φ'[r] == φ1[r], φ1'[r] == -φ1[r]/r - 2 (1 + r^2)^N e^(2 u[r]) φ[r],
      ψ'[r] == ψ1[r], ψ1'[r] == -ψ1[r]/r - 2 (1 + r^2)^N e^(2 u[r]) (2 φ[r]^2 + ψ[r]),
      K'[r] == r (1 + r^2)^N e^(2 u[r]) (2 φ[r]^2 + ψ[r]), φ[δ] == 1 - 1/2 e^(2 a) δ^2,
      φ1[δ] == -e^(2 a) δ, ψ[δ] == -1/2 e^(2 a) δ^2, ψ1[δ] == -e^(2 a) δ, K[δ] == e^(2 a) δ^2},
     {u, v, m, φ, φ1, ψ, ψ1, K}, {r, δ, rmax}]]], {N, 2, Nmax}]
```

```
KPlot[30];
```



■ FIGURE 13:

```
KPlot[130];
```

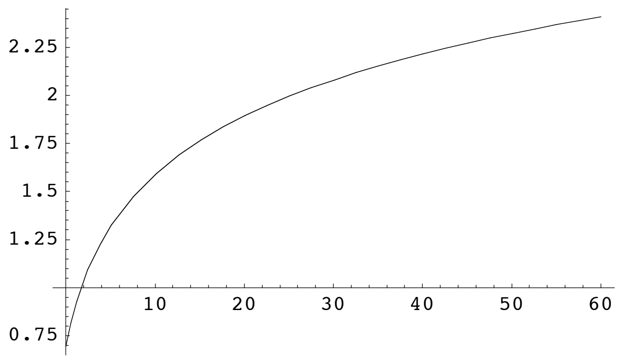


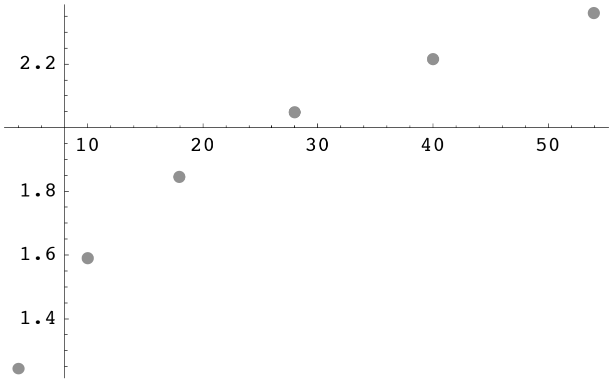
Multiplicity results for the assigned Gauss curvature problem in R^2 (4 / 5)

Jean Dolbeault, Maria Esteban,
and Gabriella Tarantello

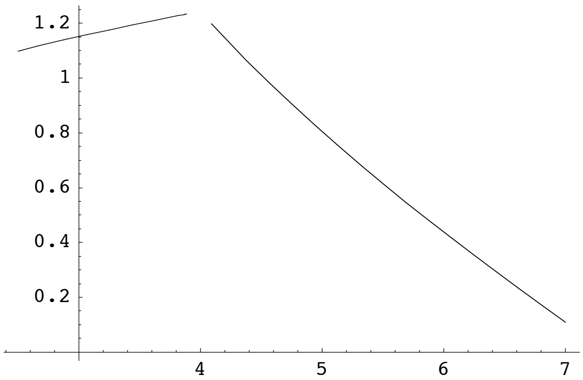
Note that all computations are done for the nonlinearity: e^{2u}
The goal is to compute the bifurcation diagram (a,N) at level α=N+2 in terms of the bifurcation parameter N

```
Off[NDSolve::"mxst"]
Off[NDSolve::"nlnum"]
Off[Power::"infy"]
Off[∞::"indet"]
Off[Plot::"plnr"]

η = 10^-4;
ϵ[a_, ξ_] := Min[ξ ⅇ^(-2 a), 10^-6]
rmax = 10^7;
shift = Log[2] / 2;

Fvalue[N_, a_] :=
 m[rmax] /. Module[{δ = ϵ[a, η]}, NDSolve[{u'[r] == v[r], u[δ] == a - 1/4 ⅇ^(2 a) δ^2,
      v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^(2 u[r]), v[δ] == -1/2 ⅇ^(2 a) δ, m'[r] == r (1 + r^2)^N ⅇ^(2 u[r]),
      m[δ] == 1/2 ⅇ^(2 a) δ^2}, {u, v, m}, {r, δ, rmax}]][[1]]

Fcrit[N_, amin_, amax_, a_, s_] := FcritIter[N, amin, amax, a, s, Fvalue[N, a]]

FcritIter[N_, amin_, amax_, a_, s_, f_] :=
 If[a + s < amin || a + s > amax, {}, Module[{g = Fvalue[N, a + s]},
   If[Abs[g - (N + 2)] < 10^-8, a + s, If[(f - (N + 2)) (g - (N + 2)) > 0,
     FcritIter[N, amin, amax, a + s, s, g], FcritIter[N, amin, amax, a, s / 2, f]]]]]

P1 = Plot[1/2 Log[2 (N + 2)], {N, 0, 60}];

