Constants

$$S[d\_] := \frac{2\,\pi^{\frac{d}{2}}}{\text{Gamma}[\frac{d}{2}]}$$

$$\text{FullSimplify}\Big[S[d]\ \text{Integrate}\Big[(1+t^2)^{\frac{1}{m-1}}\,t^{d-1},\ \{t,0,\infty\},\ \text{Assumptions}\to \text{Re}[d]>0\ \&\&\ \text{Re}\Big[\frac{2+d\,(-1+m)}{2-2\,m}\Big]>-1\ \&\&\ \text{Re}\Big[\frac{1+d\,(-1+m)+m}{1-m}\Big]>-1\ \&\&\ \text{Re}\Big[\frac{d\,(-1+m)+2\,m}{-1+m}\Big]<2\ \&\&\ \text{Re}\Big[\frac{-3+d+m-d\,m}{-1+m}\Big]>1\ \&\&\ \text{Re}\Big[\frac{-5+d+3\,m-d\,m}{-1+m}\Big]>1\ \&\&\ \text{Re}\Big[\frac{8-d-6\,m+d\,m}{2-2\,m}\Big]>1\ \&\&\ \text{Re}\Big[\frac{6-d-4\,m+d\,m}{2-2\,m}\Big]>1\ \&\&\ \text{Re}\Big[\frac{4-d-2\,m+d\,m}{2-2\,m}\Big]>1\Big]\Big]$$

$$\frac{\pi^{d/2}\,\text{Gamma}[-\frac{d}{2}+\frac{1}{1-m}]}{\text{Gamma}[\frac{1}{1-m}]}$$

$$\text{FullSimplify}\Big[S[d]\ \text{Integrate}\Big[(1+t^2)^{\frac{1}{m-1}}\,t^{d+1},\ \{t,0,\infty\},\ \text{Assumptions}\to m<1\ \&\&\ \text{Re}[d]>0\ \&\&\ \frac{\text{Re}[d]}{2+\text{Re}[d]}<m\Big]\Big]$$

$$\frac{d\,\pi^{d/2}\,\text{Gamma}[-\frac{d}{2}-\frac{m}{-1+m}]}{2\,\text{Gamma}[\frac{1}{1-m}]}$$

$$\text{RangeEig}[l\_, k\_] := \text{Solve}\Big[2\,(l+2\,k)-4\,k\left(l+k+\frac{d}{2}-1\right)(1-m) == (1-m)\left(\frac{d}{2}+\frac{1}{m-1}-1\right)^2,\ m\Big]$$

```
RangeEig[l, k]
m /. RangeEig[1, 0][[2]]
m /. RangeEig[0, 2][[2]]
m /. RangeEig[2, 0][[2]]
```

$$\Big\{\Big\{m\to\frac{8-6\,d+d^2-24\,k+8\,d\,k+16\,k^2-4\,l+16\,k\,l-4\,\sqrt{-2\,l+d\,l+l^2}}{4-4\,d+d^2-16\,k+8\,d\,k+16\,k^2+16\,k\,l}\Big\},\ \Big\{m\to\frac{8-6\,d+d^2-24\,k+8\,d\,k+16\,k^2-4\,l+16\,k\,l+4\,\sqrt{-2\,l+d\,l+l^2}}{4-4\,d+d^2-16\,k+8\,d\,k+16\,k^2+16\,k\,l}\Big\}\Big\}$$

