Forward self-similar solutions

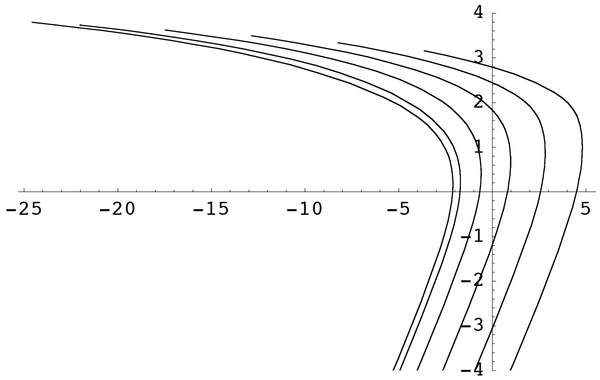
Plot des solutions et diagramme de bifurcation

```
Off[General::"spell1"]
```

```
ϵ = 10^-8;
rmax = 10;
smin = -10;
smax = 20;
```

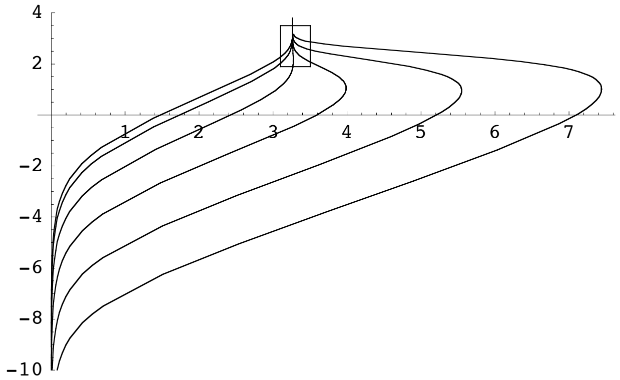
$$F[\tau\_, DS\_] := ParametricPlot\Big[\{w[rmax], Log[s - w[rmax]]\} \;/.\; NDSolve\Big[\Big\{w''[r] == -\frac{w'[r]}{r} - \frac{\tau}{2}\, r\, w'[r] - e^{w[r]-\frac{r^2}{4}},\ w[\epsilon] == s - \frac{\epsilon^2}{4} e^s,\ w'[\epsilon] == -\frac{\epsilon}{2} e^s\Big\}, \{w, w'\}, \{r, \epsilon, rmax\}\Big], \{s, smin, smax\}, DisplayFunction \to DS\Big]$$

$$Show[Table[F[10^\alpha, Identity], \{\alpha, -2, 3\}], DisplayFunction \text{ -> } \$DisplayFunction, PlotRange \to \{All, \{-4, 4\}\}];$$

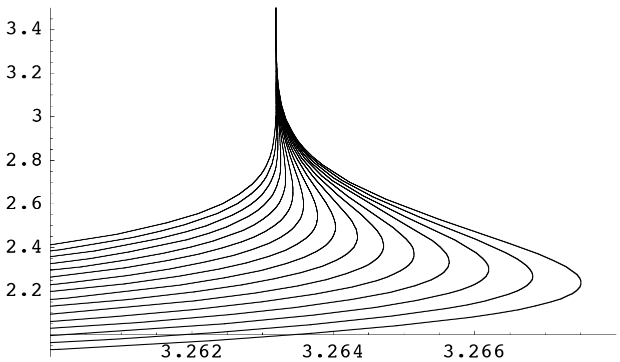


$$G[\tau\_, DS\_] := ParametricPlot\Big[\{Log[1 + m[rmax]], Log[s - w[rmax]]\} \;/.\; NDSolve\Big[\Big\{w''[r] == -\frac{w'[r]}{r} - \frac{\tau}{2}\, r\, w'[r] - e^{w[r]-\frac{r^2}{4}},\ w[\epsilon] == s - \frac{\epsilon^2}{4} e^s,\ w'[\epsilon] == -\frac{\epsilon}{2} e^s,\ m'[r] == 2\pi r\, e^{w[r]-\frac{r^2}{4}},\ m[\epsilon] == \frac{\epsilon^2}{2} e^s\Big\}, \{w, w', m\}, \{r, \epsilon, rmax\}\Big], \{s, smin, smax\}, DisplayFunction \to DS\Big]$$

```
Show[Table[G[10^α, Identity], {α, -2, 3}],
  ListPlot[{{3.1, 1.9}, {3.5, 1.9}, {3.5, 3.5}, {3.1, 3.5}, {3.1, 1.9}},
   PlotJoined → True, DisplayFunction → Identity],
  DisplayFunction -> $DisplayFunction, PlotRange → {All, {-10, 4}}];
```