P2 = ListPlot[Table[{k (k + 1) - 2, 1/2 Log[2 k (k + 1)]}, {k, 2, 7}],
   PlotStyle → {PointSize[0.02], GrayLevel[0.5]}];
```

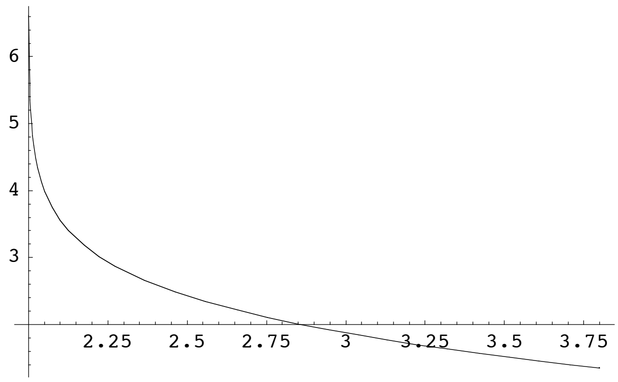




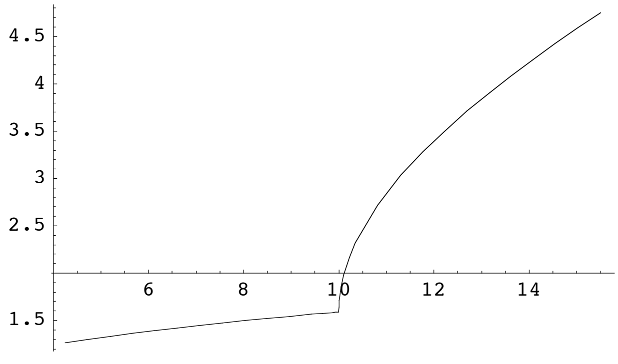
P3 = Plot[Fcrit[N, -15, 15, 0, 0.1], {N, 2.5, 7}];



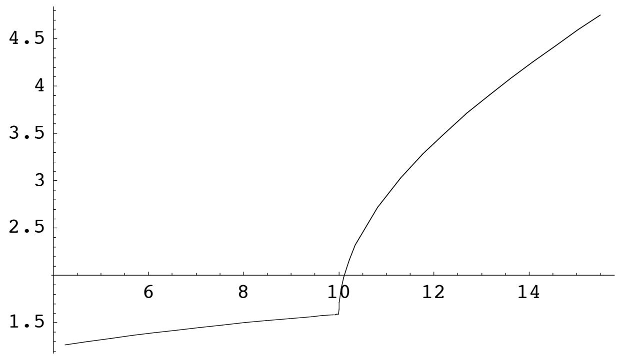
P4 = Plot$\left[\textbf{Fcrit}\left[\textbf{N, -15, 15, }\frac{1}{2}\textbf{ Log[2 (N + 2)] + 0.1, 0.1}\right]\textbf{, \{N, 1.5, 3.8\}}\right]$**;**



P5 = Plot[Fcrit[N, -15, 15, 10, -0.1], {N, 4.25, 15.5}];



```
P5 = Plot[Fcrit[N, -15, 15, 10, -0.1], {N, 4.25, 15.5}];
```




```
Fcrit[14.5, -15, 15, 10, -0.1]
Fcrit[15.5, -15, 15, 10, -0.1]
N[4.754375505447405 - 4.408464598655719]
```

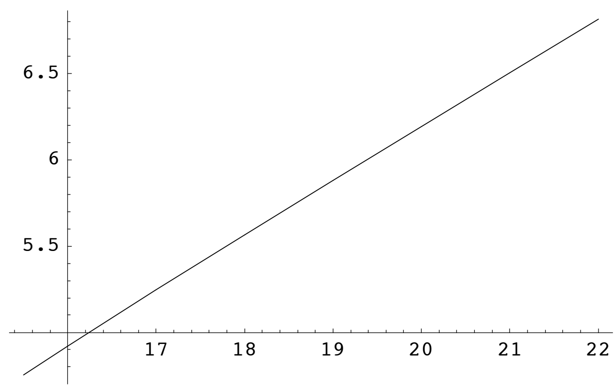
```
Tbl = Table[{k, 0.35 (k - 15.5) + 4.755}, {k, 16, 22}]
```

```
{{16, 4.93}, {17, 5.28}, {18, 5.63}, {19, 5.98}, {20, 6.33}, {21, 6.68}, {22, 7.03}}
```

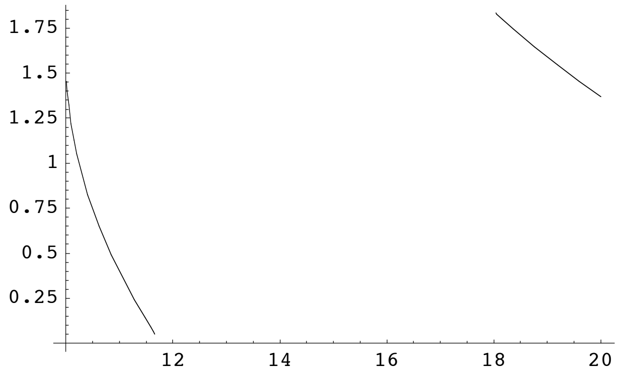
```
Table[{Tbl[[j]][[1]], Fcrit[Tbl[[j]][[1]], -15, 15, Tbl[[j]][[2]], -0.01]},
 {j, 1, Length[Tbl]}]
```