$$\frac{4+4\,\sqrt{-1+d}-6\,d+d^2}{4-4\,d+d^2}$$

$$\frac{4+d}{6+d}$$

$$\frac{4\,\sqrt{2}\,\sqrt{d}-6\,d+d^2}{4-4\,d+d^2}$$

Figure 1

9.5^2

90.25

```
αmin = -12;
size = 12;
hmax = 65;
iindexmax = 10;
```

```
LowerAlpha[k_, l_, d_] :=
 α /. Solve[-2 α (l + 2 k) - 4 k (l + k + d/2 - 1) == ((d + 2 α - 2)/2)^2, α][[1]]

LambdaDot[k_, l_, d_] := {α, -2 α (l + 2 k) - 4 k (l + k + d/2 - 1)} /. α -> LowerAlpha[k, l, d]

LambdaCont[d_] := Plot[((d + 2 α - 2)/2)^2, {α, αmin, 0}, DisplayFunction → Identity];

Lambda[k_, l_, d_] :=
 If[{k, l} != {0, 0} && Im[LowerAlpha[k, l, d]] == 0, Plot[-2 α (l + 2 k) - 4 k (l + k + d/2 - 1),
   {α, αmin, LowerAlpha[k, l, d]}, DisplayFunction → Identity], {}]

Figure[d_, DF_] :=
 Show[LambdaCont[d], Table[Lambda[k, l, d], {k, 0, size - 1}, {l, 0, size - 1}],
  PlotRange → {{αmin, 0}, {0, hmax}}, AspectRatio → 2, DisplayFunction → DF]

AddPoints[d_, DF_] := ListPlot[
  {LambdaDot[0, 1, d], LambdaDot[1, 0, d], LambdaDot[1, 1, d], LambdaDot[2, 0, d],
   LambdaDot[0, 2, d], LambdaDot[0, 3, d], LambdaDot[1, 2, d], LambdaDot[2, 1, d],
   LambdaDot[3, 0, d]}, DisplayFunction → DF, PlotStyle → PointSize[0.02]]

Essential = Show[Graphics[
    {GrayLevel[0.9], Polygon[Join[Table[{-i, (-i + 1.5)^2}, {i, 0, iindexmax, 0.1}],
       {{0, (iindexmax - 1.5)^2}}, {{0, 0}}]]}], DisplayFunction → Identity];
```

```
Show[Figure[5, Identity], Essential, AddPoints[5, Identity],
  Figure[5, Identity], DisplayFunction → $DisplayFunction, Ticks → None];
```

