Sharp rates of decay of solutions to the nonlinear fast diffusion equation via functional inequalities

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Null
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αmin = -9;
size = 10;
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```
LowerAlpha[k_, l_, d_] :=
 α /. Solve[-2 α (l + 2 k) - 4 k (l + k + d/2 - 1) == ((d + 2 α - 2)/2)^2, α][[1]]
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```
LambdaDot[k_, l_, d_] := {α, -2 α (l + 2 k) - 4 k (l + k + d/2 - 1)} /. α -> LowerAlpha[k, l, d]
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```
LambdaCont[d_] := Plot[((d + 2 α - 2)/2)^2, {α, αmin, 0}, DisplayFunction → Identity];
```

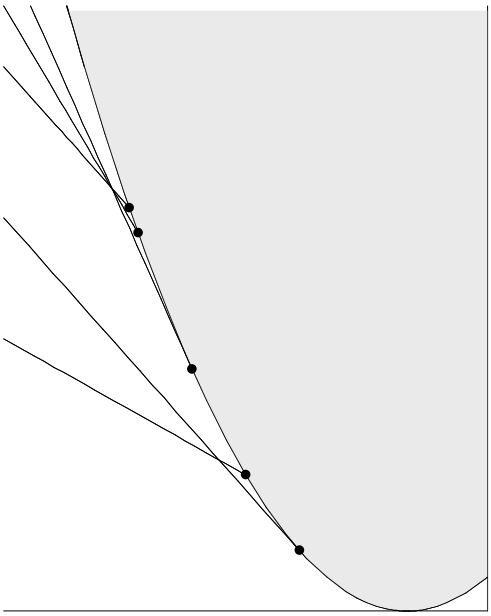
```
Lambda[k_, l_, d_] :=
 If[{k, l} != {0, 0} && Im[LowerAlpha[k, l, d]] == 0, Plot[-2 α (l + 2 k) - 4 k (l + k + d/2 - 1),
   {α, αmin, LowerAlpha[k, l, d]}, DisplayFunction → Identity], {}]
```

```
Figure[d_, DF_] :=
 Show[LambdaCont[d], Table[Lambda[k, l, d], {k, 0, size - 1}, {l, 0, size - 1}],
  PlotRange → {{αmin, 0}, {0, 40}}, AspectRatio → 1.25, DisplayFunction → DF]
```

```
AddPoints[d_, DF_] := ListPlot[
  {LambdaDot[0, 1, d], LambdaDot[1, 0, d], LambdaDot[1, 1, d], LambdaDot[2, 0, d],
   LambdaDot[0, 2, d]}, DisplayFunction → DF, PlotStyle → PointSize[0.02]]
```

```
Essential = Show[
   Graphics[{GrayLevel[0.9], Polygon[Join[Table[{-i, (-i + 1.5)^2}, {i, 0, 7.8, 0.1}],
       {{0, 39.69}}, {{0, 0}}]]}], DisplayFunction → Identity];
```

```
Show[Figure[5, Identity], Essential, AddPoints[5, Identity],
  Figure[5, Identity], DisplayFunction → $DisplayFunction, Ticks → None];
```



```
F0[d_, DF_] := Plot[(1 - m) (d/2 + 1/(m - 1) - 1)^2,
   {m, -5, 1}, PlotRange → {All, {0, 10}}, DisplayFunction → DF];
Essential = Show[Graphics[{GrayLevel[0.9],
     Polygon[Join[Table[{m, (1 - m) (1.5 + 1/(m - 1))^2}, {m, -5, 0.93, 0.01}],
       {{-0.2, 9.68}}]]}], DisplayFunction → Identity];
RectEnl = ListPlot[{{1/3, 0}, {1/3, 9.6} {1, 9.6}, {1, 0}},
   PlotJoined → True, DisplayFunction → Identity];
Show[F0[5, Identity], RectEnl, Essential, F0[5, Identity], RectEnl,
  DisplayFunction → $DisplayFunction, PlotRange → {All, {0, 9.68}}];
```

```
Solve[2 (l + 2 k) - 4 k (l + k + d/2 - 1) (1 - m) == (1 - m) (d/2 + 1/(m - 1) - 1)^2, m]
size = 4;
StartM[k_, l_, d_] := Max[(8 - 6 d + d^2 - 24 k + 8 d k + 16 k^2 - 4 l + 16 k l - 4 Sqrt[-2 l + d l + l^2]) /
    (4 - 4 d + d^2 - 16 k + 8 d k + 16 k^2 + 16 k l),
   (8 - 6 d + d^2 - 24 k + 8 d k + 16 k^2 - 4 l + 16 k l + 4 Sqrt[-2 l + d l + l^2]) /
    (4 - 4 d + d^2 - 16 k + 8 d k + 16 k^2 + 16 k l)]
StartPts[k_, l_, d_] := {{m, (1 - m) (d/2 + 1/(m - 1) - 1)^2}} /. m -> StartM[k, l, d]
TblPts[d_] := Table[ListPlot[StartPts[k, l, d], PlotStyle → PointSize[0.012],
    DisplayFunction → Identity], {k, 0, size - 1}, {l, 0, size - 1}];
F1[k_, l_, d_, DF_] := Module[{mcalcmin = StartM[k, l, d]},
  Plot[2 (l + 2 k) - 4 k (l + k + d/2 - 1) (1 - m), {m, mcalcmin, 1},
   PlotRange → {All, {0, 10}}, DisplayFunction → DF]]
Tbl[d_] := Table[F1[k, l, d, Identity], {k, 0, size - 1}, {l, 0, size - 1}];
Essential =
  Show[Graphics[{GrayLevel[0.9], Polygon[Join[Table[{m, (1 - m) (1.5 + 1/(m - 1))^2},
       {m, 1/3, 0.93, 0.01}], {{-0.2, 9.68}}]]}], DisplayFunction → Identity];
Show[F0[5, Identity], Tbl[5], Essential, TblPts[5], F0[5, Identity],
  DisplayFunction → $DisplayFunction, PlotRange → {{1/3, 1}, {0, 9.68}}];
```

$$\left\{\left\{m \to \frac{8 - 6\,d + d^2 - 24\,k + 8\,d\,k + 16\,k^2 - 4\,l + 16\,k\,l - 4\sqrt{-2\,l + d\,l + l^2}}{4 - 4\,d + d^2 - 16\,k + 8\,d\,k + 16\,k^2 + 16\,k\,l}\right\},\right.$$
$$\left.\left\{m \to \frac{8 - 6\,d + d^2 - 24\,k + 8\,d\,k + 16\,k^2 - 4\,l + 16\,k\,l + 4\sqrt{-2\,l + d\,l + l^2}}{4 - 4\,d + d^2 - 16\,k + 8\,d\,k + 16\,k^2 + 16\,k\,l}\right\}\right\}$$