```
{{16, 4.92146}, {17, 5.24763}, {18, 5.56649},
 {19, 5.88087}, {20, 6.19266}, {21, 6.50313}, {22, 6.81313}}
```

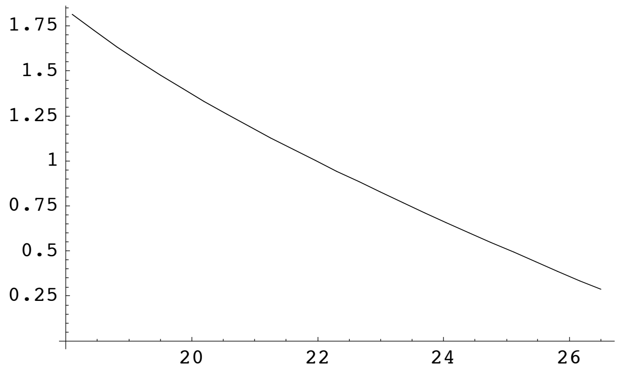
```
P5bis = ListPlot[
   {{15.5, 4.754375505447405`}, {16, 4.921460971832275`}, {17, 5.247627897262573`},
    {18, 5.566491928100586`}, {19, 5.880874819755556`}, {20, 6.192664613723758`},
    {21, 6.503131899833683`}, {22, 6.813128986358647`}}, PlotJoined → True];
```



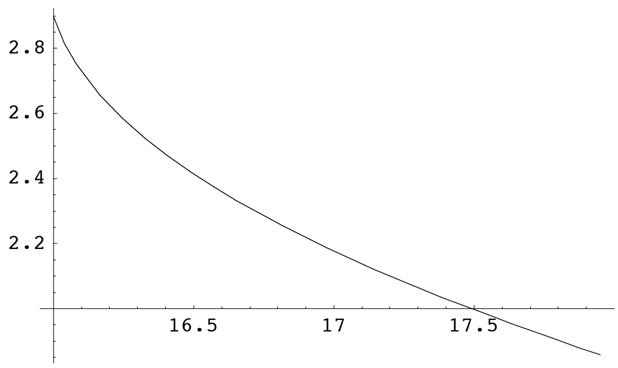
```
P6 = Plot[Fcrit[N, 0, 20, 1/2 Log[2 (N + 2)] - 0.01, -0.1], {N, 10, 20}];
```



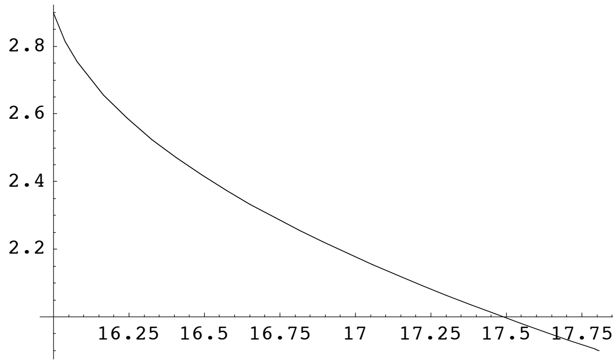

```
P7 = Plot[Fcrit[N, 0, 20, 1/2 Log[2 (N + 2)] - 0.01, -0.1], {N, 18.1, 26.5}];
```



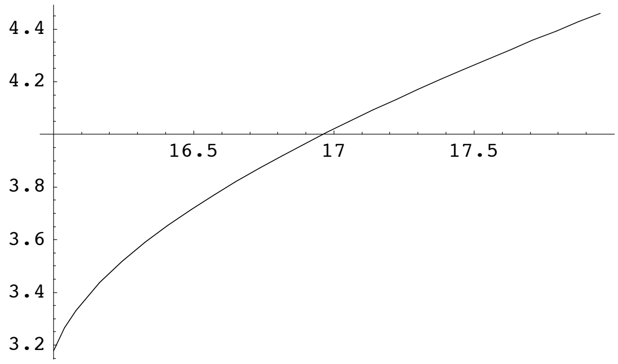

```
P8a = Plot[Fcrit[N, 0, 20, 1/2 Log[2 (N + 2)] + 0.01, +0.1], {N, 16, 17.95}];
```



```
P8b = Plot[Fcrit[N, 0, 20, 2.9, -0.1], {N, 16, 17.95}];
```




```
P8c = Plot[Fcrit[N, 0, 20, 2.9, 0.1], {N, 16, 17.95}];
```



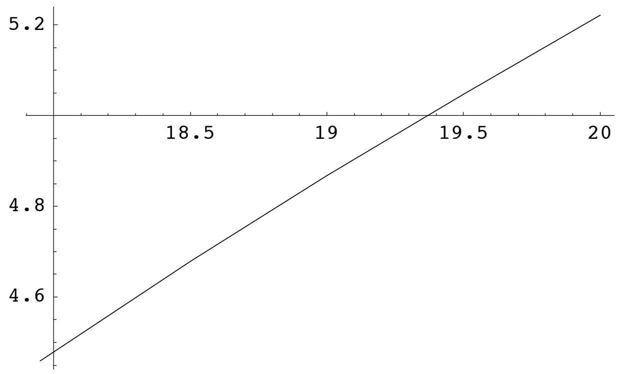

```
Tbl = Table[{k, 4.480217659473419 + 2 (k - 18) (4.480217659473419 - 4.264164304733277)},
  {k, 18, 20, 0.5}]
```

```
{{18, 4.48022}, {18.5, 4.69627}, {19., 4.91232}, {19.5, 5.12838}, {20., 5.34443}}
```

```
Table[{Tbl[[j]][[1]], Fcrit[Tbl[[j]][[1]], 2, 8, Tbl[[j]][[2]], -0.1]},
 {j, 1, Length[Tbl]}]
```

```
{{18, {}}, {18.5, 4.67947}, {19., 4.86744}, {19.5, 5.04741}, {20., 5.22152}}
```