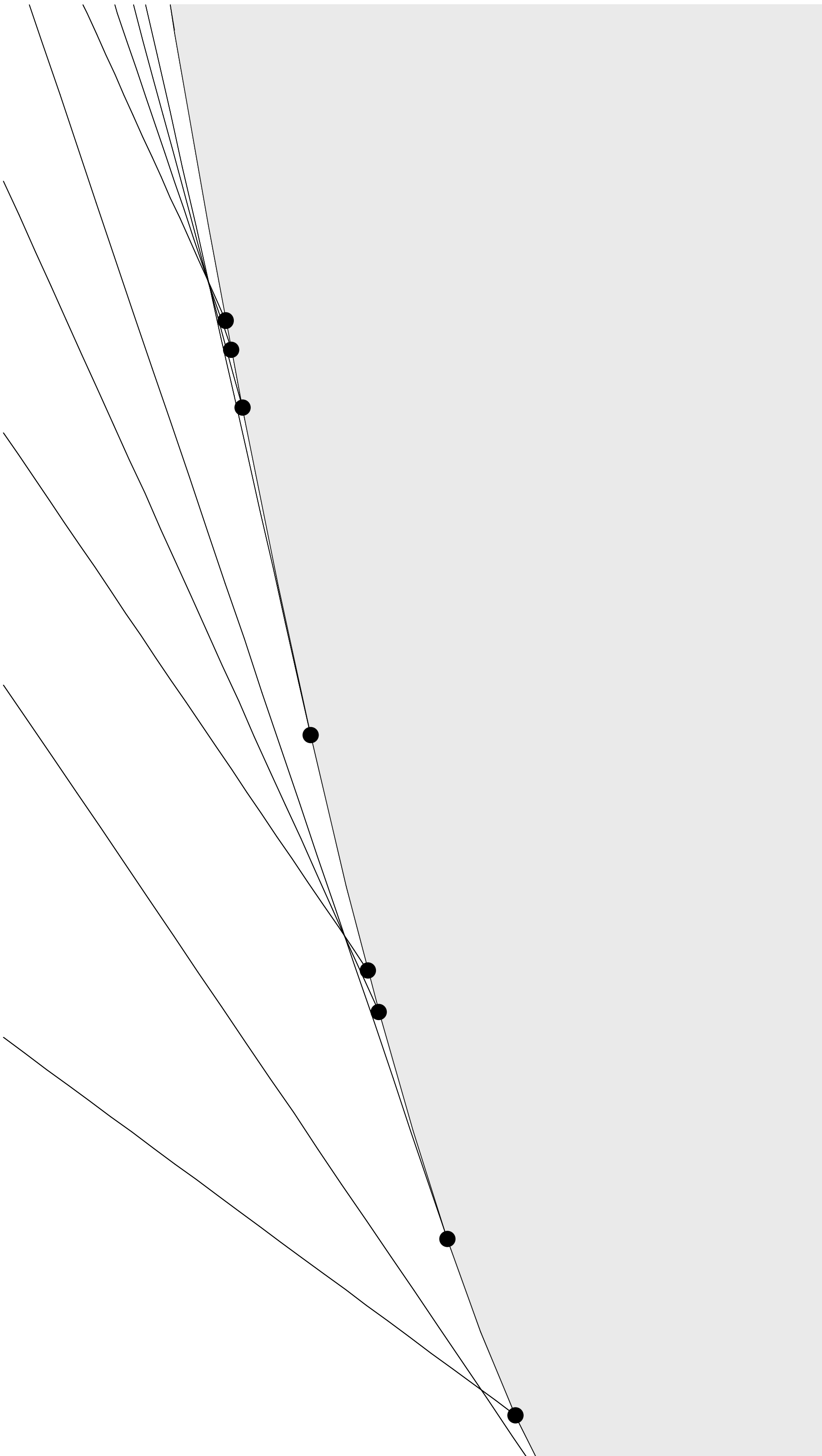


Figure 2

```
size = 7;
StartM[k_, l_, d_] := Max[(8 - 6 d + d^2 - 24 k + 8 d k + 16 k^2 - 4 l + 16 k l - 4 Sqrt[-2 l + d l + l^2]) /
    (4 - 4 d + d^2 - 16 k + 8 d k + 16 k^2 + 16 k l),
   (8 - 6 d + d^2 - 24 k + 8 d k + 16 k^2 - 4 l + 16 k l + 4 Sqrt[-2 l + d l + l^2]) /
    (4 - 4 d + d^2 - 16 k + 8 d k + 16 k^2 + 16 k l)]
StartPts[k_, l_, d_] := {{m, (1 - m) (d/2 + 1/(m - 1) - 1)^2} /. m -> StartM[k, l, d]}


F0[d_, DF_] := Plot[(1 - m) (d/2 + 1/(m - 1) - 1)^2, {m, -5, 1},
   PlotRange → {All, {0, 10}}, DisplayFunction → DF, PlotPoints → 400];
Essential = Show[Graphics[{GrayLevel[0.9],
      Polygon[Join[Table[{m, (1 - m) (1.5 + 1/(m - 1))^2}, {m, -5, 0.93, 0.01}],
        {{-0.2, 9.68}}]]}], DisplayFunction → Identity];
RectEnl = ListPlot[{{1/3, 0}, {1/3, 9.6} {1, 9.6}, {1, 0}},
   PlotJoined → True, DisplayFunction → Identity];
TblPts[d_] := Table[ListPlot[StartPts[k, l, d], PlotStyle → PointSize[0.02],
    DisplayFunction → Identity], {k, 0, size - 1}, {l, 0, size - 1}];
F1[k_, l_, d_, DF_] := Module[{mcalcmin = StartM[k, l, d]},
  Plot[2 (1 + 2 k) - 4 k (1 + k + d/2 - 1) (1 - m), {m, mcalcmin, 1},
   PlotRange → {All, {0, 10}}, DisplayFunction → DF]]
Tbl[d_] := Table[F1[k, l, d, Identity], {k, 0, size - 1}, {l, 0, size - 1}];
Essential =
  Show[Graphics[{GrayLevel[0.9], Polygon[Join[Table[{m, (1 - m) (1.5 + 1/(m - 1))^2},
         {m, 1/3, 0.93, 0.01}], {{-0.2, 9.68}}]]}], DisplayFunction → Identity];
VAxe = ListPlot[{{0, 0}, {1, 0}, {1, 10}}, PlotJoined → True,
   DisplayFunction → Identity];
Show[F0[5, Identity], Tbl[5], Essential, TblPts[5], F0[5, Identity], VAxe,
  DisplayFunction → $DisplayFunction, PlotRange → {{3/5, 1.02}, {0, 6.9}},
  AspectRatio → 2, Ticks → None, Axes → True];
```