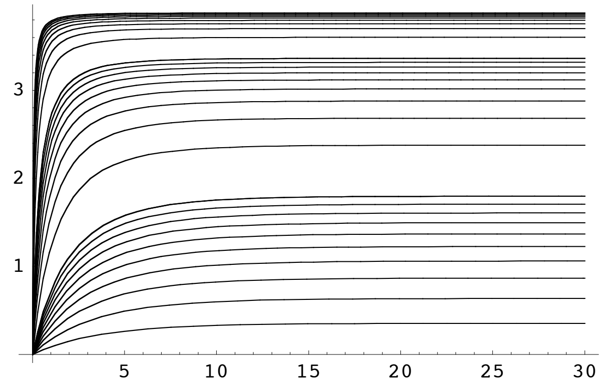

```
Show[Table[G[a, Identity], {a, 0.6, 0.9, 0.02}],
  DisplayFunction -> $DisplayFunction, PlotRange → {{3.26, 3.268}, {1.9, 3.5}}];
```

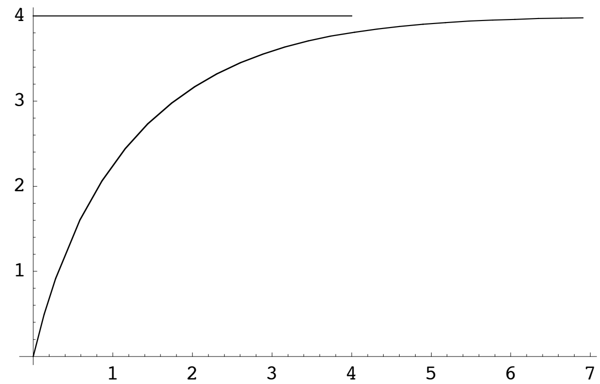


Cumulated densities

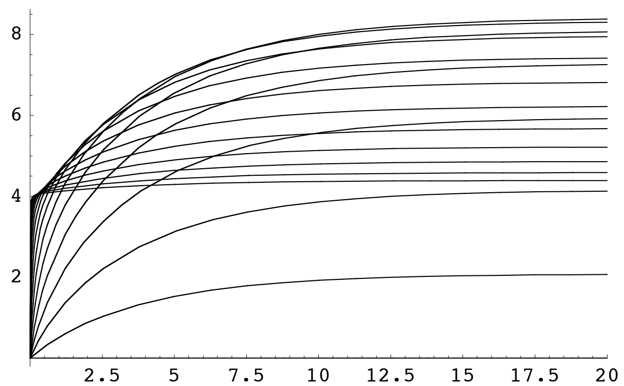
```
ϵ = 0.000001;

F[a_, DS_] := Plot[ϕ[y] /.
   NDSolve[{ψ'[x] + ψ[x] / 4 + ψ[x] S[x] / (2 x) == 0, ϕ'[x] == ψ[x], S'[x] + τ S[x] / 4 == ψ[x],
     ψ[ϵ] == a - a (1 + 2 a) ϵ / 4, ϕ[ϵ] == a ϵ - a (1 + 2 a) ϵ^2 / 8, S[ϵ] == a ϵ},
    {ϕ, ψ, S}, {x, ϵ, ymax}], {y, ϵ, ymax}, DisplayFunction → DS]

G[a_] := ϕ[ymax] /.
  NDSolve[{ψ'[x] + ψ[x] / 4 + ψ[x] S[x] / (2 x) == 0, ϕ'[x] == ψ[x], S'[x] + τ S[x] / 4 == ψ[x],
     ψ[ϵ] == a - a (1 + 2 a) ϵ / 4, ϕ[ϵ] == a ϵ - a (1 + 2 a) ϵ^2 / 8,
     S[ϵ] == a ϵ}, {ϕ, ψ, S}, {x, ϵ, ymax}][[1]]
```

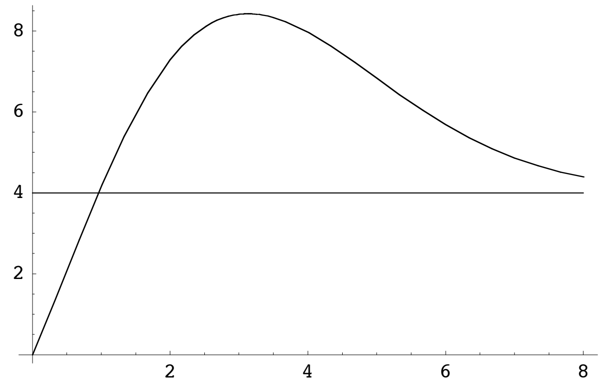
```
τ = 0.1;
ymax = 30;

Show[Table[F[a*0.1, Identity], {a, 1, 10}], Table[F[a, Identity], {a, 1, 10}],
  Table[F[a*10, Identity], {a, 1, 10}], DisplayFunction → $DisplayFunction];

Show[ListPlot[{{0, 4}, {4, 4}}, PlotJoined → True, DisplayFunction → Identity],
  Plot[G[e^a - 1], {a, 0, Log[1000]}, DisplayFunction → Identity],
  DisplayFunction → $DisplayFunction, PlotRange → All];
```