```
P8d = ListPlot[{{17.95, 4.459494104385343`},
    {18.5`, 4.67946547269821`}, {19.`, 4.867435526847836`},
    {19.5`, 5.047410070896143`}, {20.`, 5.221516907215115`}}, PlotJoined → True];
```

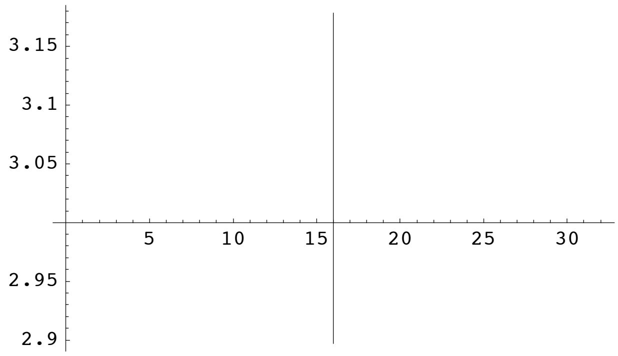


```
Fcrit[16, 0, 20, 2.9, 0.01]
Fcrit[16, 0, 20, 2.9, -0.01]
```

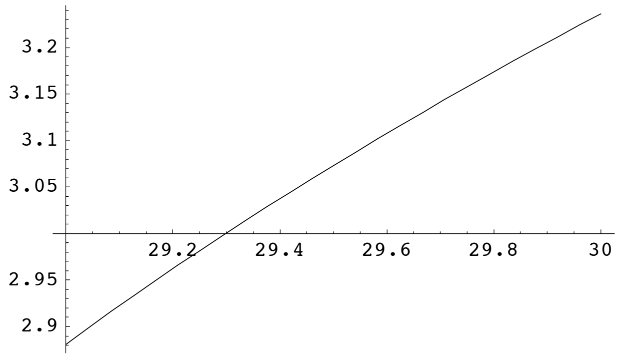
3.17809

2.89731

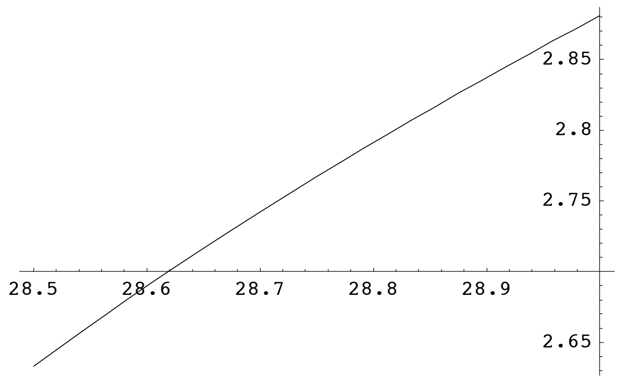
```
P8e = ListPlot[{{16, 2.8973097229003906}, {16, 3.178091278076166}}, PlotJoined → True];
```



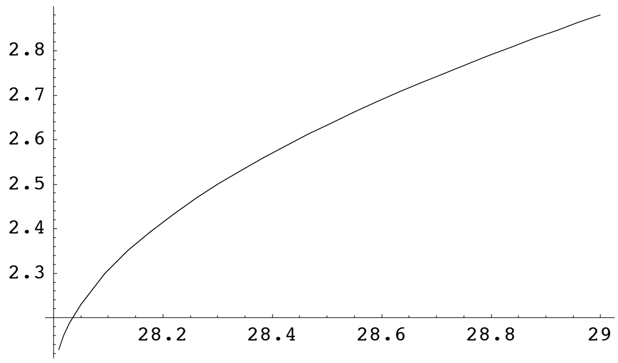

```
P9a = Plot[Fcrit[N, 0, 20, 2.5, 0.1], {N, 29, 30}];
```



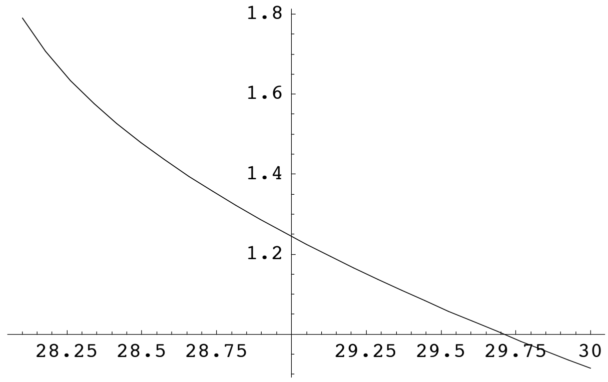

```
P9b = Plot[Fcrit[N, 0, 30, 2.4, 0.1], {N, 28.5, 29}];
```



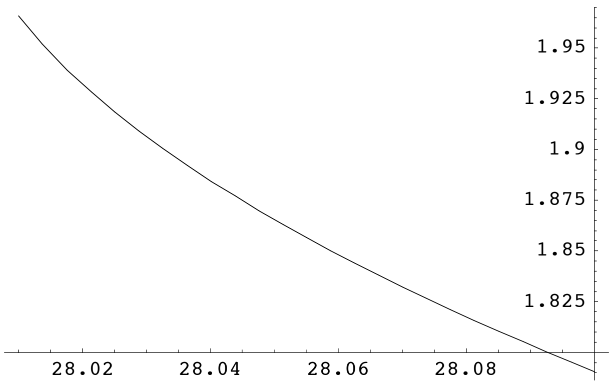
```
P9c = Plot[Fcrit[N, 0, 3, 1/2 Log[2 (N + 2)] + 0.001, 0.01], {N, 28.01, 29}];
```