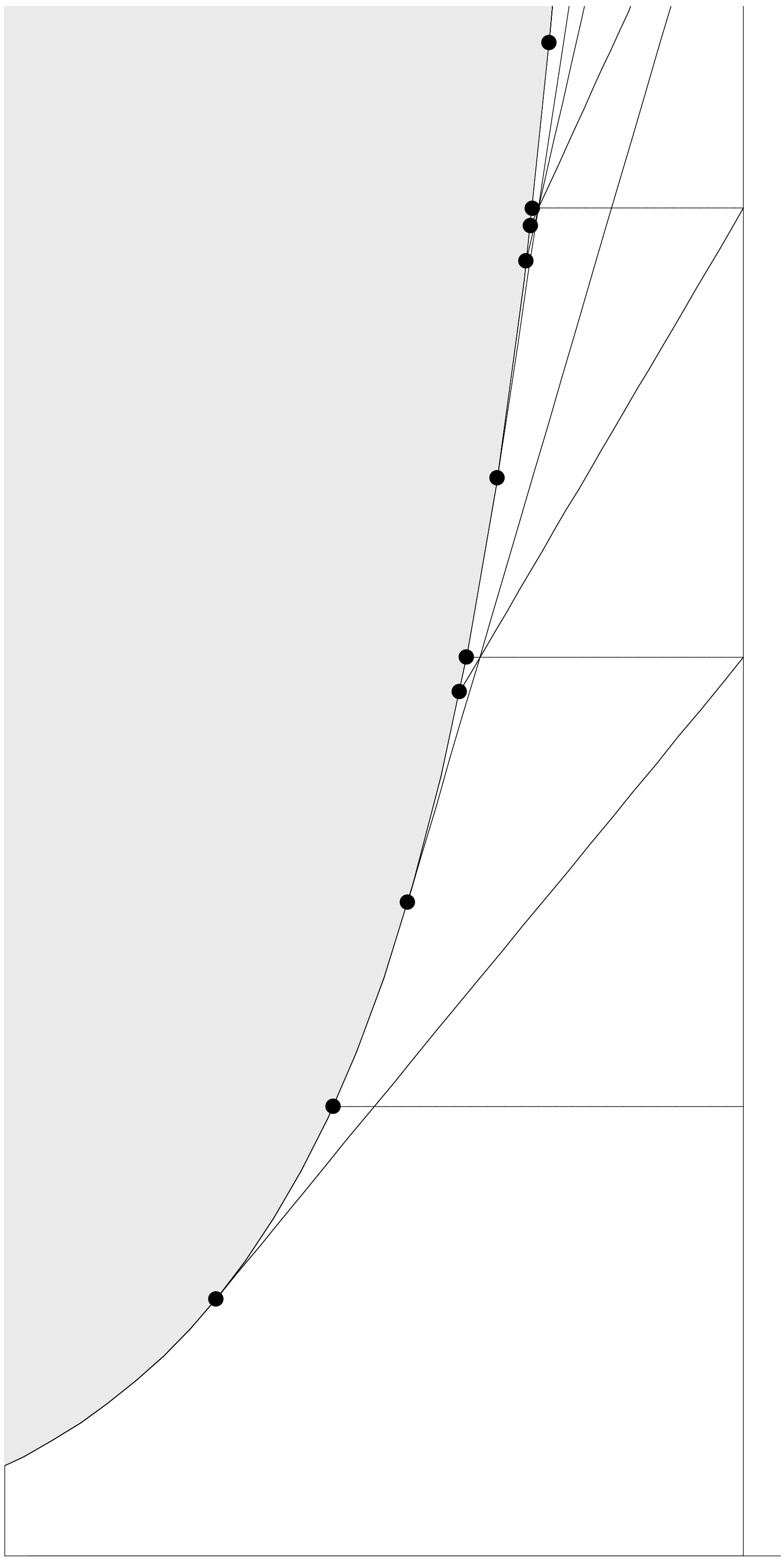


Figure 3

```
size = 5; eps = 0.02; Thk = 4;
GreenCurve[d_] :=
 Show[Plot[(1 - m) (d/2 + 1/(m - 1) - 1)^2 - 2 eps, {m, (d - 2)/d, d/(d + 2)}, DisplayFunction → Identity,
   PlotStyle → {RGBColor[0, 1, 0], AbsoluteThickness[Thk]}],
  ListPlot[{{0.88, 0.7}, {0.91, 0.7}}, PlotJoined → True, DisplayFunction → Identity,
   PlotStyle → {RGBColor[0, 1, 0], AbsoluteThickness[Thk]}],
  ListPlot[{{d/(d + 2), Evaluate[(1 - m) (d/2 + 1/(m - 1) - 1)^2 - 2 eps /. m -> d/(d + 2)]},
    {(d - 1 + eps)/d, 2}, {1, 2}}, PlotJoined → True, DisplayFunction → Identity,
   PlotStyle → {RGBColor[0, 1, 0], AbsoluteThickness[Thk]}], PlotRange → All]
GreenCurve[5]; BlueCurve[d_] := Show[Plot[(1 - m) (d/2 + 1/(m - 1) - 1)^2 - eps,
   {m, (d - 2)/d, d/(d + 2)}, DisplayFunction → Identity,
   PlotStyle → {RGBColor[0, 0, 1], AbsoluteThickness[Thk]}],
  ListPlot[{{d/(d + 2), Evaluate[(1 - m) (d/2 + 1/(m - 1) - 1)^2 - eps /. m -> d/(d + 2)]}, {1, 4 - eps}},
   PlotJoined → True, DisplayFunction → Identity,