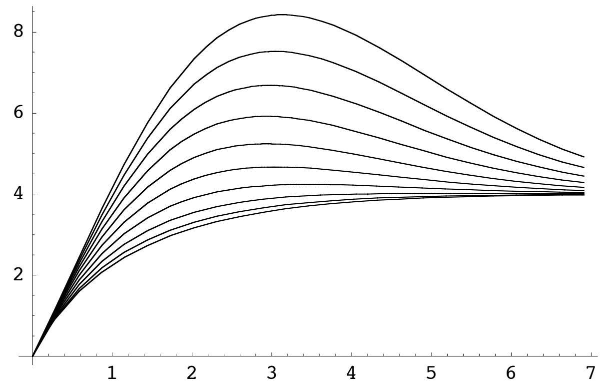

```
τ = 10;
ymax = 30;

Show[Table[F[e^a - 1, Identity], {a, 0, 8, 0.5}],
  DisplayFunction → $DisplayFunction, PlotRange → {{0, 20}, All}];

Show[ListPlot[{{0, 4}, {8, 4}}, PlotJoined → True, DisplayFunction → Identity],
  Plot[G[e^a - 1], {a, 0, Log[3000]}, DisplayFunction → Identity],
  DisplayFunction → $DisplayFunction, PlotRange → All];
```



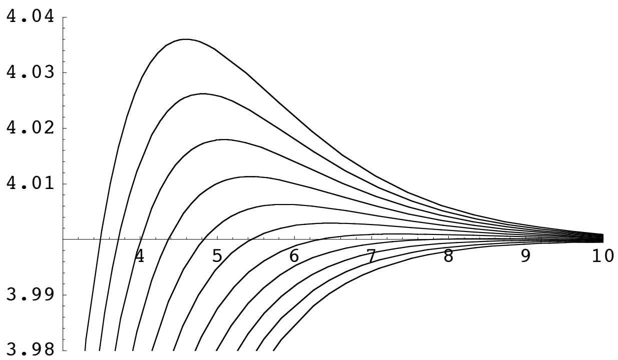


Masse comme fonction de a, pour différentes valeurs de τ

```
ymax = 30;

H[a_, t_] := ϕ[ymax] /.
  NDSolve[{ψ'[x] + ψ[x] / 4 + ψ[x] S[x] / (2 x) == 0, ϕ'[x] == ψ[x], S'[x] + t S[x] / 4 == ψ[x],
     ψ[ϵ] == a - a (1 + 2 a) ϵ / 4, ϕ[ϵ] == a ϵ - a (1 + 2 a) ϵ^2 / 8,
     S[ϵ] == a ϵ}, {ϕ, ψ, S}, {x, ϵ, ymax}][[1]]

Show[Table[Plot[H[ⅇ^a - 1, 0.1 s^2], {a, 0, Log[1000]}, DisplayFunction → Identity],
   {s, 1, 10}], DisplayFunction → $DisplayFunction];
```




```
Show[
  Table[Plot[H[ⅇ^a - 1, tt], {a, 0, 10}, DisplayFunction → Identity], {tt, 0.5, 1, 0.05}],
  DisplayFunction → $DisplayFunction, PlotRange → {{3, 10}, {3.98, 4.04}}];
```



Masse maximale

```
Off[General::"spell1"]
ϵ = 10^-6;
ymax = 40;
amax = 10;
HH[a_, t_] := H[e^a - 1, t]
η = 0.00000000001;
Minter[t_, Mt_, a_, δ_] := Module[{h = HH[a, t]}, If[Abs[h - Mt] < η && a < amax,
   {a, Mt}, If[h > Mt, Minter[t, h, a + δ, δ], Minter[t, h, a - δ / 2, -δ / 2]]]]
M[t_, δ_] := Minter[t, HH[δ, t], 2 δ, δ]

Plot[Max[{4, M[t, 1][[2]]}], {t, 0.5, 1}];
```

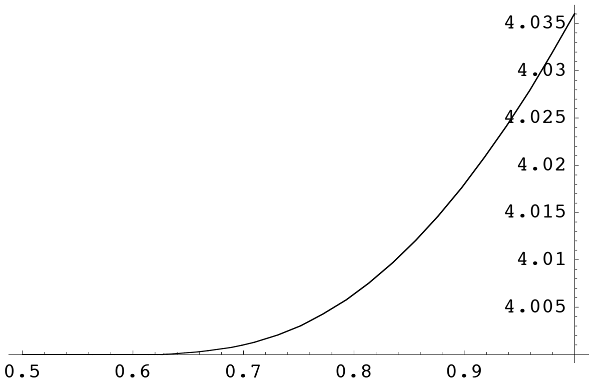


Figure 0

```
Show[ListPlot[{{4, 0}, {4, 0.5}}, PlotJoined → True, DisplayFunction → Identity],
  ParametricPlot[{Max[{4, M[t, 1][[2]]}], t}, {t, 0.5, 8}, DisplayFunction → Identity],
  DisplayFunction → $DisplayFunction, PlotRange → {{0, 8}, {0, 8}}];
```