```
P10 = Plot[Fcrit[N, 0, 20, 1/2 Log[2 (N + 2)] - 0.01, -0.1], {N, 28.1, 30}];
```

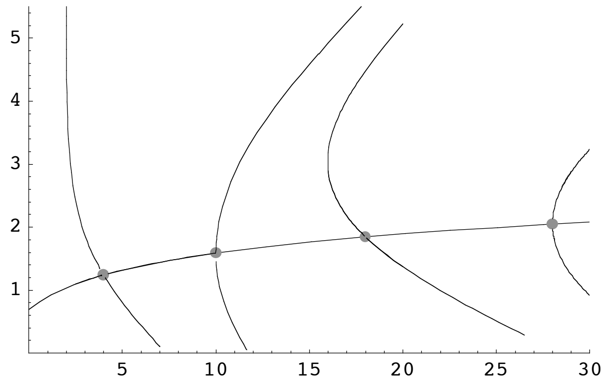



```
P10bis = Plot[Fcrit[N, 0, 20, 1/2 Log[2 (N + 2)] - 0.001, -0.01], {N, 28.01, 28.1}];
```



■ FIGURE 14:

```
Show[P1, P2, P3, P4, P5, P5bis, P6, P7, P8a, P8b, P8c, P8d,
  P8e, P9a, P9b, P9c, P10, P10bis, PlotRange → {{0, 30}, {0, 5.5}}];
```

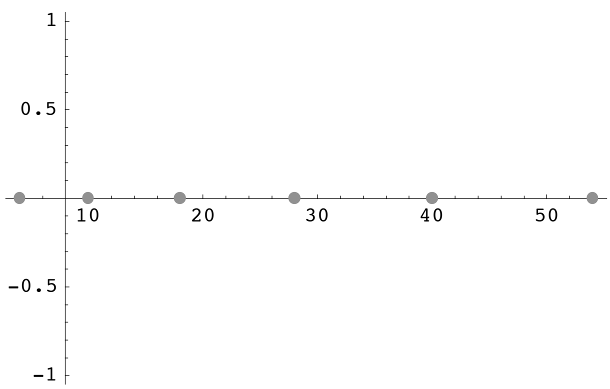


Multiplicity results for the assigned Gauss curvature problem in R^2 (5 / 5)

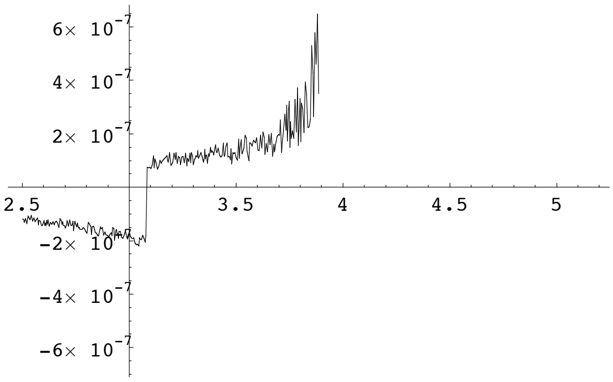
Jean Dolbeault, Maria Esteban, and Gabriella Tarantello

Note that all computations are done for the nonlinearity: e^{2u}
The goal is to compute the bifurcation diagram (a,N) at level α=N+2 in terms of the bifurcation parameter N

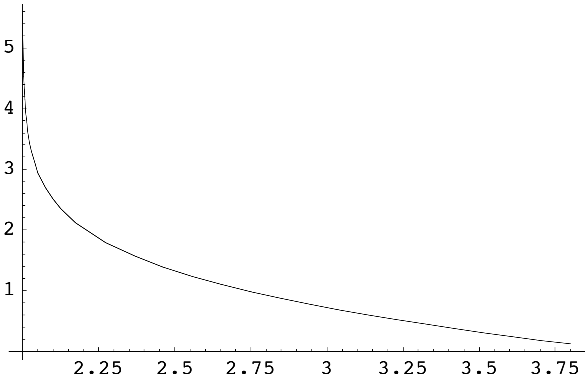
```
Off[NDSolve::"mxst"]
Off[NDSolve::"nlnum"]
Off[Power::"infy"]
Off[∞::"indet"]
Off[Plot::"plnr"]

η = 10^-4;
ϵ[a_, ξ_] := Min[ξ ⅇ^(-2 a), 10^-6]
rmax = 10^7;
shift = Log[2] / 2;

Fvalue[N_, a_] :=
 m[rmax] /. Module[{δ = ϵ[a, η]}, NDSolve[{u'[r] == v[r], u[δ] == a - 1/4 ⅇ^(2 a) δ^2,
      v'[r] == -u'[r]/r - (1 + r^2)^N ⅇ^(2 u[r]), v[δ] == -1/2 ⅇ^(2 a) δ, m'[r] == r (1 + r^2)^N ⅇ^(2 u[r]),
      m[δ] == 1/2 ⅇ^(2 a) δ^2}, {u, v, m}, {r, δ, rmax}]][[1]]

Fcrit[N_, amin_, amax_, a_, s_] := FcritIter[N, amin, amax, a, s, Fvalue[N, a]]

FcritIter[N_, amin_, amax_, a_, s_, f_] :=
 If[a + s < amin || a + s > amax, {}, Module[{g = Fvalue[N, a + s]},
   If[Abs[g - (N + 2)] < 10^-8, a + s - 1/2 Log[2 (N + 2)], If[(f - (N + 2)) (g - (N + 2)) > 0,
     FcritIter[N, amin, amax, a + s, s, g], FcritIter[N, amin, amax, a, s / 2, f]]]]]