   PlotStyle → {RGBColor[0, 0, 1], AbsoluteThickness[Thk]}],
  ListPlot[{{0.88, 1}, {0.91, 1}}, PlotJoined → True, DisplayFunction → Identity,
   PlotStyle → {RGBColor[0, 0, 1], AbsoluteThickness[Thk]}], PlotRange → All]
BlueCurve[5];
RedCurve[d_] :=
 Show[Plot[(1 - m) (d/2 + 1/(m - 1) - 1)^2, {m, (d - 2)/d, (d + 4)/(d + 6)}, DisplayFunction → Identity,
   PlotStyle → {RGBColor[1, 0, 0], AbsoluteThickness[Thk]}],
  ListPlot[{{(d + 4)/(d + 6), Evaluate[(1 - m) (d/2 + 1/(m - 1) - 1)^2 /. m -> (d + 4)/(d + 6)]}, {(d + 1)/(d + 2), 4}, {1, 4}},
   PlotJoined → True, DisplayFunction → Identity,
   PlotStyle → {RGBColor[1, 0, 0], AbsoluteThickness[Thk]}],
  ListPlot[{{0.88, 1.3}, {0.91, 1.3}}, PlotJoined → True, DisplayFunction → Identity,
   PlotStyle → {RGBColor[1, 0, 0], AbsoluteThickness[Thk]}], PlotRange → All]
RedCurve[5];
Show[F0[5, Identity], Tbl[5], Essential, TblPts[5], F0[5, Identity], VAxe,
  GreenCurve[5], BlueCurve[5], RedCurve[5], DisplayFunction → $DisplayFunction,
  PlotRange → {{3/5, 1.02}, {0, 4.7}}, AspectRatio → 0.7, Ticks → None, Axes → True];
```