P2 = ListPlot[Table[{k (k + 1) - 2, 0}, {k, 2, 7}],
   PlotStyle → {PointSize[0.02], GrayLevel[0.5]}];
```



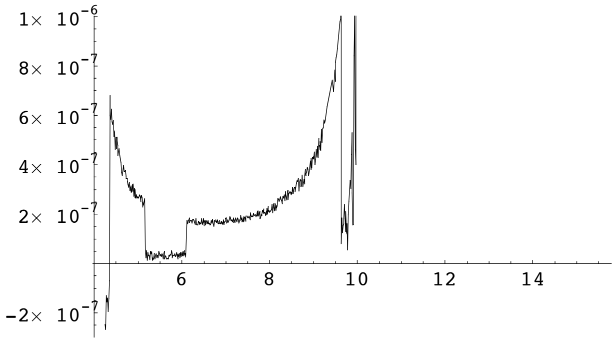
P3 = Plot[Fcrit[N, -15, 15, 0, 0.1], {N, 2.5, 7}];



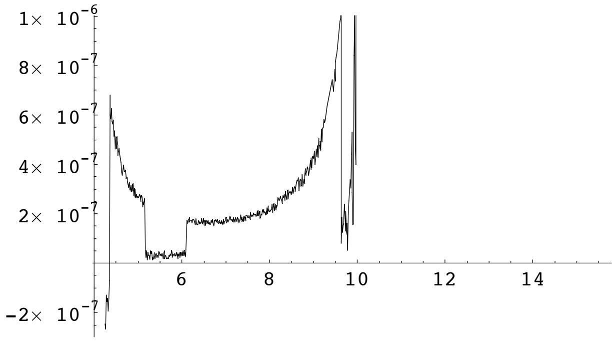
P4 = Plot$\left[\textbf{Fcrit}\left[\textbf{N, -15, 15, }\frac{1}{2}\textbf{ Log[2 (N + 2)] + 0.1, 0.1}\right]\textbf{, \{N, 1.5, 3.8\}}\right]$**;**



P5 = Plot[Fcrit[N, -15, 15, 10, -0.1], {N, 4.25, 15.5}];



P5 = Plot[Fcrit[N, -15, 15, 10, -0.1], {N, 4.25, 15.5}];



```
Fcrit[14.5, -15, 15, 10, -0.1]
Fcrit[15.5, -15, 15, 10, -0.1]
N[4.754375505447405 - 4.408464598655719]
```

```
2.66021
```

```
2.9767
```

```
0.345911
```

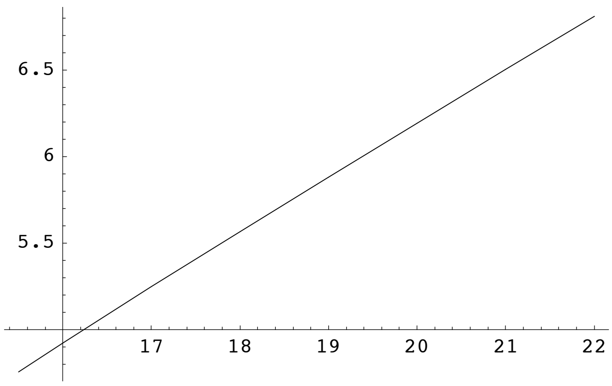
```
Tbl = Table[{k, 0.35 (k - 15.5) + 4.755}, {k, 16, 22}]
```

```
{{16, 4.93}, {17, 5.28}, {18, 5.63}, {19, 5.98}, {20, 6.33}, {21, 6.68}, {22, 7.03}}
```

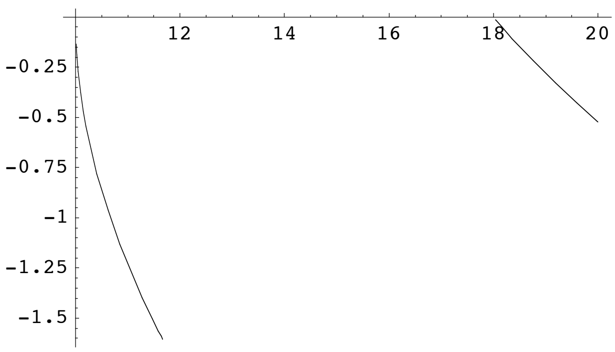
```
Table[{Tbl[[j]][[1]], Fcrit[Tbl[[j]][[1]], -15, 15, Tbl[[j]][[2]], -0.01]},
 {j, 1, Length[Tbl]}]
```

```
{{16, 3.1297}, {17, 3.42883}, {18, 3.72205},
 {19, 4.01204}, {20, 4.30057}, {21, 4.58881}, {22, 4.87753}}
```

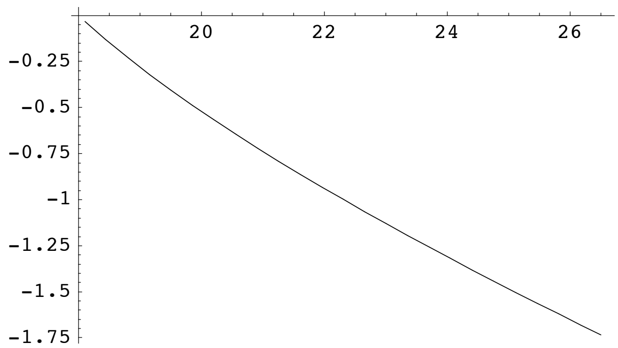
```
P5bis = ListPlot[
   {{15.5, 4.754375505447405`}, {16, 4.921460971832275`}, {17, 5.247627897262573`},
    {18, 5.566491928100586`}, {19, 5.880874819755556`}, {20, 6.192664613723758`},
    {21, 6.503131899833683`}, {22, 6.813128986358647`}}, PlotJoined → True];
```



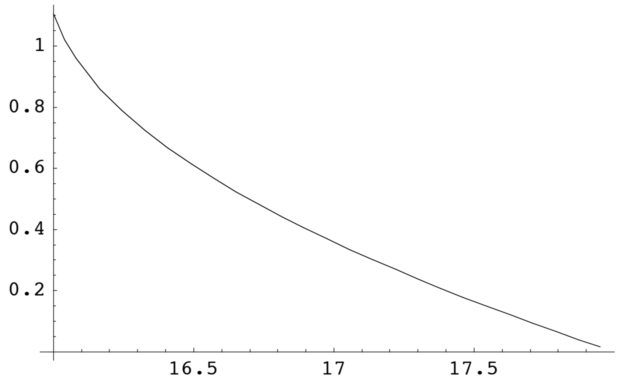

```
P6 = Plot[Fcrit[N, 0, 20, 1/2 Log[2 (N + 2)] - 0.01, -0.1], {N, 10, 20}];
```



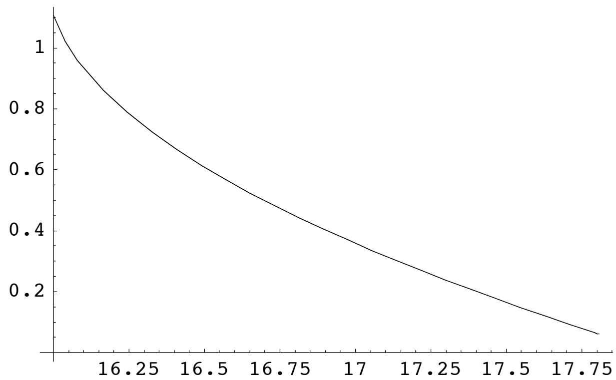
```
P7 = Plot[Fcrit[N, 0, 20, 1/2 Log[2 (N + 2)] - 0.01, -0.1], {N, 18.1, 26.5}];
```



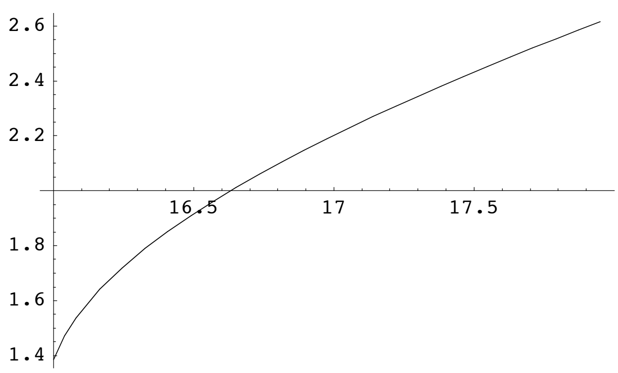

```
P8a = Plot[Fcrit[N, 0, 20, 1/2 Log[2 (N + 2)] + 0.01, +0.1], {N, 16, 17.95}];
```




```
P8b = Plot[Fcrit[N, 0, 20, 2.9, -0.1], {N, 16, 17.95}];
```




```
P8c = Plot[Fcrit[N, 0, 20, 2.9, 0.1], {N, 16, 17.95}];
```



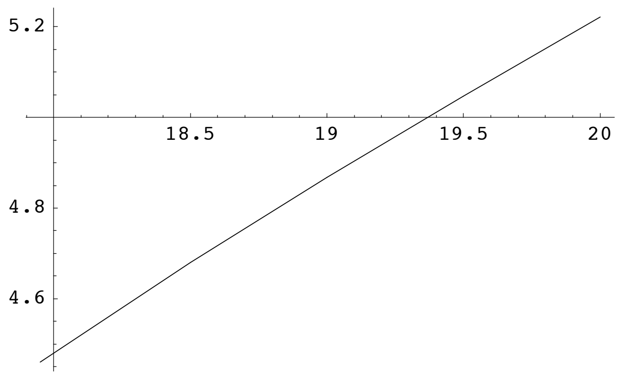
```
Tbl = Table[{k, 4.480217659473419 + 2 (k - 18) (4.480217659473419 - 4.264164304733277)},
  {k, 18, 20, 0.5}]
```

{{18, 4.48022}, {18.5, 4.69627}, {19., 4.91232}, {19.5, 5.12838}, {20., 5.34443}}

```
Table[{Tbl[[j]][[1]], Fcrit[Tbl[[j]][[1]], 2, 8, Tbl[[j]][[2]], -0.1]},
 {j, 1, Length[Tbl]}]
```

{{18, {}}, {18.5, 2.82268}, {19., 2.9986}, {19.5, 3.16681}, {20., 3.32942}}

```
P8d = ListPlot[{{17.95, 4.459494104385343`},
    {18.5`, 4.67946547269821`}, {19.`, 4.867435526847836`},
    {19.5`, 5.047410070896143`}, {20.`, 5.221516907215115`}}, PlotJoined → True];
```

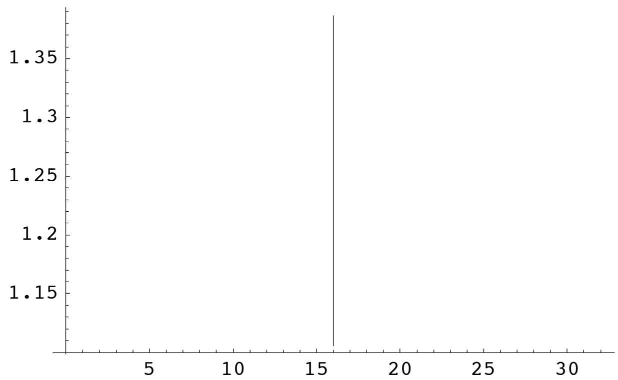