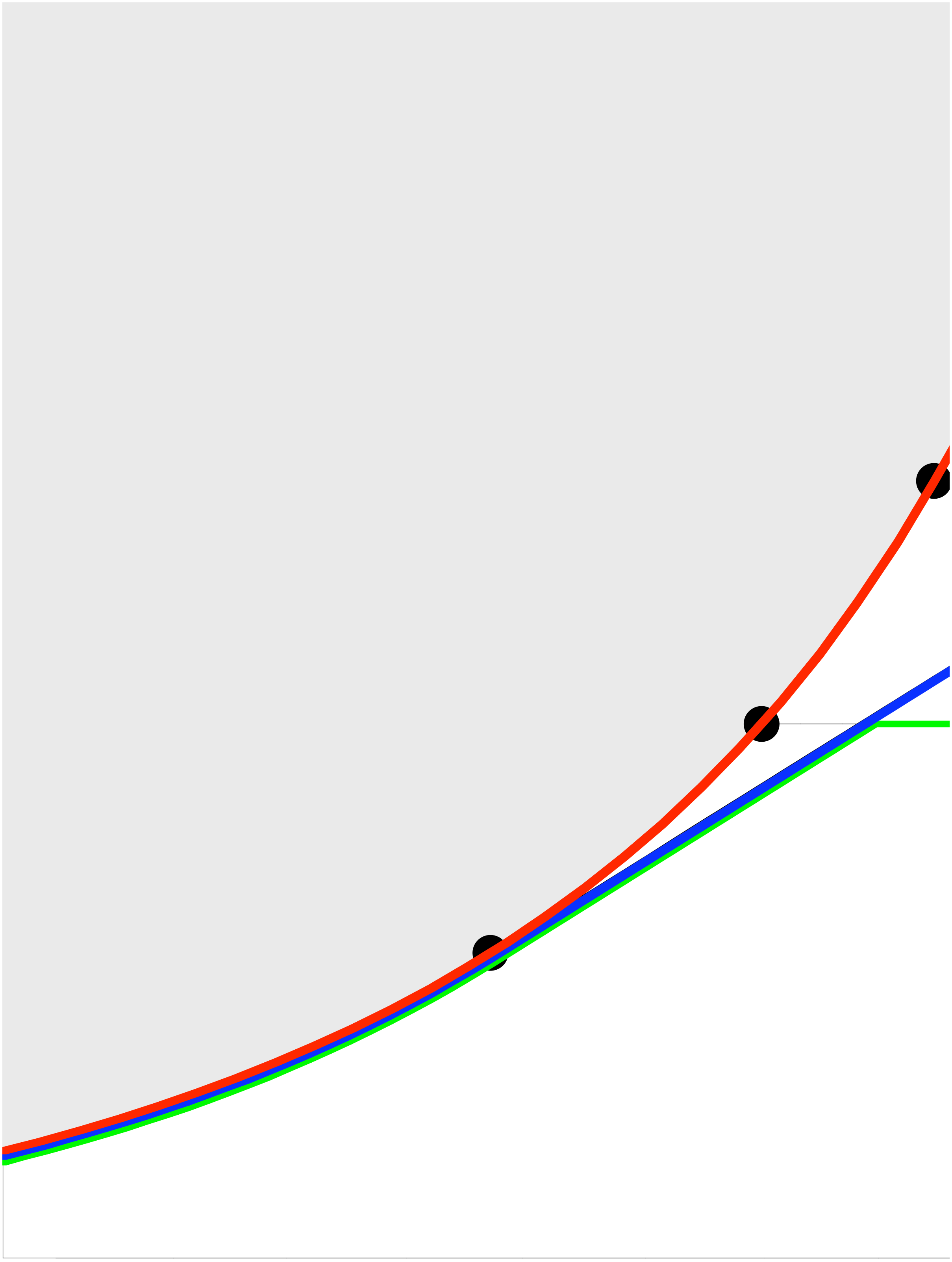