```
Fcrit[16, 0, 20, 2.9, 0.01]
Fcrit[16, 0, 20, 2.9, -0.01]
```

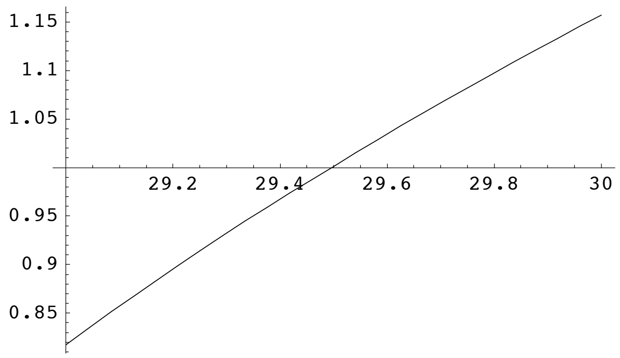
1.38633

1.10555

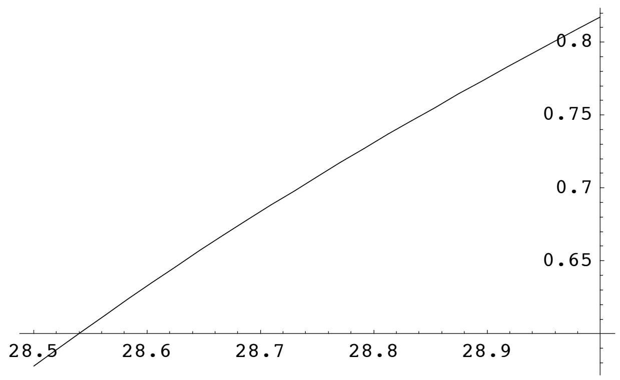
$$P8e = ListPlot\left[\left\{\left\{16, 2.8973097229003906 - \frac{1}{2} Log[2 (16 + 2)]\right\},\left\{16, 3.178091278076166 - \frac{1}{2} Log[2 (16 + 2)]\right\}\right\}, PlotJoined \to True\right];$$



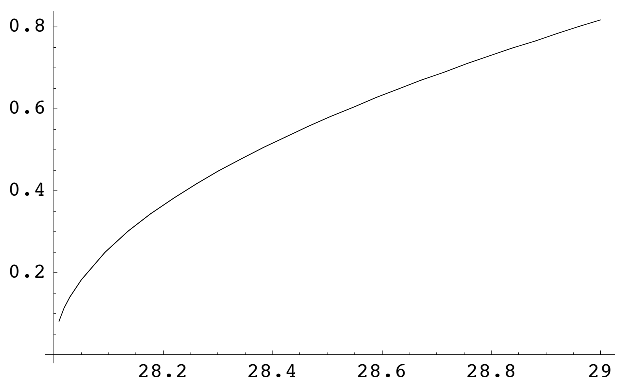
```
P9a = Plot[Fcrit[N, 0, 20, 2.5, 0.1], {N, 29, 30}];
```



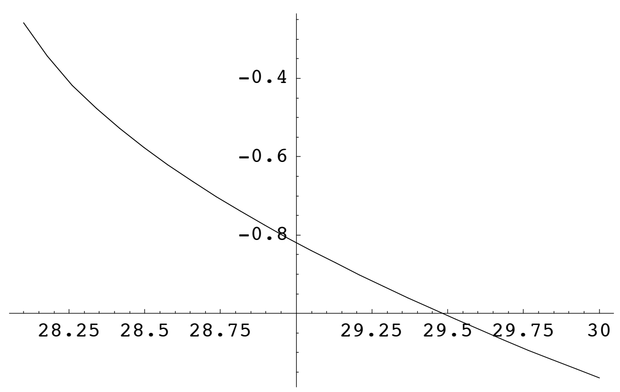

```
P9b = Plot[Fcrit[N, 0, 30, 2.4, 0.1], {N, 28.5, 29}];
```



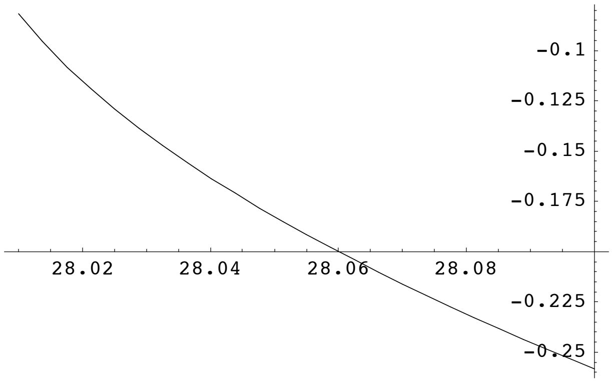
$$\text{P9c = Plot}\left[\text{Fcrit}\left[N, 0, 3, \frac{1}{2}\text{Log}[2\ (N+2)] + 0.001, 0.01\right], \{N, 28.01, 29\}\right];$$



$$\text{P10 = Plot}\left[\text{Fcrit}\left[N, 0, 20, \frac{1}{2}\text{Log}[2\ (N+2)] - 0.01, -0.1\right], \{N, 28.1, 30\}\right];$$



```
P10bis = Plot[Fcrit[N, 0, 20, 1/2 Log[2 (N + 2)] - 0.001, -0.01], {N, 28.01, 28.1}];
```



■ FIGURE 14bis:

```
Show[P2, P3, P4, P5, P6, P7, P8a, P8b, P8c, P8e,
  P9a, P9b, P9c, P10, P10bis, PlotRange → {{0, 30}, {-1.5, 2}}];
```