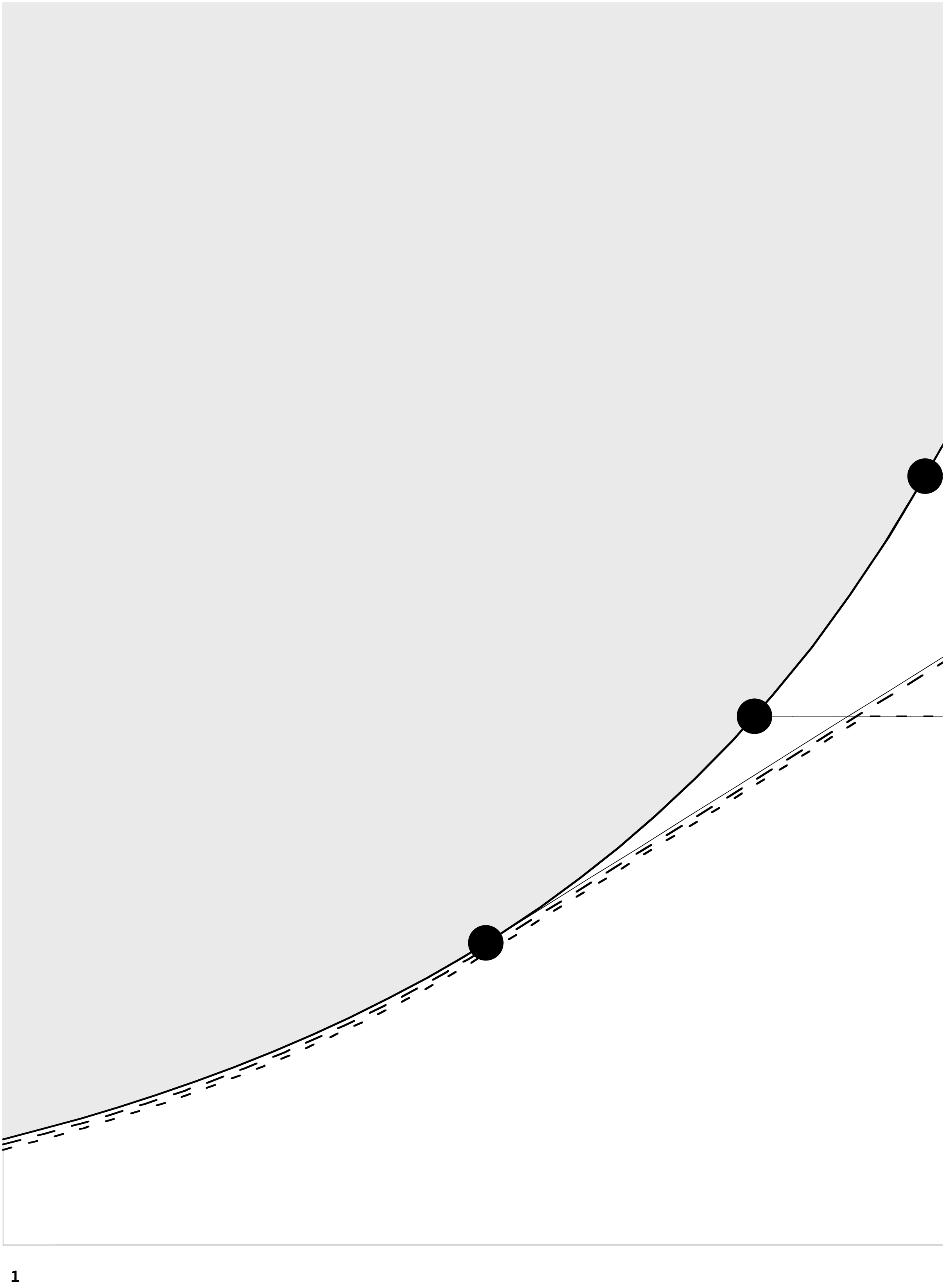


Figure 3 - black and white

```
size = 5; eps = 0.02; Thk = 1; D1 = {0.005, 0.01}; D2 = {0.01, 0.01};
GreenCurve[d_] := Show[Plot[(1 - m) (d/2 + 1/(m - 1) - 1)^2 - 2 eps, {m, (d - 2)/d, d/(d + 2)},
    DisplayFunction → Identity, PlotStyle → {Dashing[D1], AbsoluteThickness[Thk]}],
   ListPlot[{{0.88, 0.7}, {0.91, 0.7}}, PlotJoined → True, DisplayFunction → Identity,
    PlotStyle → {Dashing[D1], AbsoluteThickness[Thk]}],
   ListPlot[{{d/(d + 2), Evaluate[(1 - m) (d/2 + 1/(m - 1) - 1)^2 - 2 eps /. m -> d/(d + 2)]},
     {(d - 1 + eps)/d, 2}, {1, 2}}, PlotJoined → True, DisplayFunction → Identity,
    PlotStyle → {Dashing[D1], AbsoluteThickness[Thk]}], PlotRange → All]
GreenCurve[5]; BlueCurve[d_] := Show[Plot[(1 - m) (d/2 + 1/(m - 1) - 1)^2 - eps,
    {m, (d - 2)/d, d/(d + 2)}, DisplayFunction → Identity,
    PlotStyle → {Dashing[D2], AbsoluteThickness[Thk]}],
   ListPlot[{{d/(d + 2), Evaluate[(1 - m) (d/2 + 1/(m - 1) - 1)^2 - eps /. m -> d/(d + 2)]}, {1, 4 - eps}},
    PlotJoined → True, DisplayFunction → Identity,
    PlotStyle → {Dashing[D2], AbsoluteThickness[Thk]}],
   ListPlot[{{0.88, 1}, {0.91, 1}}, PlotJoined → True, DisplayFunction → Identity,
    PlotStyle → {Dashing[D2], AbsoluteThickness[Thk]}], PlotRange → All]
BlueCurve[5];
RedCurve[d_] := Show[Plot[(1 - m) (d/2 + 1/(m - 1) - 1)^2, {m, (d - 2)/d, (d + 4)/(d + 6)},
    DisplayFunction → Identity, PlotStyle → {AbsoluteThickness[Thk]}],
   ListPlot[{{(d + 4)/(d + 6), Evaluate[(1 - m) (d/2 + 1/(m - 1) - 1)^2 /. m -> (d + 4)/(d + 6)]}, {(d + 1)/(d + 2), 4}, {1, 4}},
    PlotJoined → True, DisplayFunction → Identity,
    PlotStyle → {AbsoluteThickness[Thk]}],
   ListPlot[{{0.88, 1.3}, {0.91, 1.3}}, PlotJoined → True, DisplayFunction → Identity,
    PlotStyle → {AbsoluteThickness[Thk]}], PlotRange → All]
RedCurve[5];
Show[F0[5, Identity], Tbl[5], Essential, TblPts[5], F0[5, Identity], VAxe,
  GreenCurve[5], BlueCurve[5], RedCurve[5], DisplayFunction → $DisplayFunction,
  PlotRange → {{3/5, 1.02}, {0, 4.7}}, AspectRatio → 0.7, Ticks → None, Axes → True];
```


1